Cryptography in the Age of Quantum Computers

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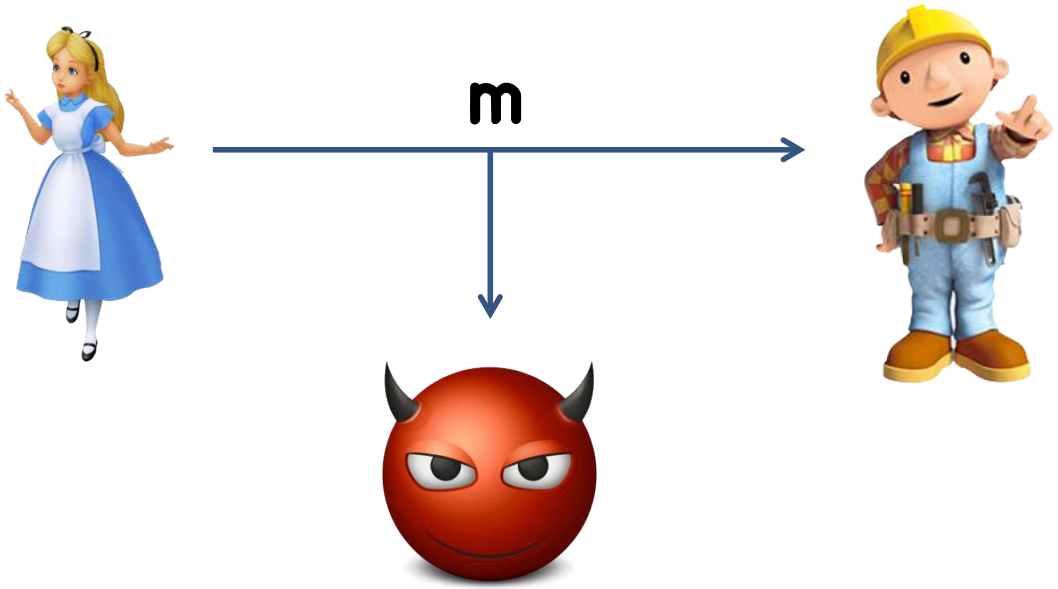
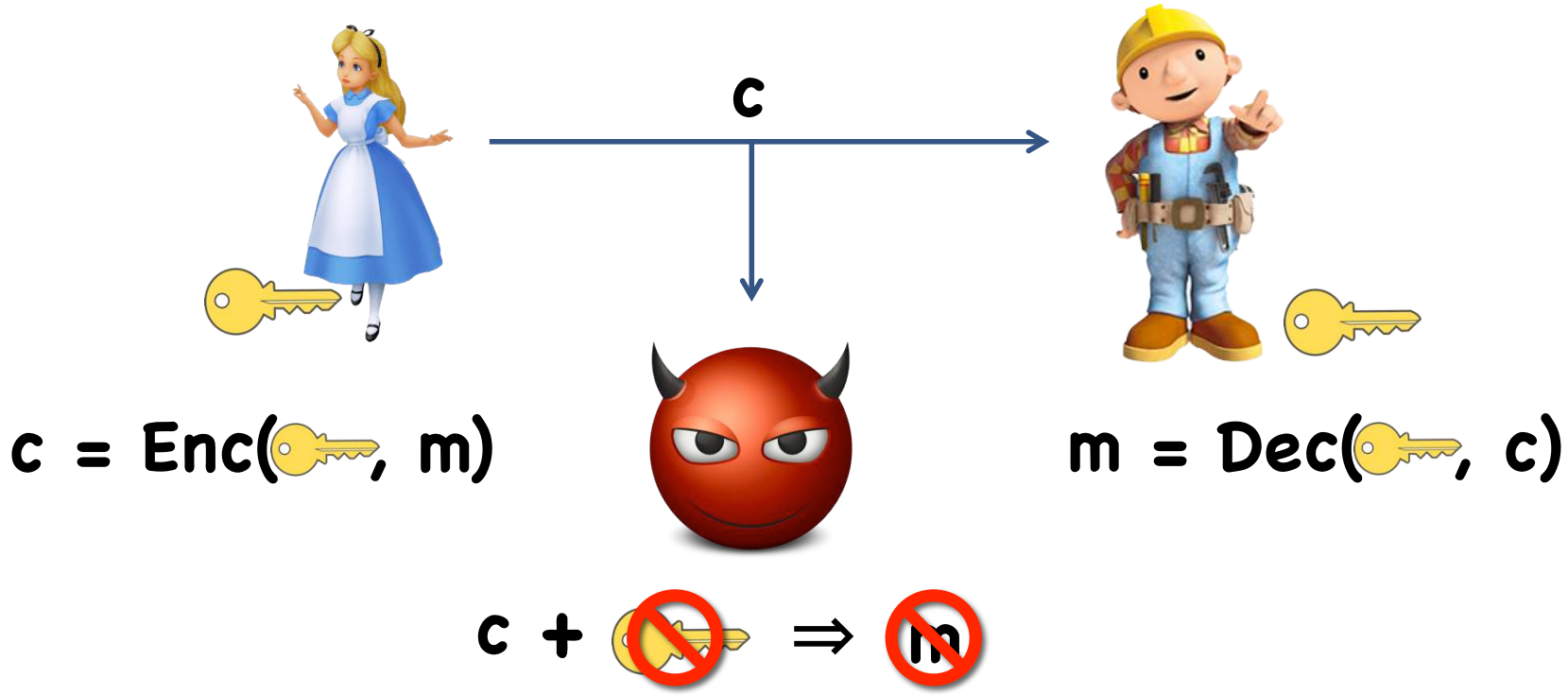
Based on joint works with:
Dan Boneh, Özgür Dagdelen, Marc Fischlin, Anja Lehmann, Christian Schaffner

Typical Crypto Application



Solution: (Private Key) Encryption



Major question: How is security defined?

Definition 1: 1-time security

For any m_0, m_1:

$$c_0 = \mathrm{Enc}(k, m_0) \approx c_1 = \mathrm{Enc}(k, m_1)$$

Statistical security: statistical closeness

- [Sha'49]: $|k| \geq |m|$

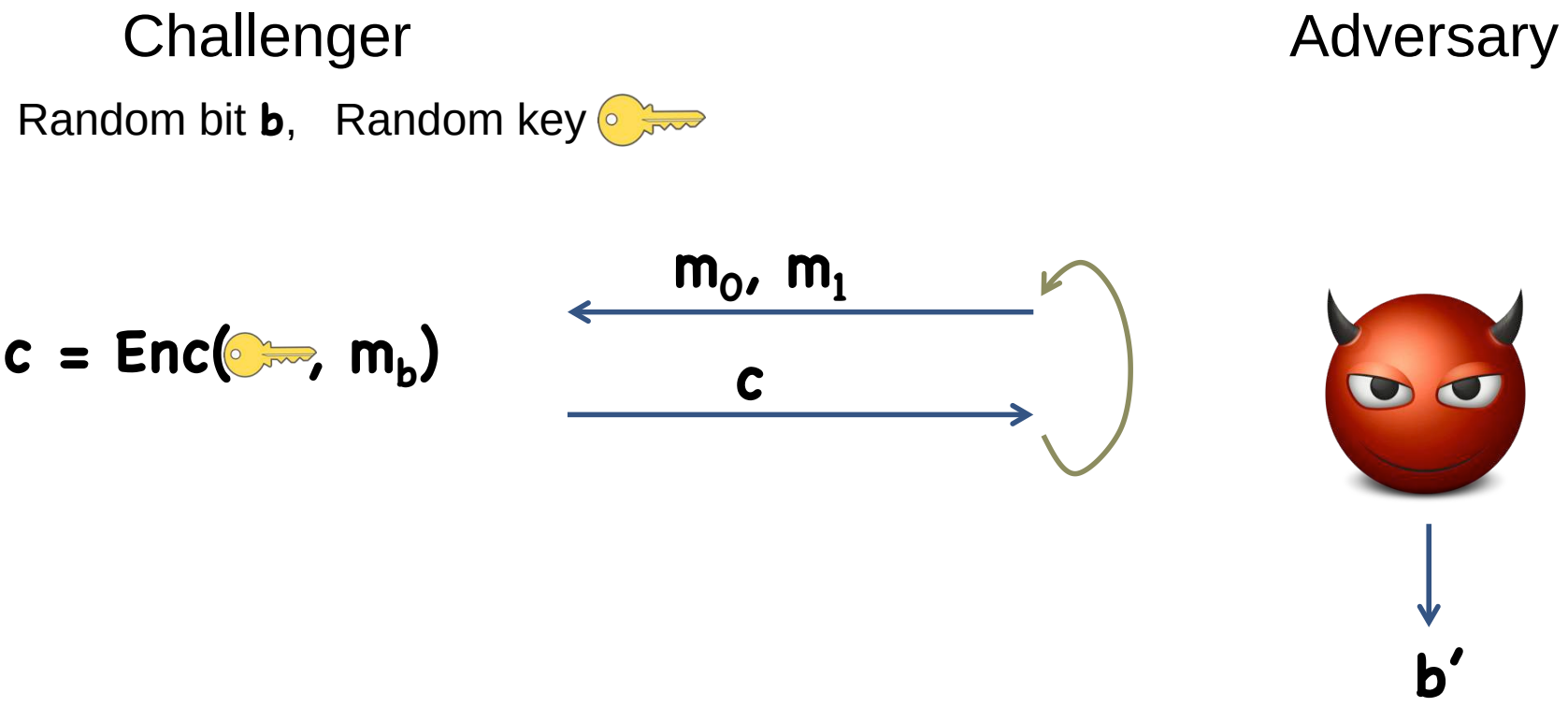
Computational security: computational indistinguishability

- Restrict adversaries running efficiently
- Now possible to have $|k| << |m|$

Question: what if I encrypt a second message?

Definition 2: CPA Security

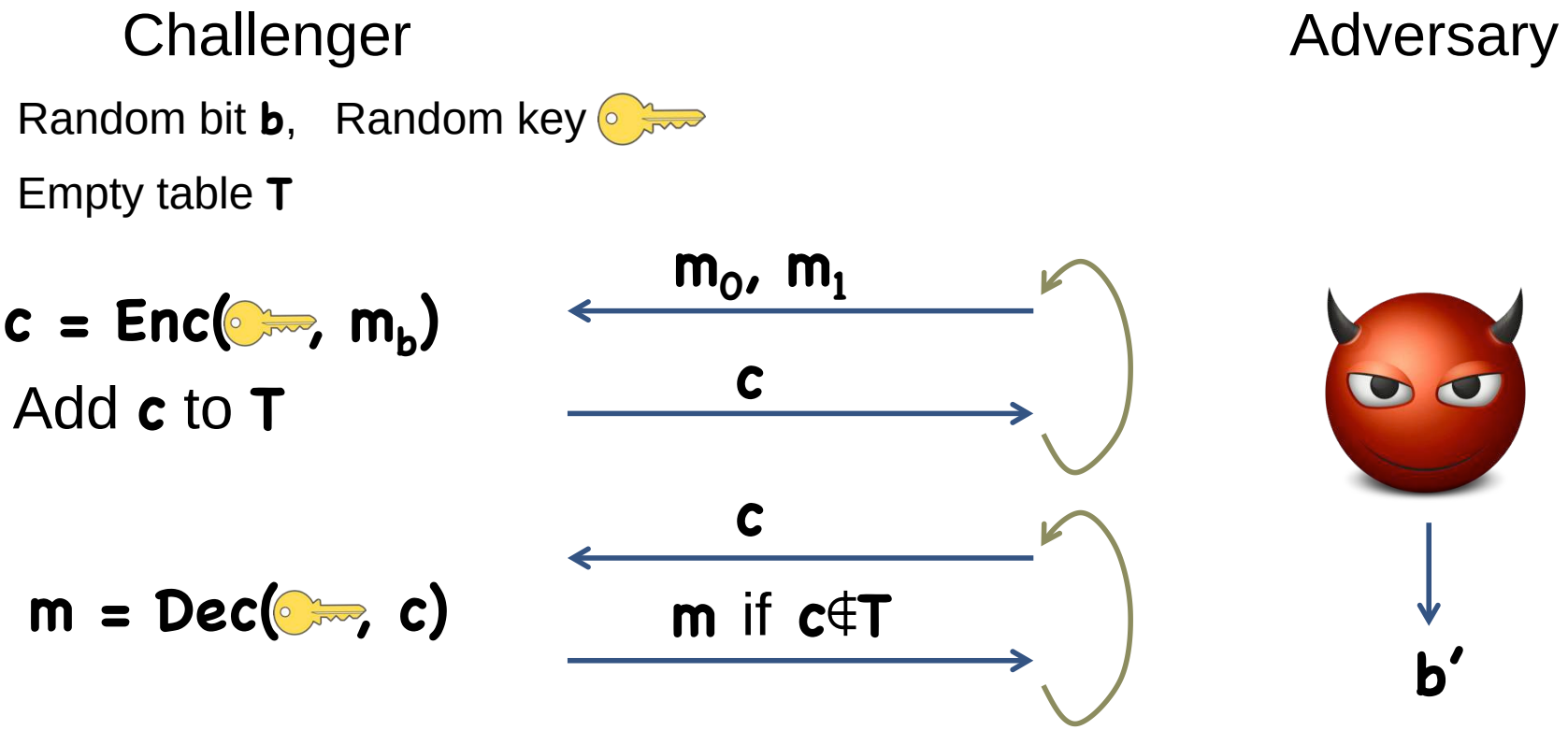
Indistinguishability under *chosen plaintext attack*



Def: CPA-Security ⇒ ∀ efficient , | Pr[b′=b] − ½ | < negl

Definition 3: CCA Security

Indistinguishability under *chosen ciphertext attack*



Def: CCA-Security ⇒ ∀ efficient , | Pr[b'=b] − ½ | < negl

Other Scenarios

Circular security:

Side-channel attacks:

Takeaway:
Models should give adversary as much power as possible

Quantum Computers

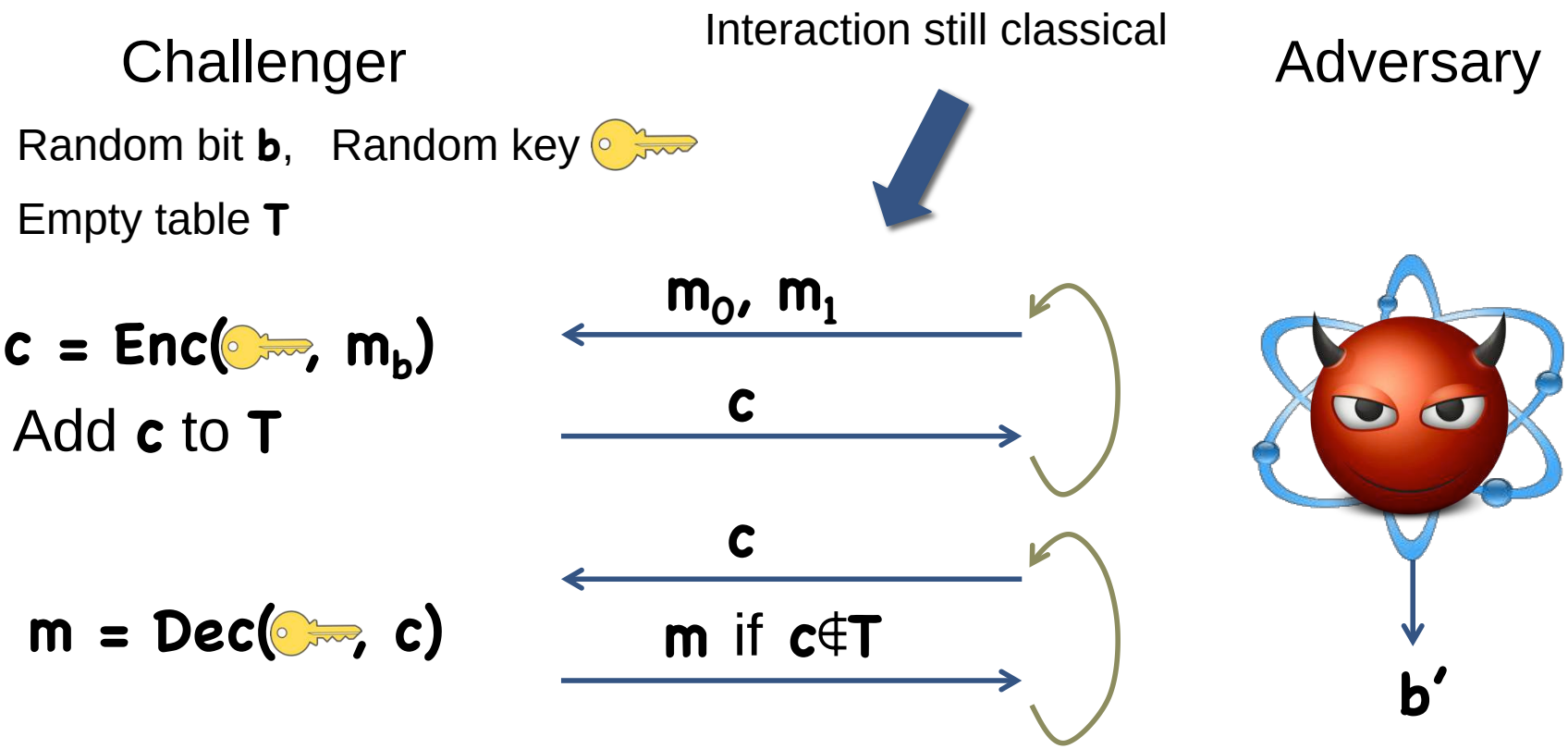
So far, assumed adversary obeys classical physics

What about quantum physics?

Quantum computing = using quantum physics to perform certain computations

- Active research area
- [Sho’94]: quantum computers can break lots of crypto

Post-Quantum CCA Security



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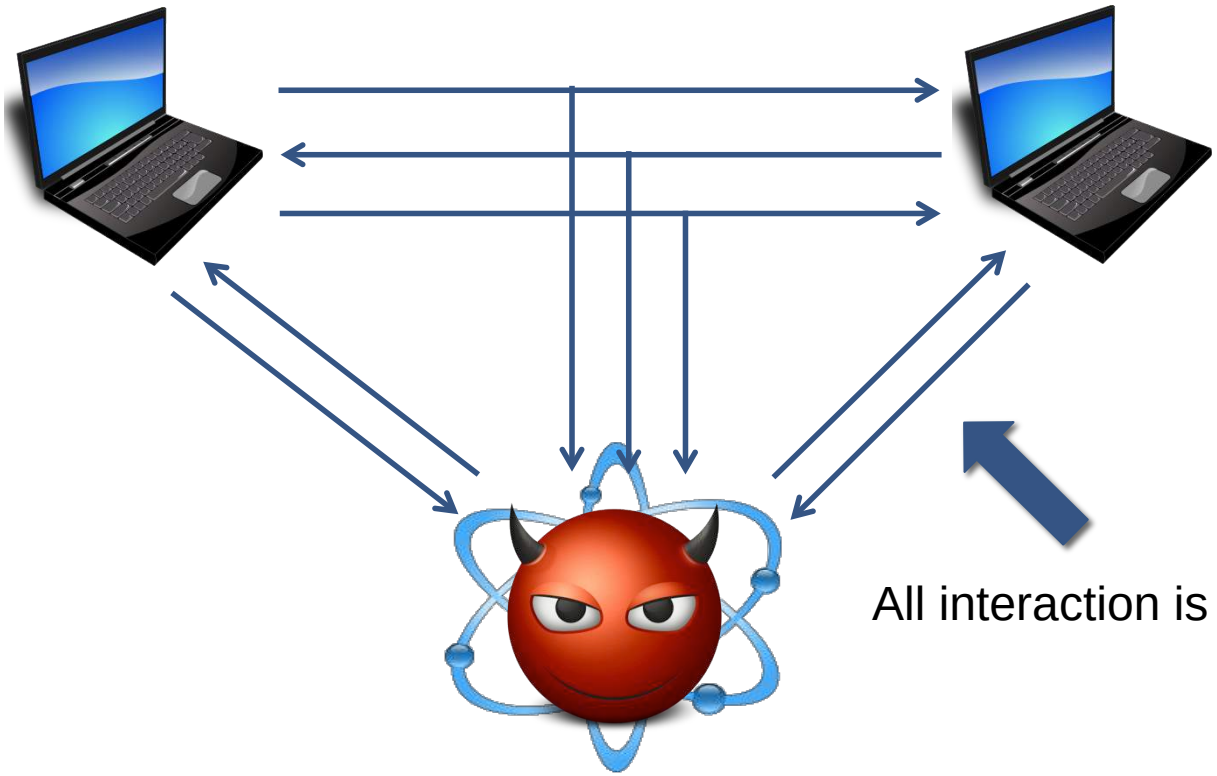
Post-Quantum Security

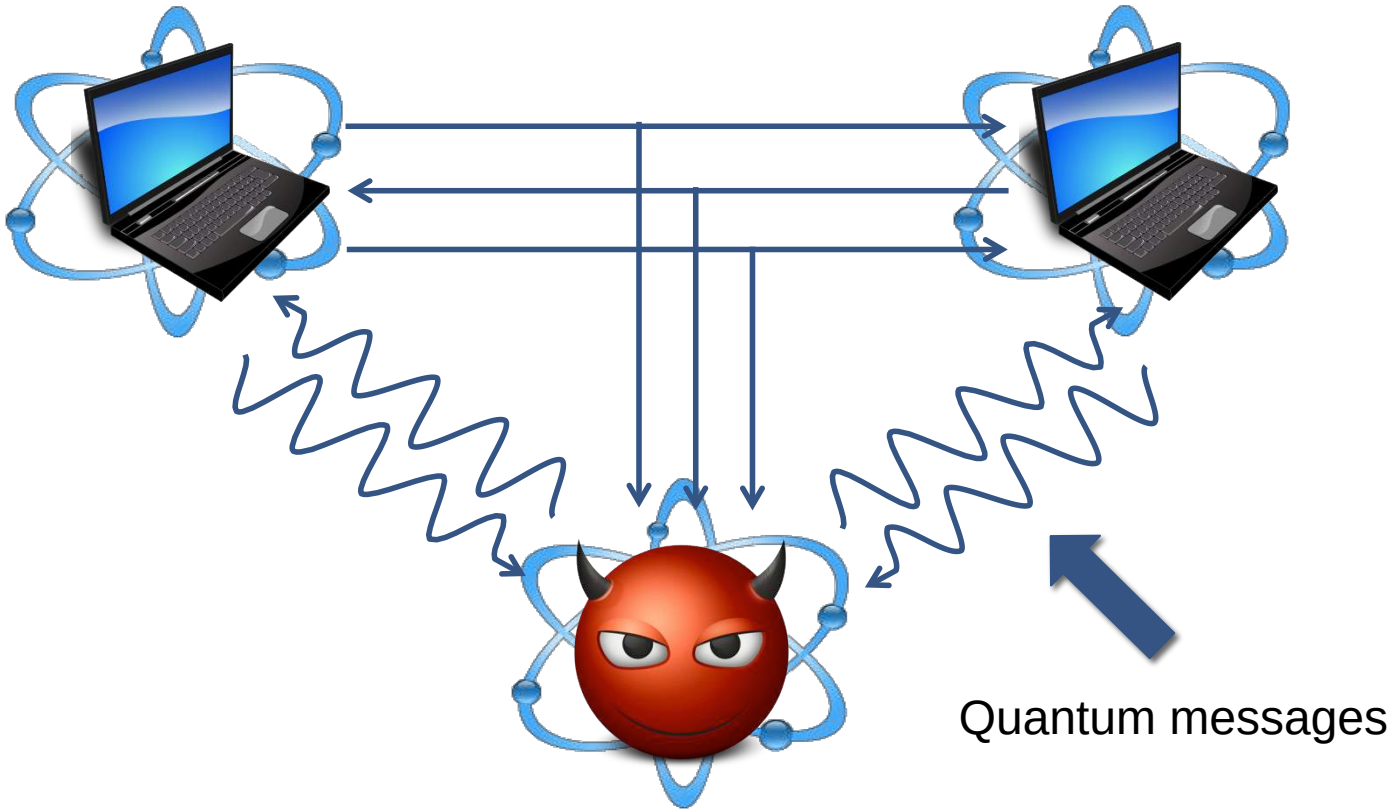
Post-quantum = end-users are classical



All interaction is classical

Full Quantum Security

Full quantum = end-users are *quantum*



Quantum Background

Quantum states:

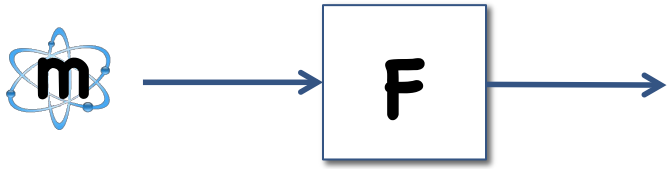
$= \text{superposition of } \mathbf{all} \text{ messages}$

$= \Sigma\, \alpha_m |m\rangle \qquad (\Sigma\, |\alpha_m|^2 = 1)$

Measurement:

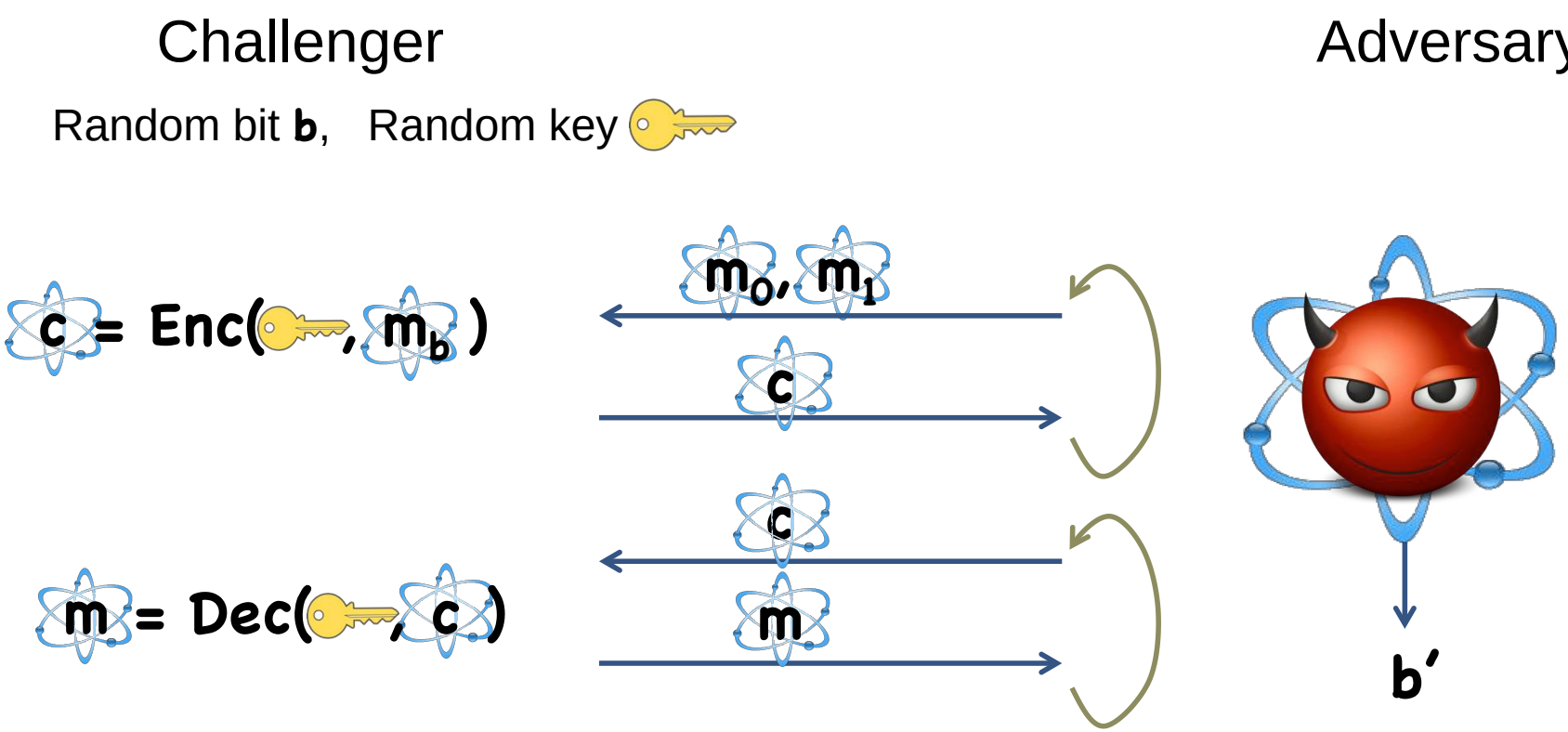
m with probability $|\alpha_m|^2$

Simulate classical ops in superposition:



$= \Sigma\, \alpha_m |F(m)\rangle$

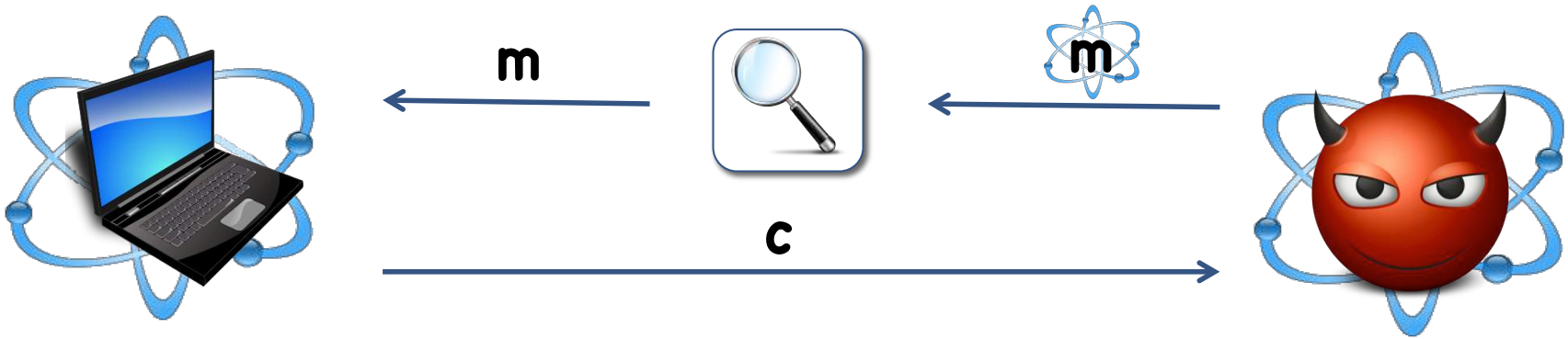
Full Quantum CCA Security?



Def: CCA-Security ⇒ ∀ efficient | Pr[b'=b] − ½ | < negl

Are Full Quantum Attacks Plausible?

Objection: can always “classicalize” by sampling



⇒ Reduce attack to post-quantum attack!

Reasons to still use full quantum notions:

- Classicalization is burden on hardware designer
- What if adversary can bypass?
- Classicalization amounts to a *hardware* assumption

This Work

[BDFLSZ'11,Zha'12a,Zha'13]: Quantum random oracle model

[Zha'12b]: Pseudorandom functions

[BZ'13a]: Message Authentication Codes

[BZ'13b]: Digital signatures and encryption

Theorem: Full-quantum security > Post-quantum security

Theorem (Informal): Full-quantum security can be obtained with "minimal" overhead w.r.t. post-quantum security

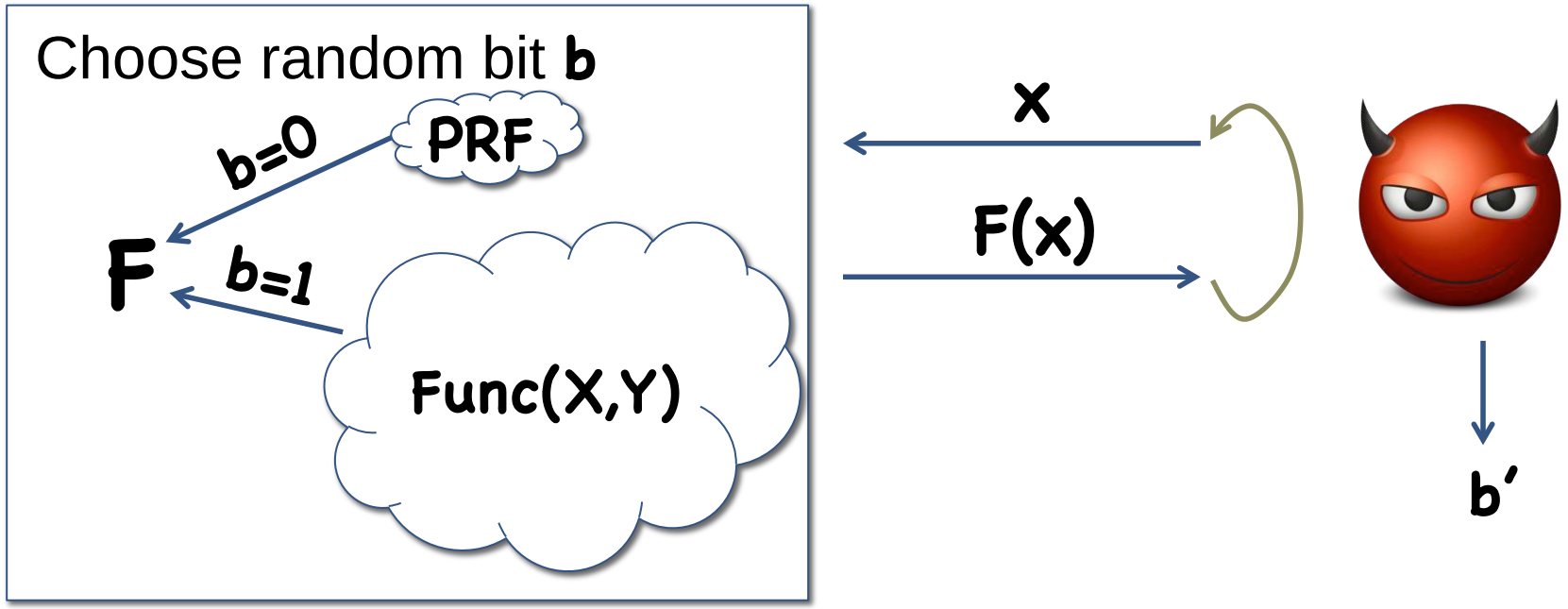
Example: Pseudorandom Functions

[GGM'84]

Efficient keyed functions that "look like" random functions

- Fundamental building block in symmetric crypto

Classical security:



Def: Security ⇒ ∀ efficient  , | Pr[b'=b] − ½ | < negl

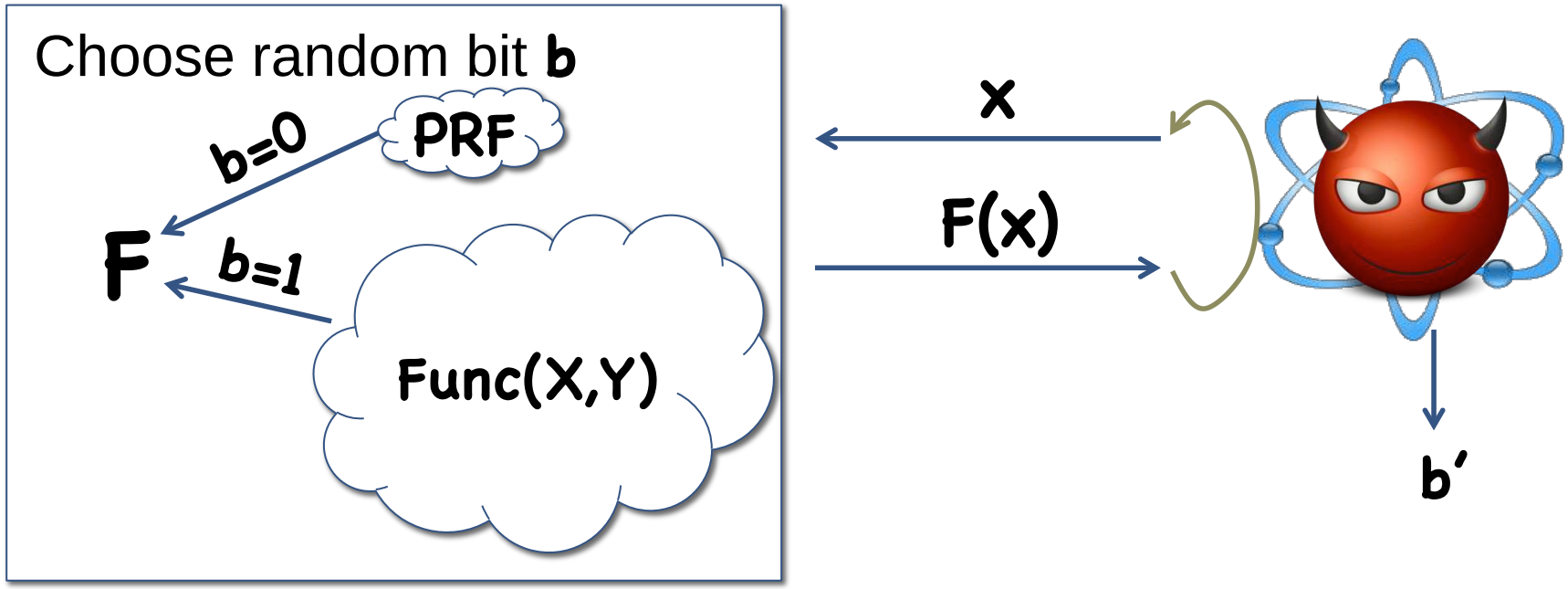
Example: Pseudorandom Functions

[GGM'84]

Efficient keyed functions that "look like" random functions

- Fundamental building block in symmetric crypto

Post-quantum security:



Def: PQ-Security ⇒ ∀ efficient , | Pr[b'=b] − ½ | < negl

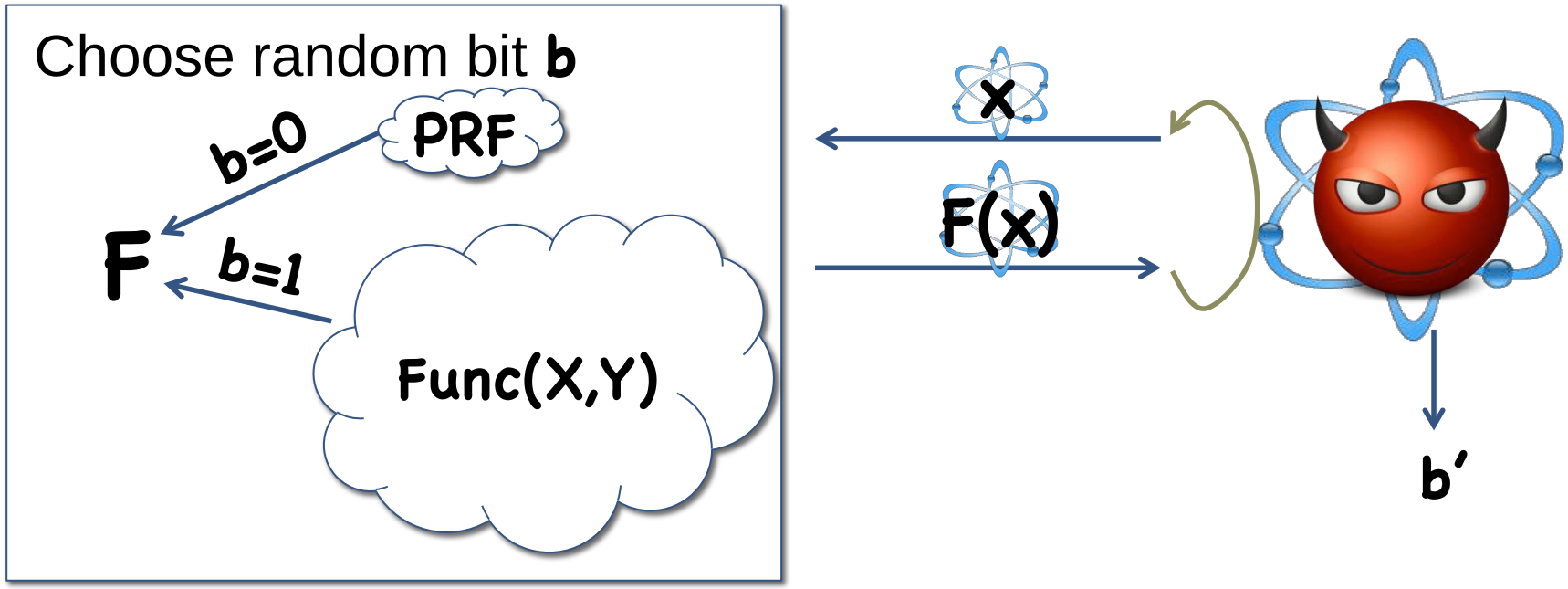
Example: Pseudorandom Functions

[GGM'84]

Efficient keyed functions that "look like" random functions

- Fundamental building block in symmetric crypto

Full-quantum security:



Def: FQ-Security ⇒ ∀ efficient , | Pr[b'=b] − ½ | < negl

How to build QPRFs?

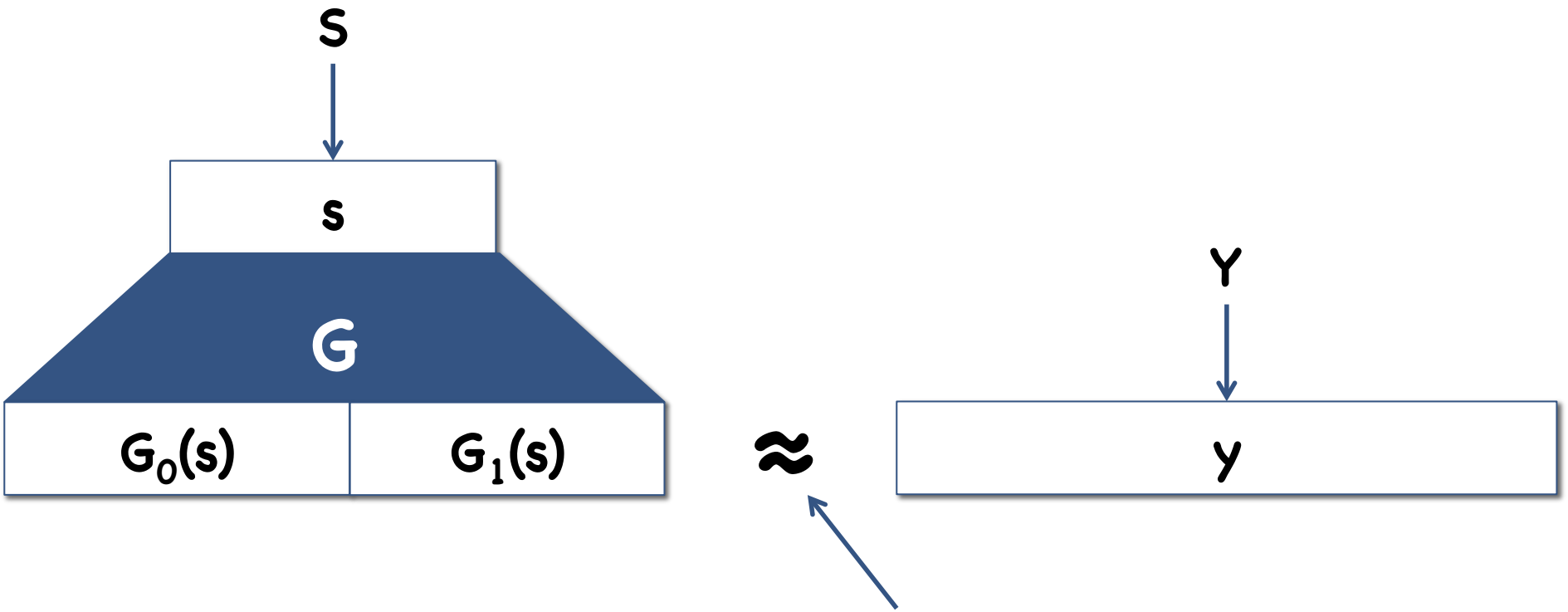
Hope that existing PQ-secure PRFs are FQ secure

Examples: GGM, NR, BPR

Questions:

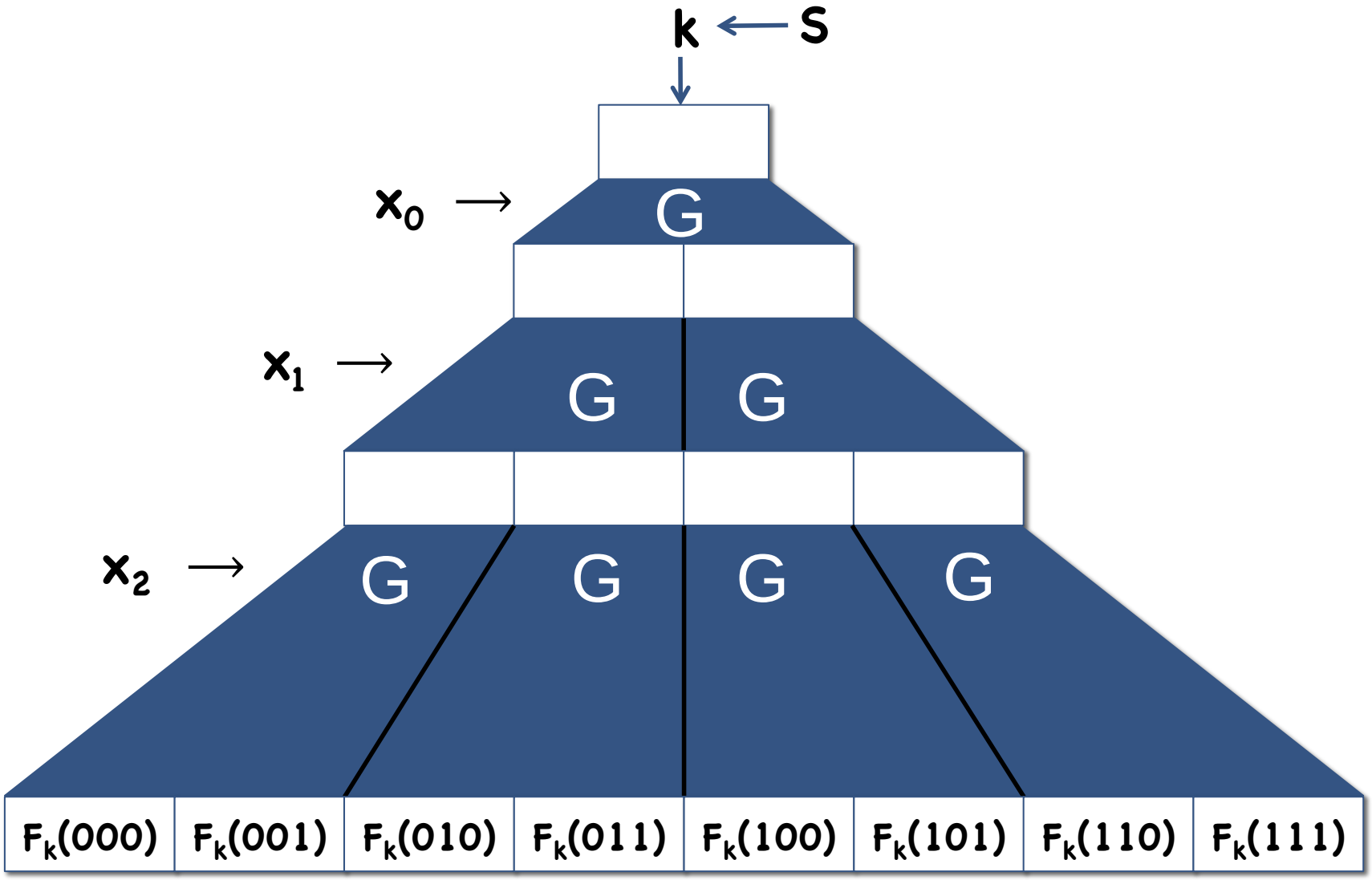
- Do classical security analyses carry over?
- If not, what new tools are needed?

Pseudorandom Generators



Indistinguishable by efficient quantum adversaries

The GGM Construction



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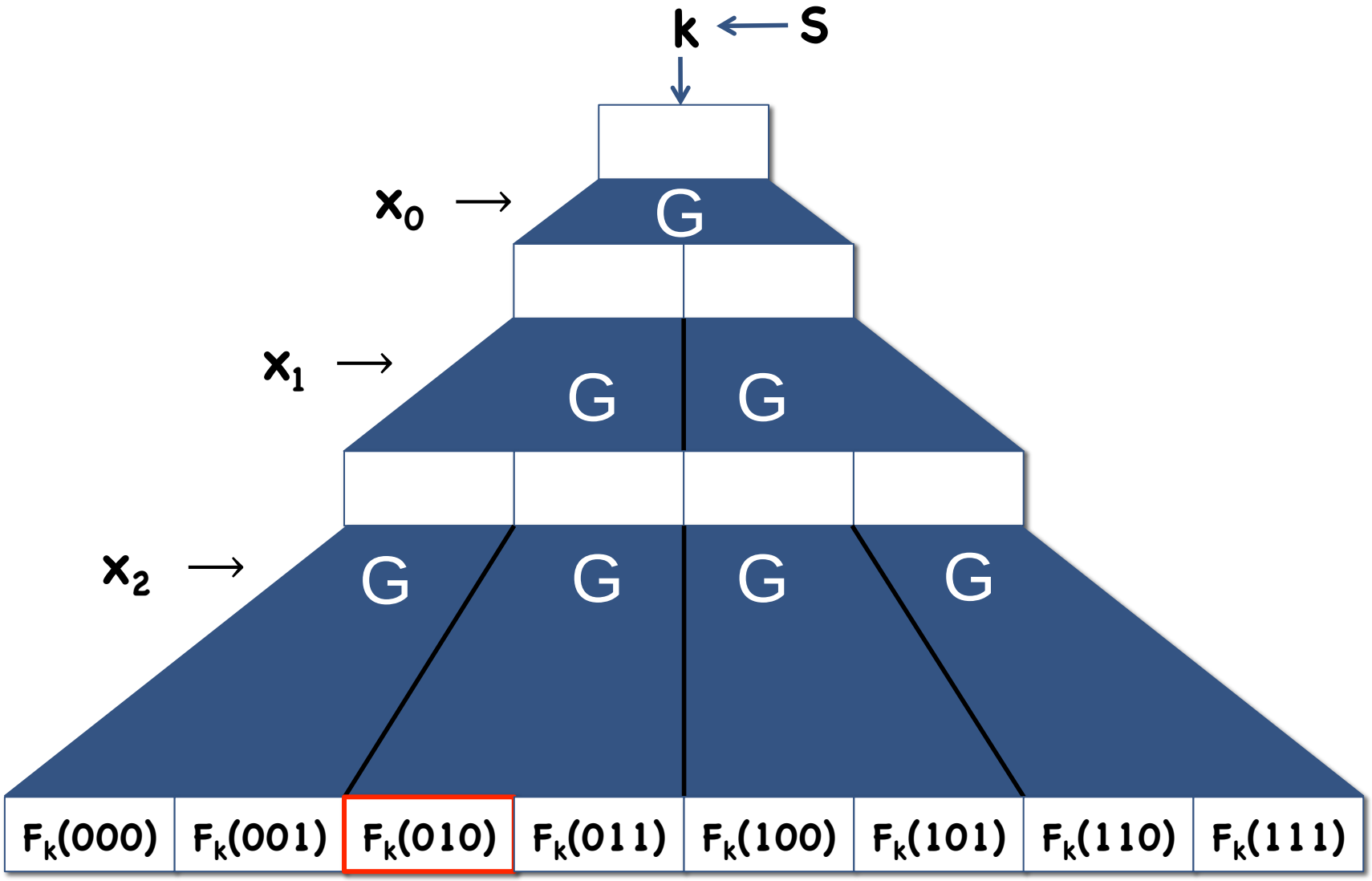

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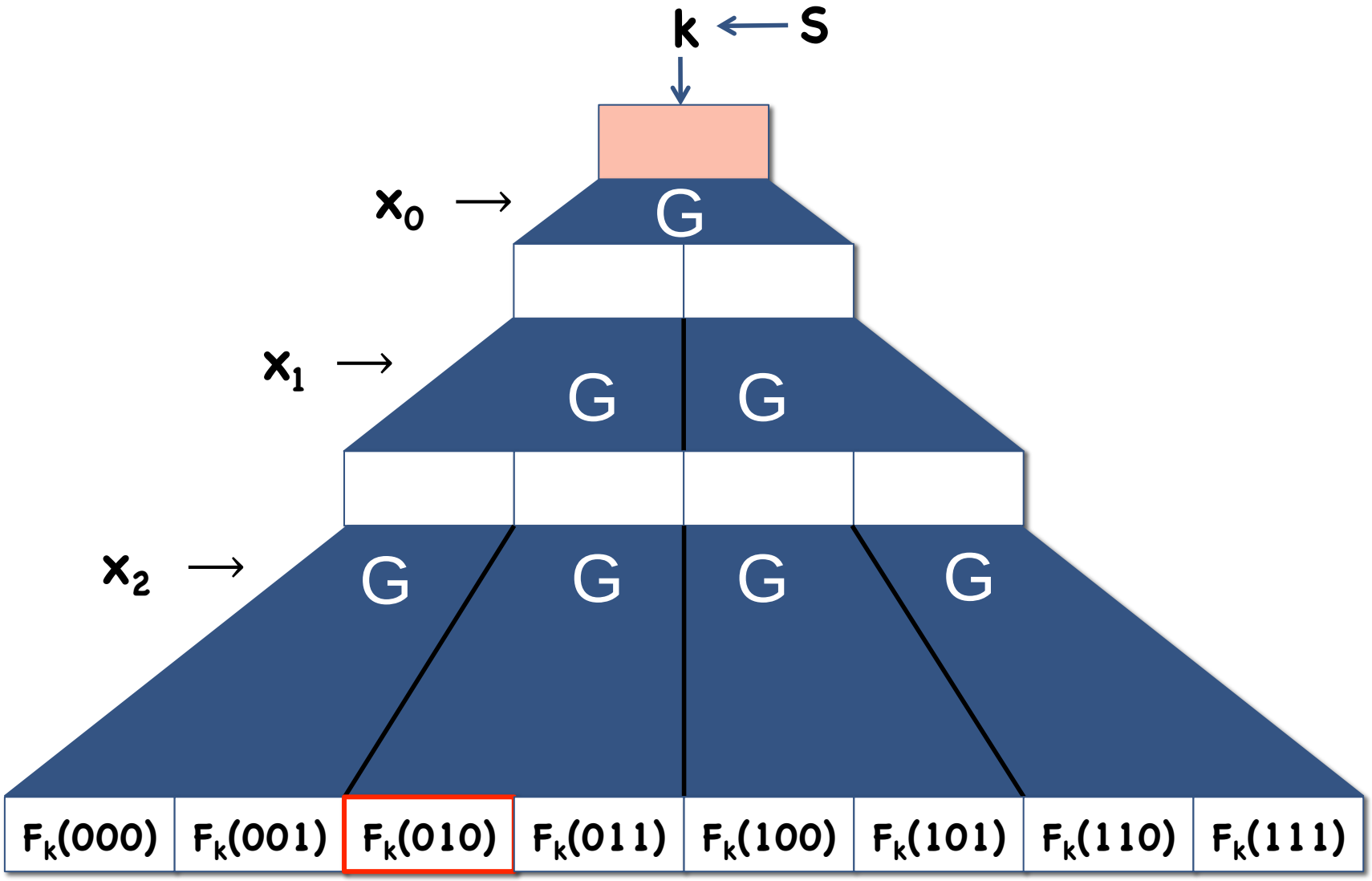

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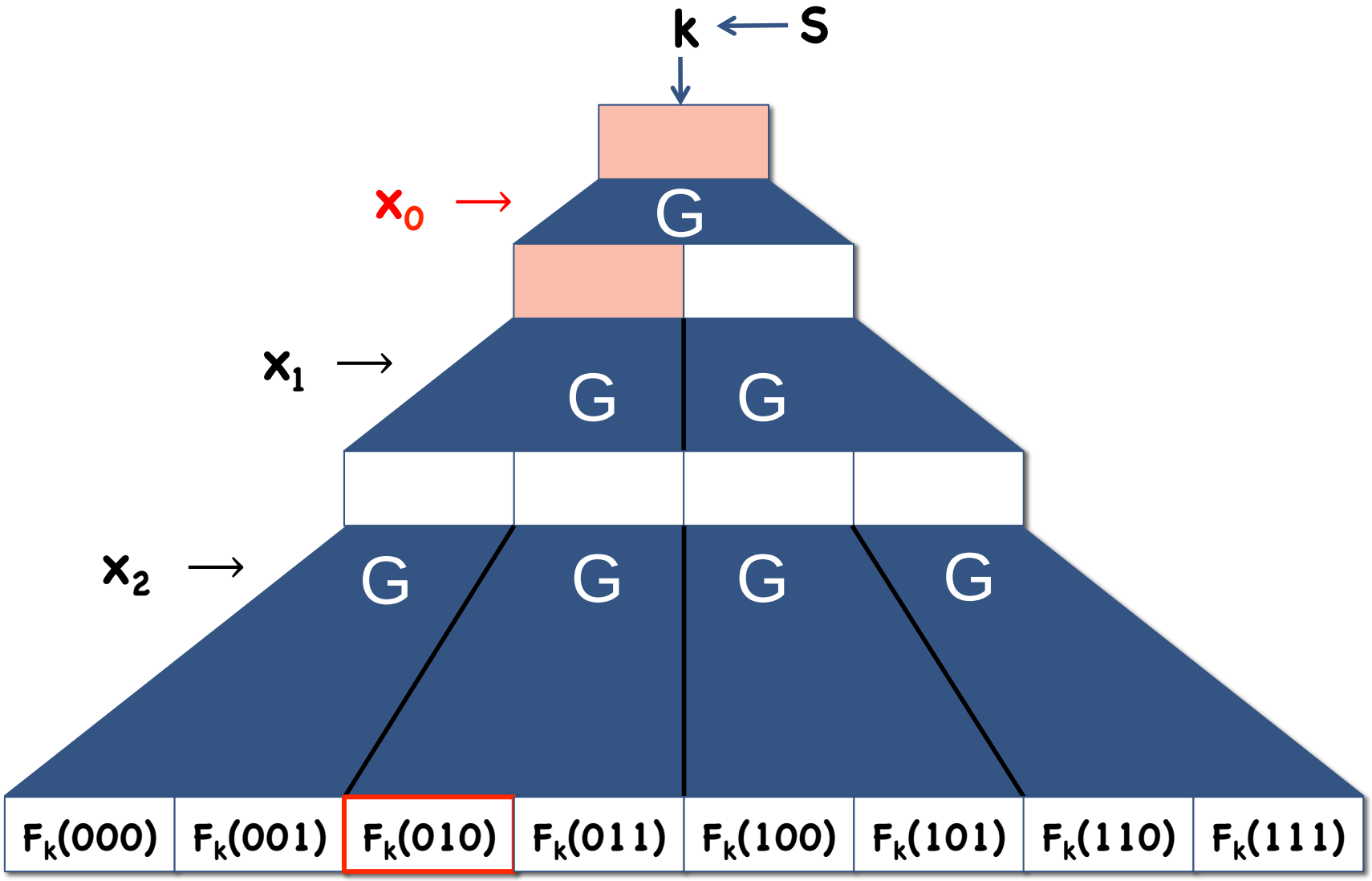

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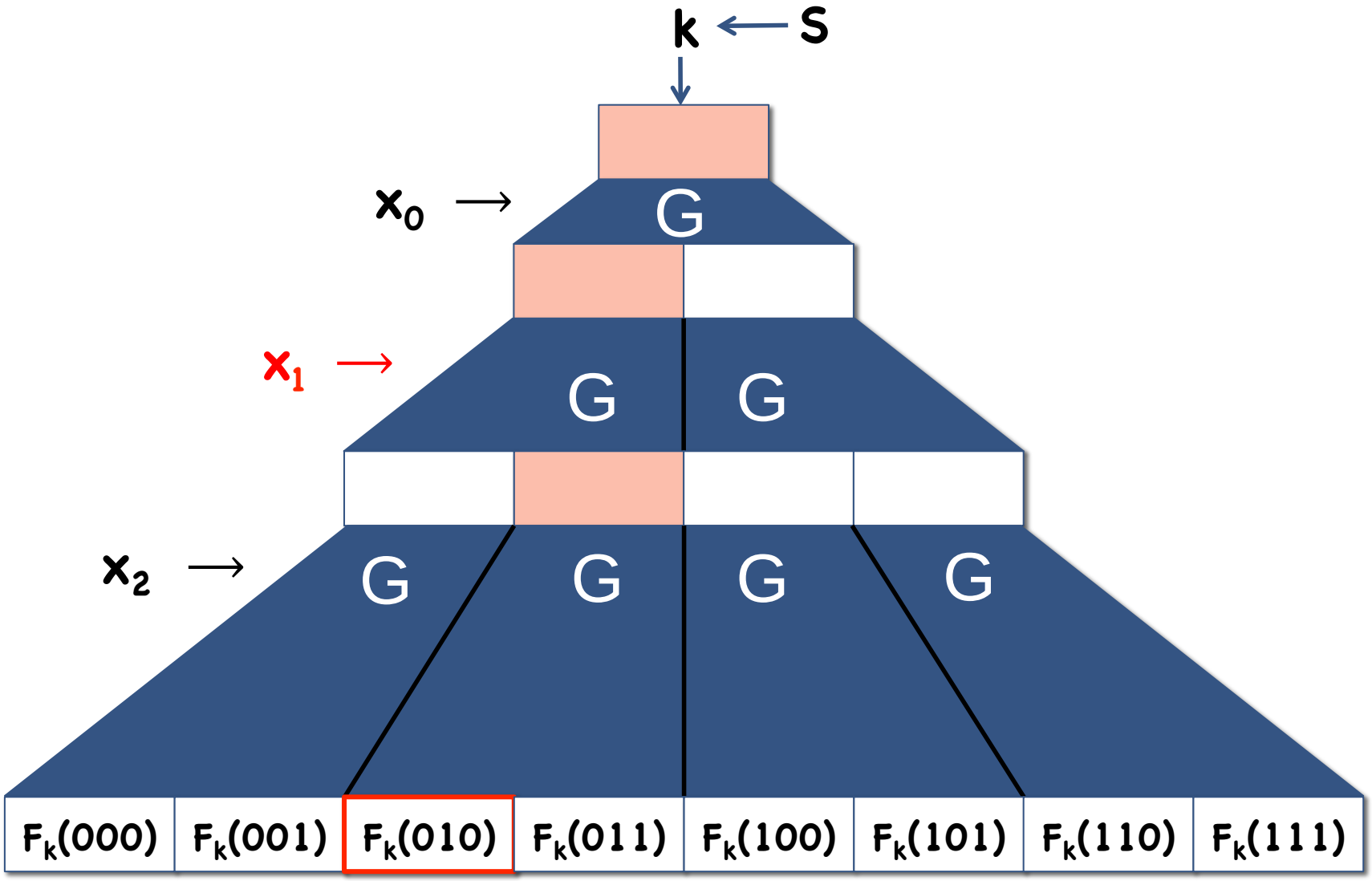

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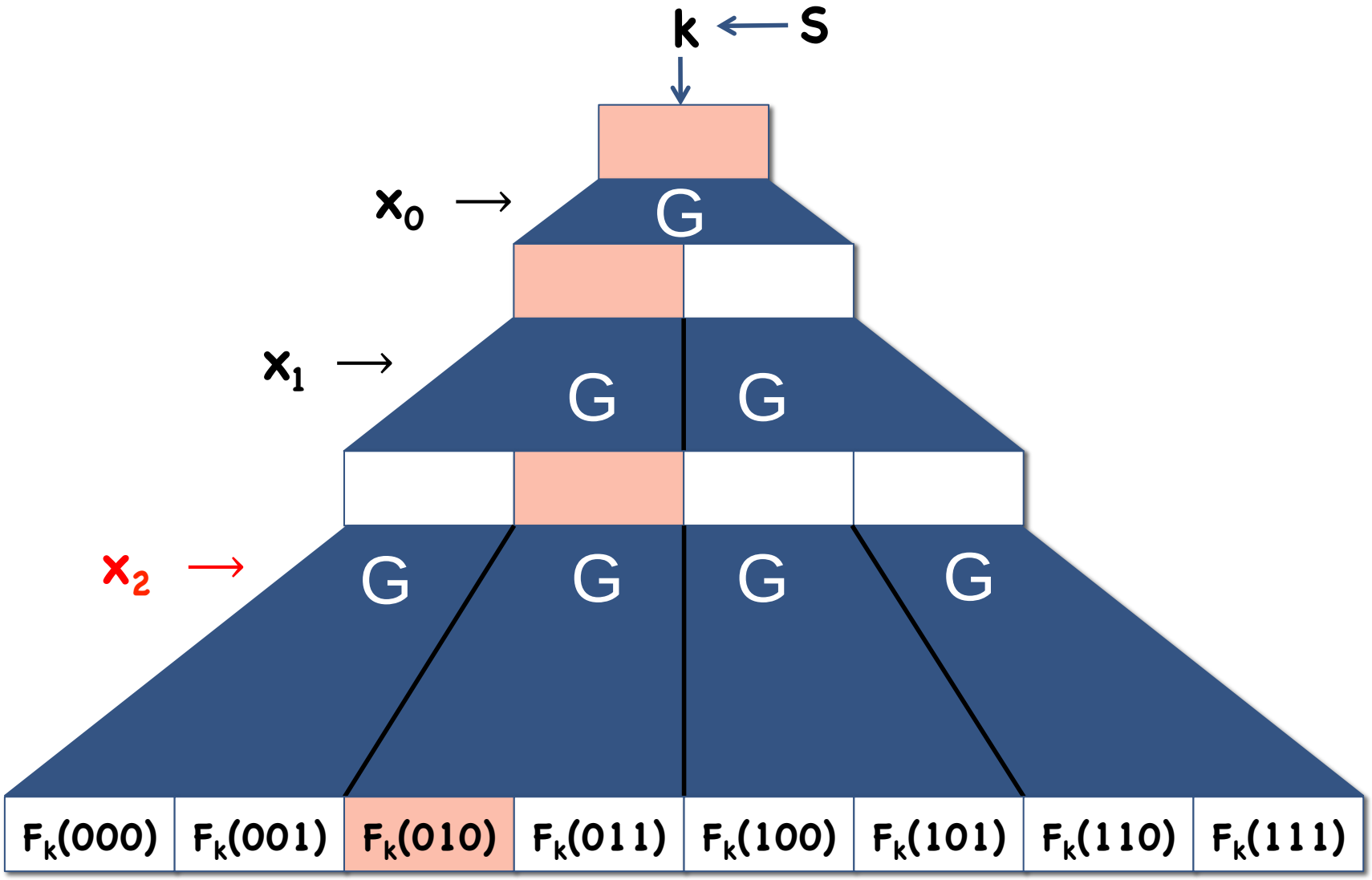


Quantum Security Proof?

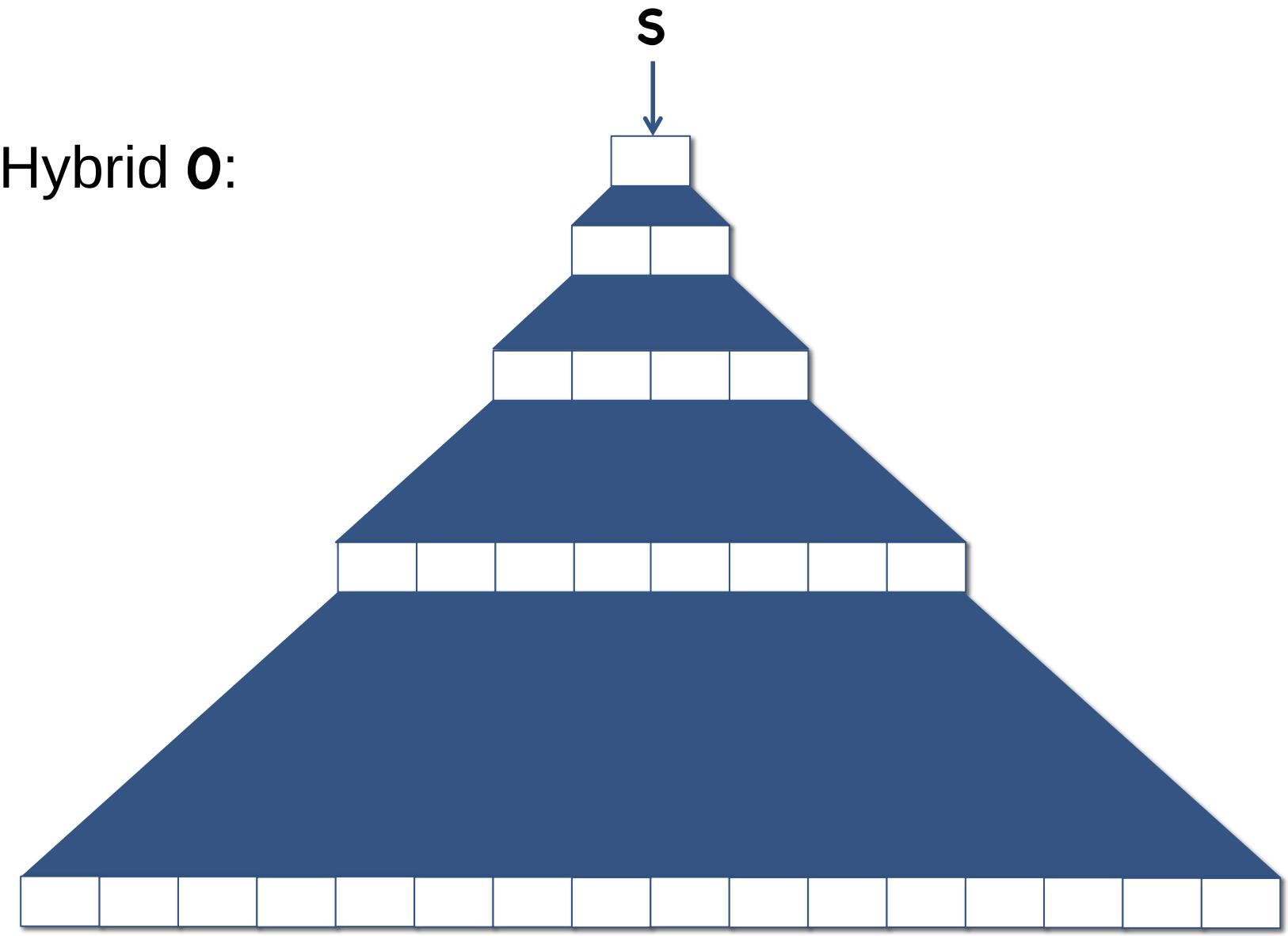
Idea: follow classical steps

- Turn PRF distinguisher into PRG distinguisher

Step 1: Hybridize over levels of tree

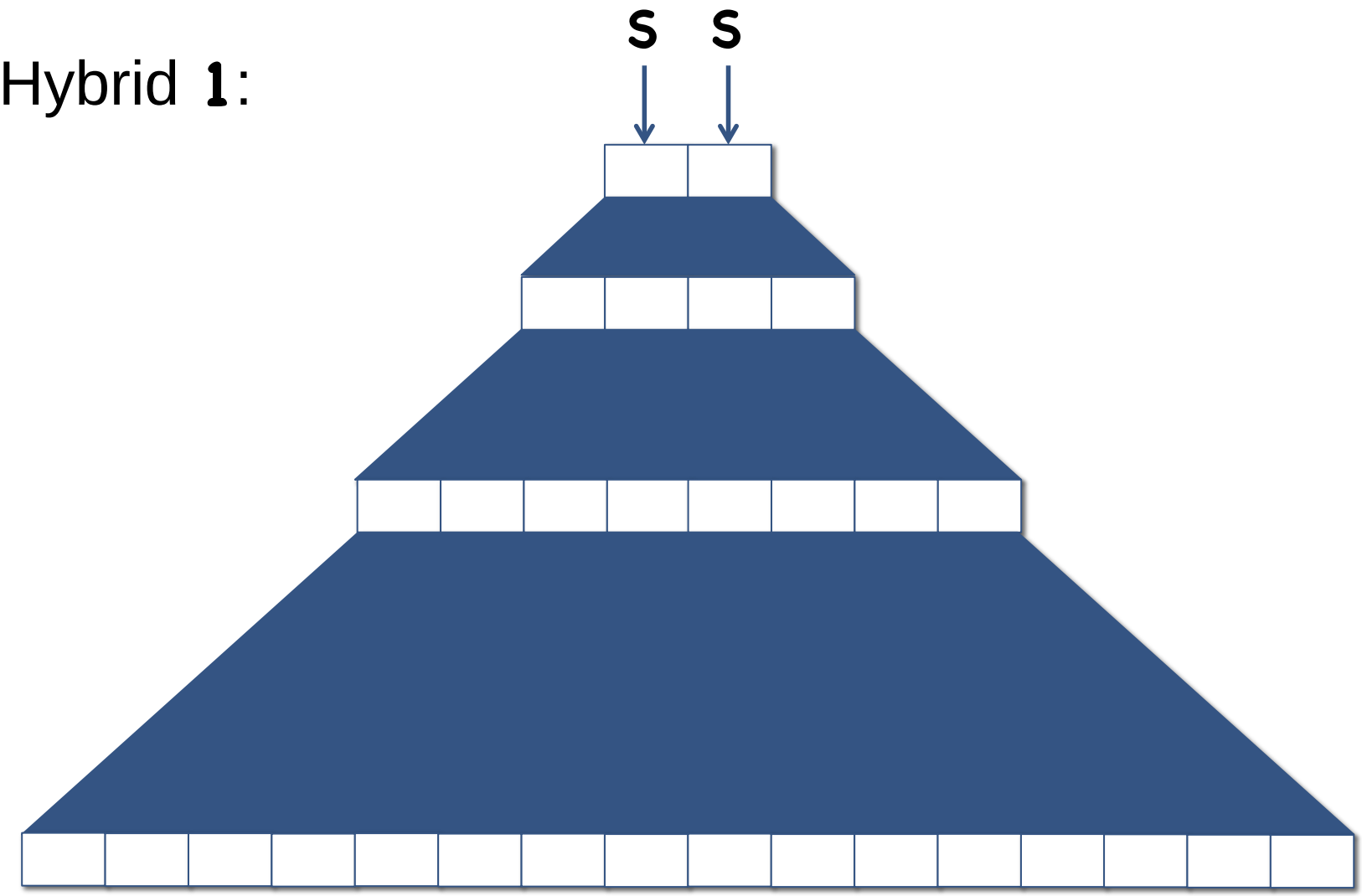
Hybridize Over Levels

Hybrid **0**:



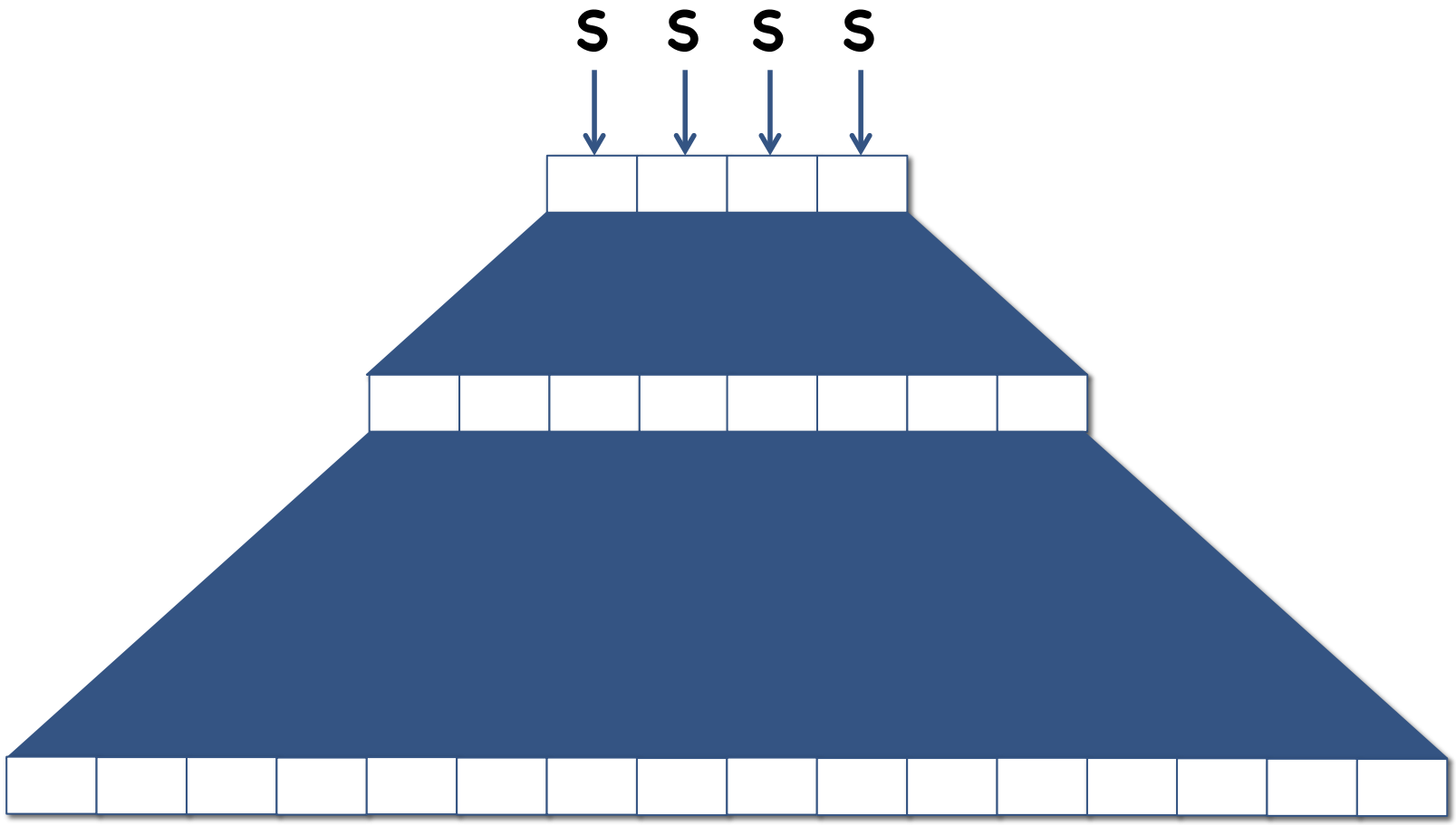
Hybridize Over Levels

Hybrid **1**:



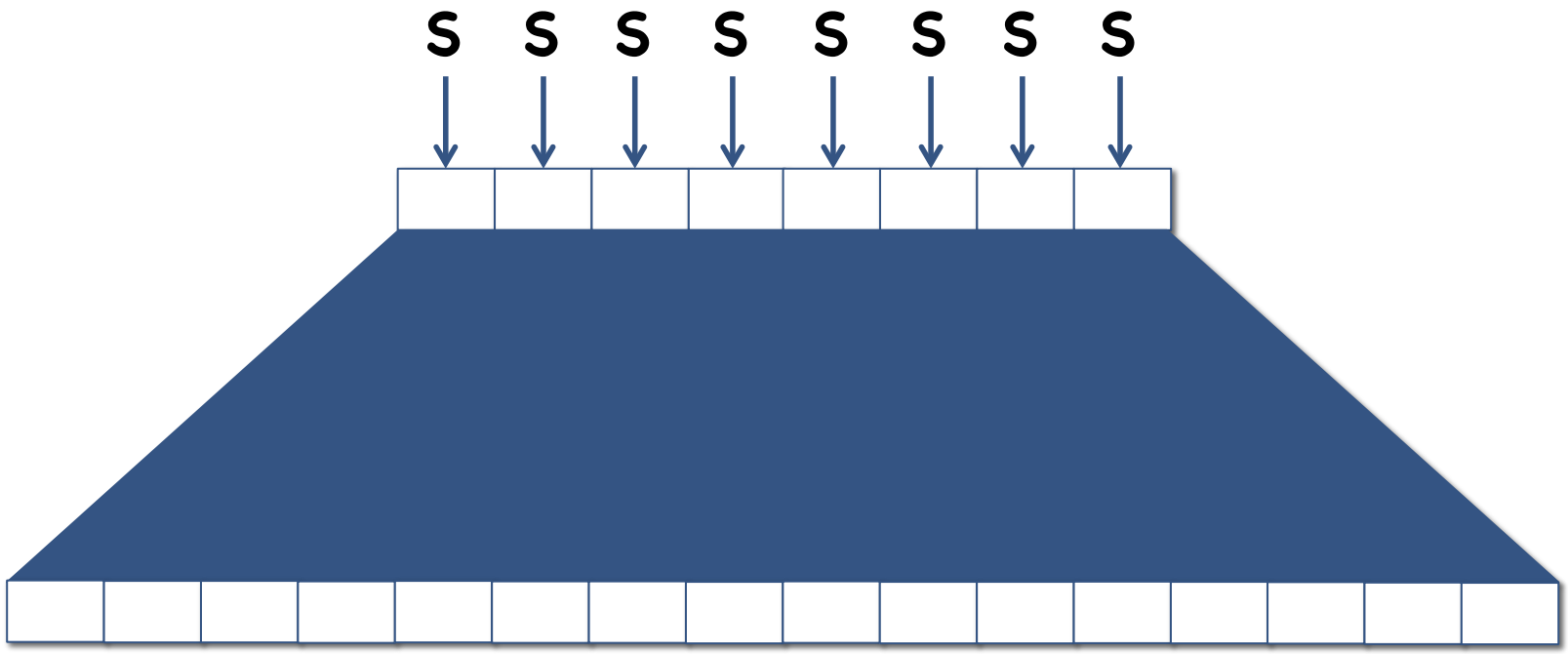
Hybridize Over Levels

Hybrid **2**:



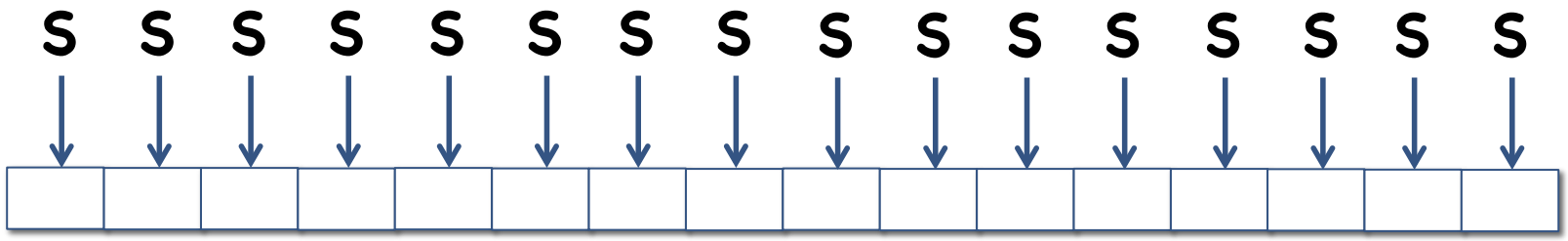
Hybridize Over Levels

Hybrid **3**:



Hybridize Over Levels

Hybrid **n**:



Hybridize Over Levels

Distinguish **PRF** from **Func(X,Y)** with adv. **ε**

↓

Distinguish two adjacent hybrids with adv. **ε/n**

n polynomial ⇒ acceptable loss

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Argument carries over to quantum setting unmodified

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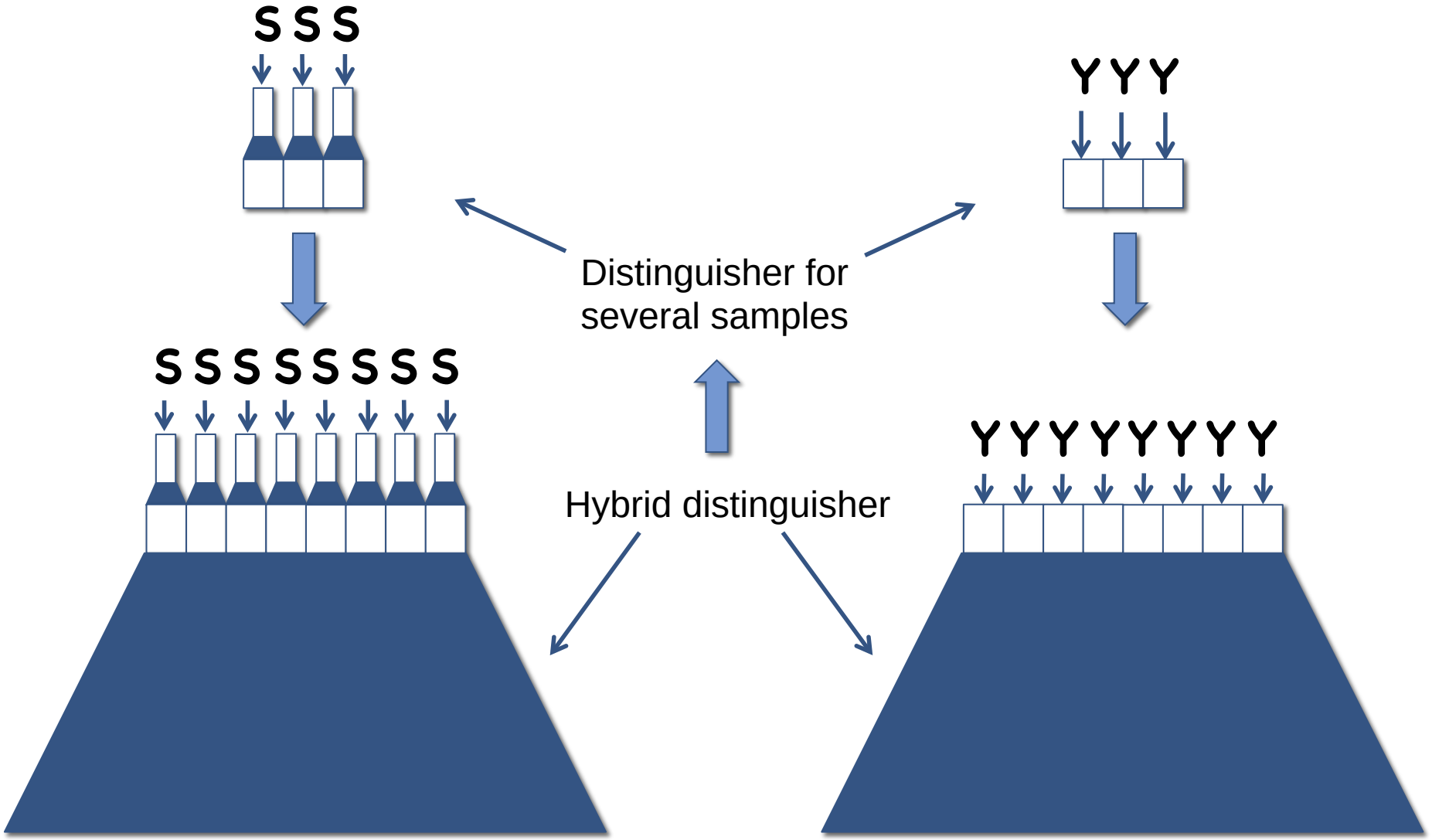
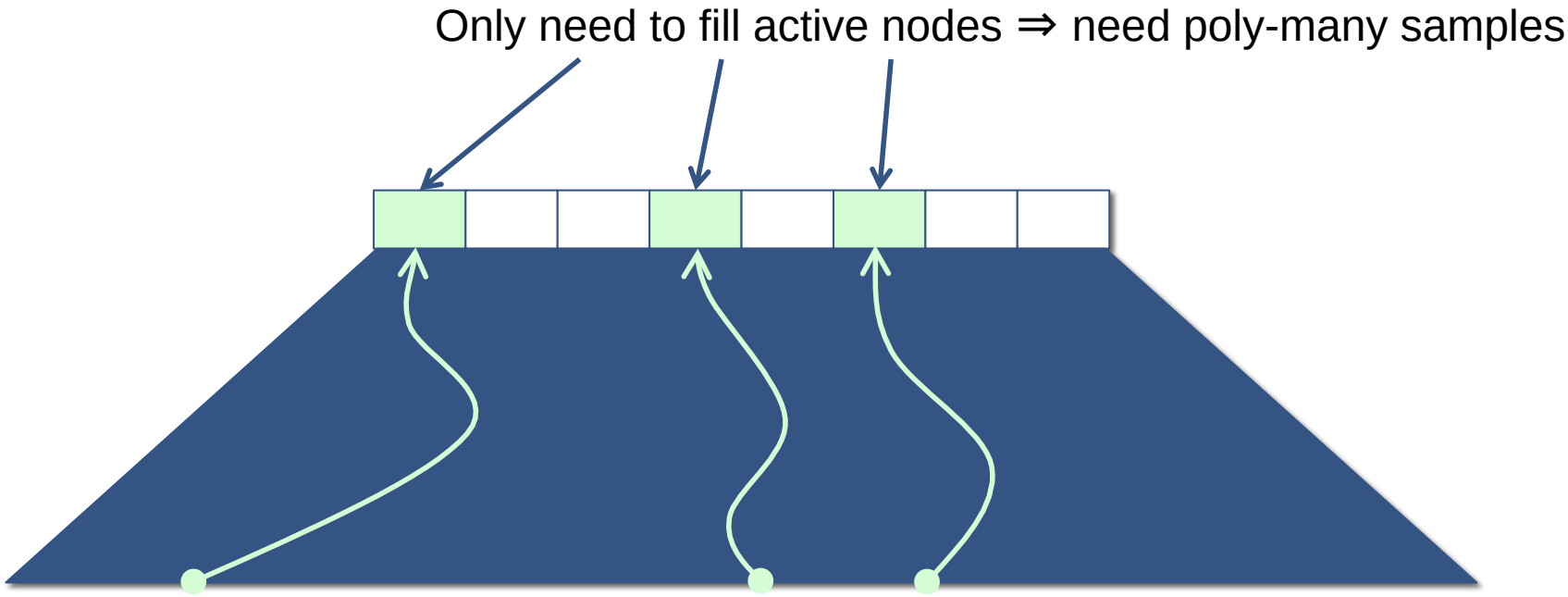
Simulating Hybrids



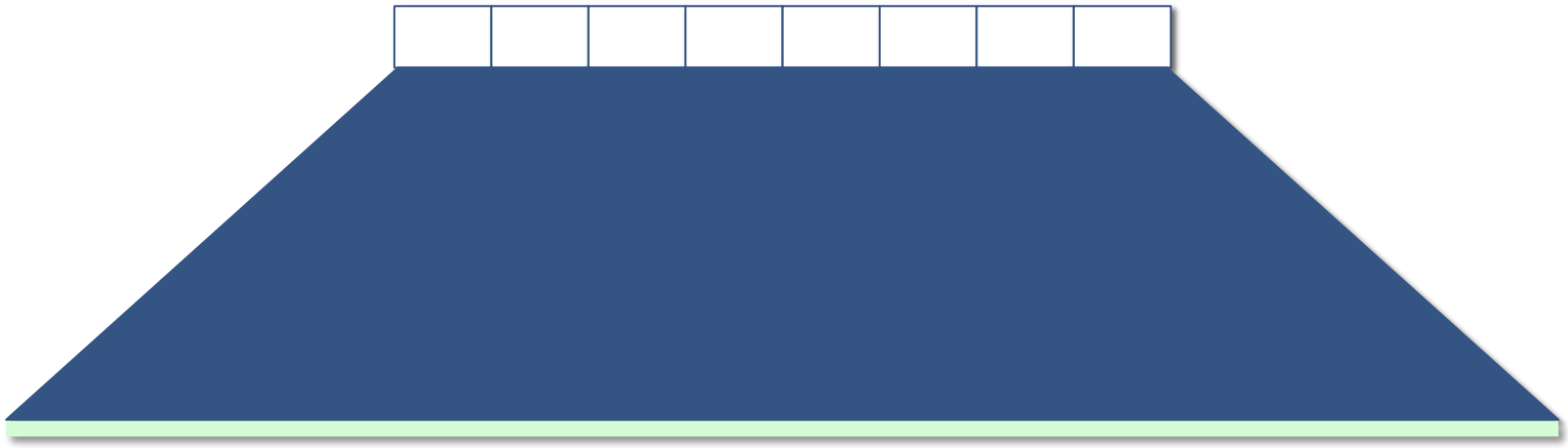
How It Was Done Classically

Active node: value used to answer query



Adversary only queries polynomial number of points

Quantum Simulation?



Adversary can query on all exponentially-many inputs

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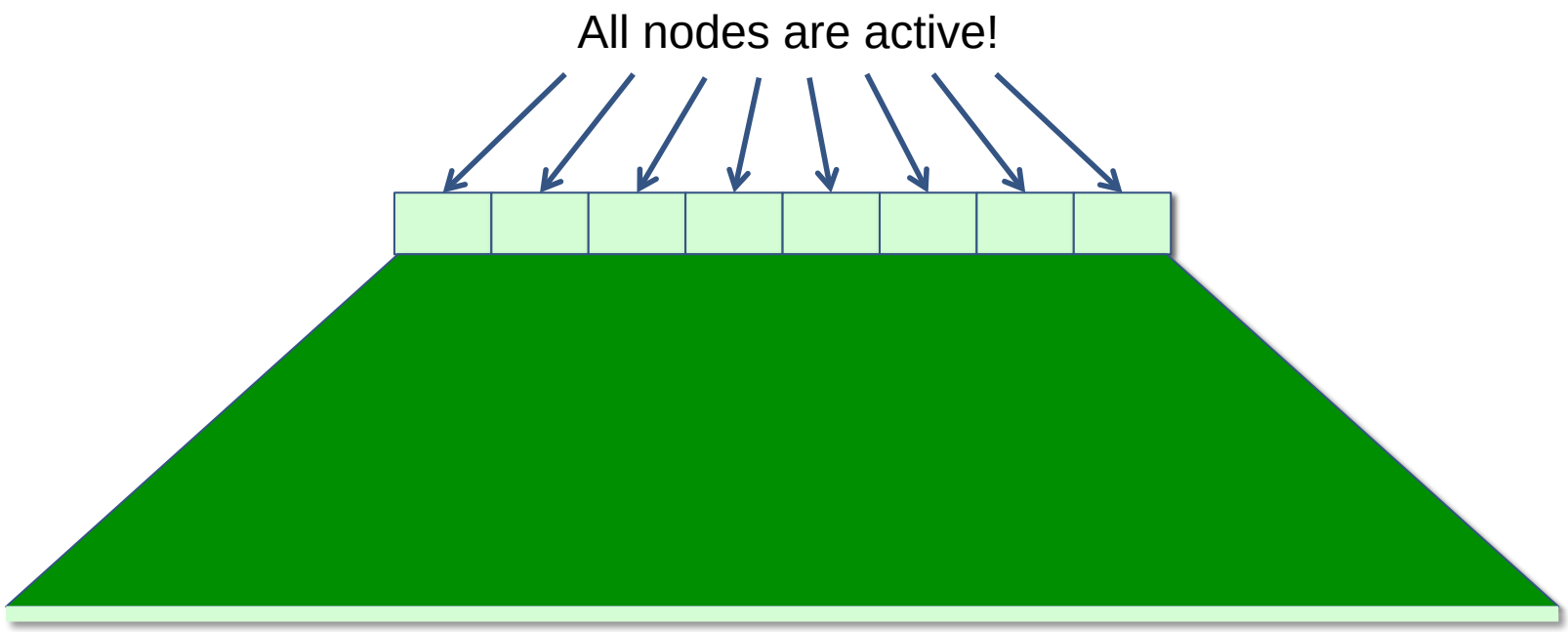

Adversary can query on all exponentially-many inputs

Need exponentially many samples to simulate!

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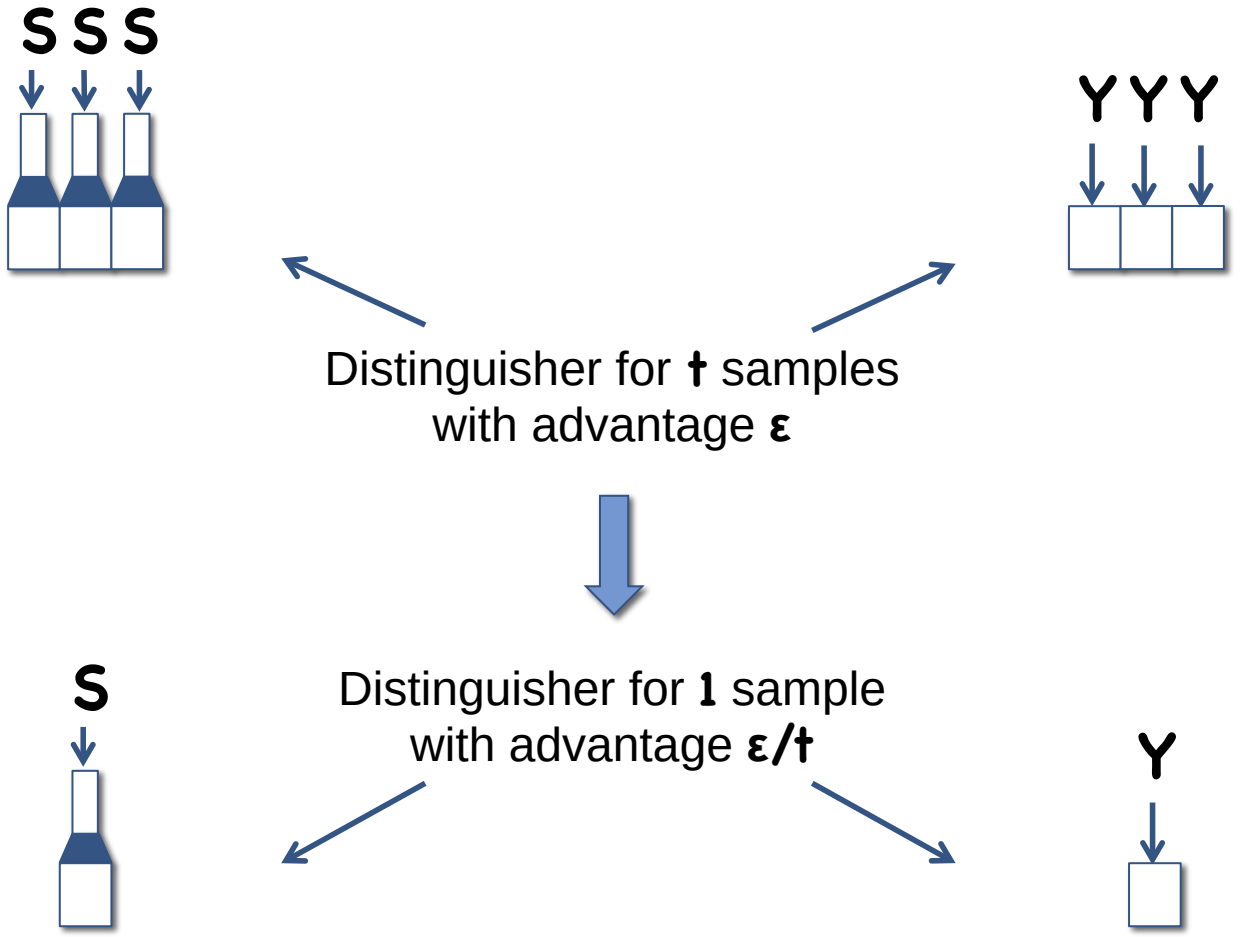
Step 3: Hybrid over samples

Hybrid Over Samples



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- Exponential samples ⇒ exponential security loss
- Can only handle poly-many samples

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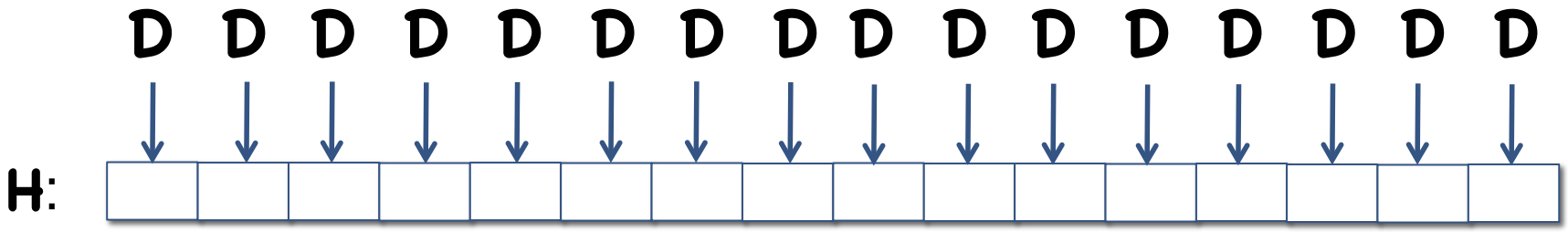
A Distribution to Simulate

Distribution D on Y $\Rightarrow$ induces distribution on functions

For all $x \in X$:

$y_x \leftarrow D$

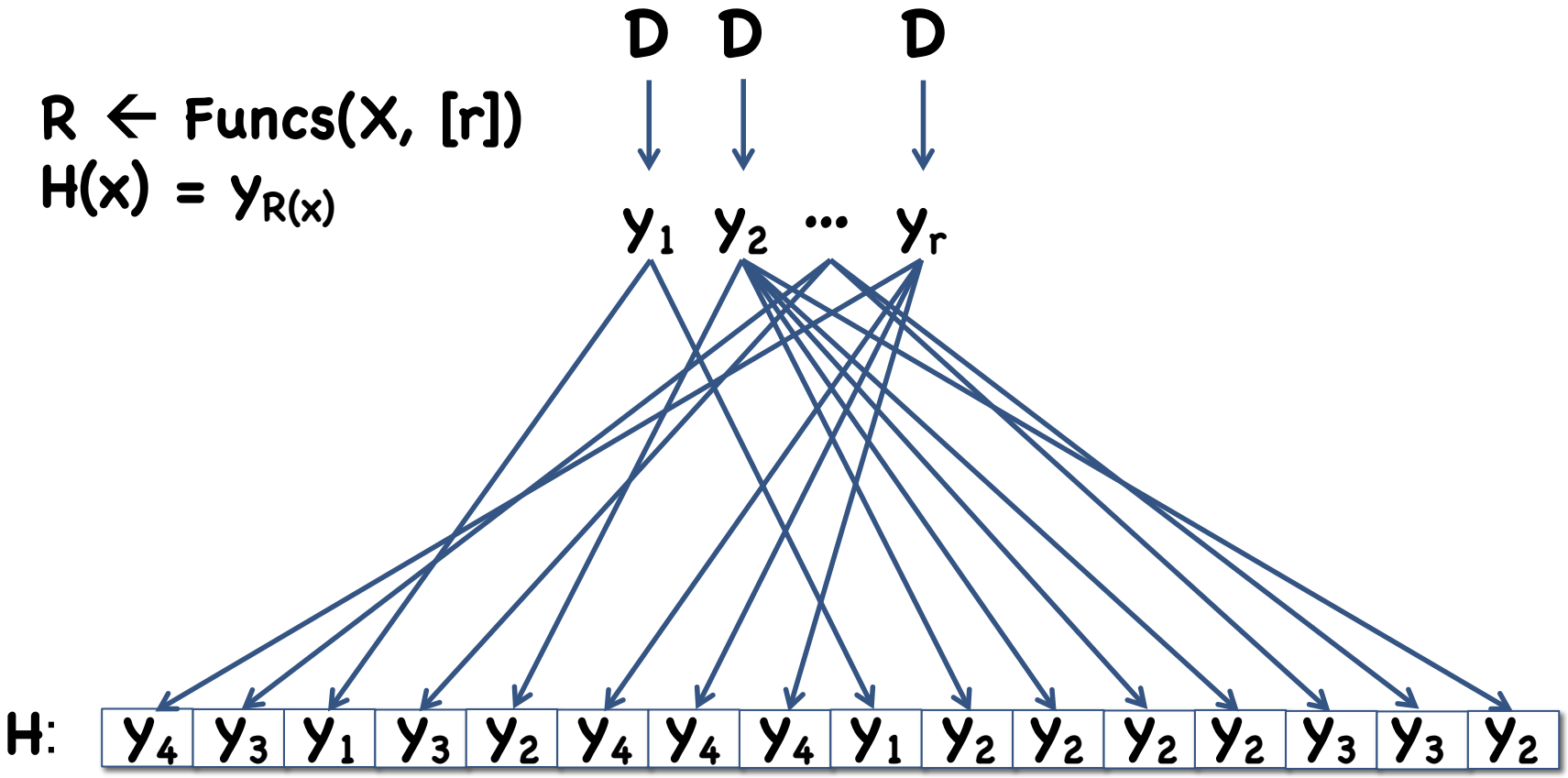
$H(x) = y_x$



$H \leftarrow D^X$

Goal: simulate using poly-many samples

Solution: Small-Range Distributions



Small-Range Distributions

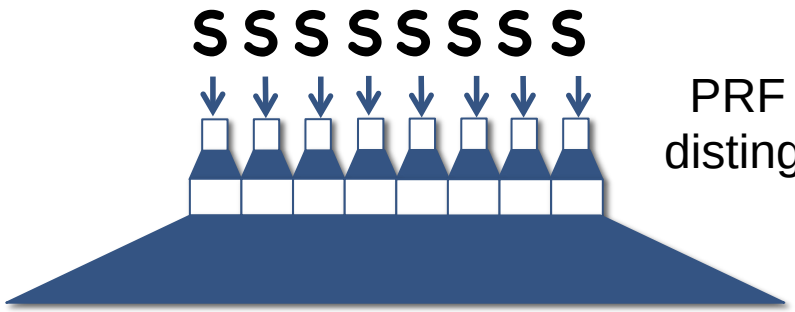
Theorem: $SR_r^X(D)$ is indistinguishable from D^X by any q-query quantum algorithm, except with advantage $O(q^3/r)$

Notes:

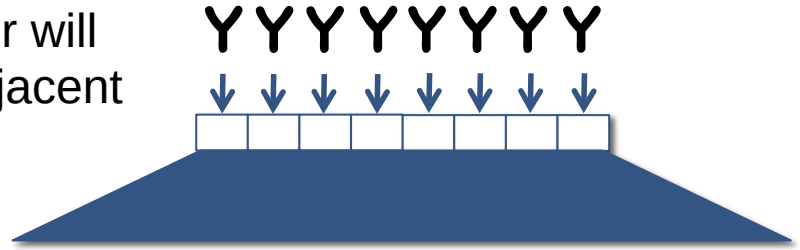
- Highly non-trivial
- Distinguishing prob not negligible, but good enough
  - We get to choose r
- Random function R not efficiently constructible

Theorem: Can simulate R using k-wise independence

Quantum GGM Proof



PRF distinguisher will distinguish two adjacent hybrids



≈ (SR distributions)

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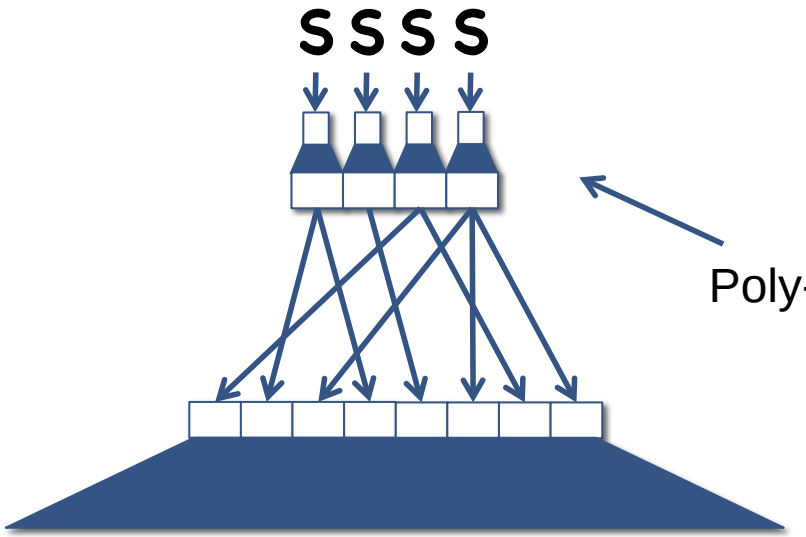
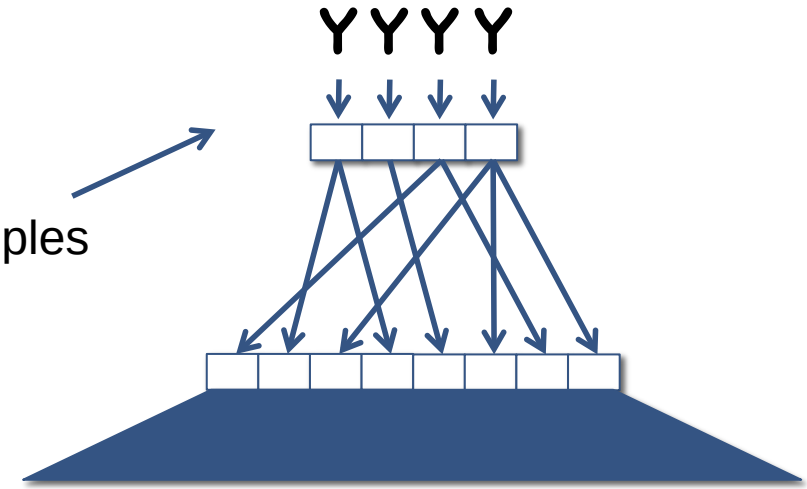


Poly-many samples



Quantum Security Proof

Idea: follow classical steps

- Turn PRF distinguisher into PRG distinguisher

Step 1: Hybridize over levels of tree ✓

Step 2: Approx. sim. hybrids using poly-many samples ✓

Step 3: Hybrid over samples ✓

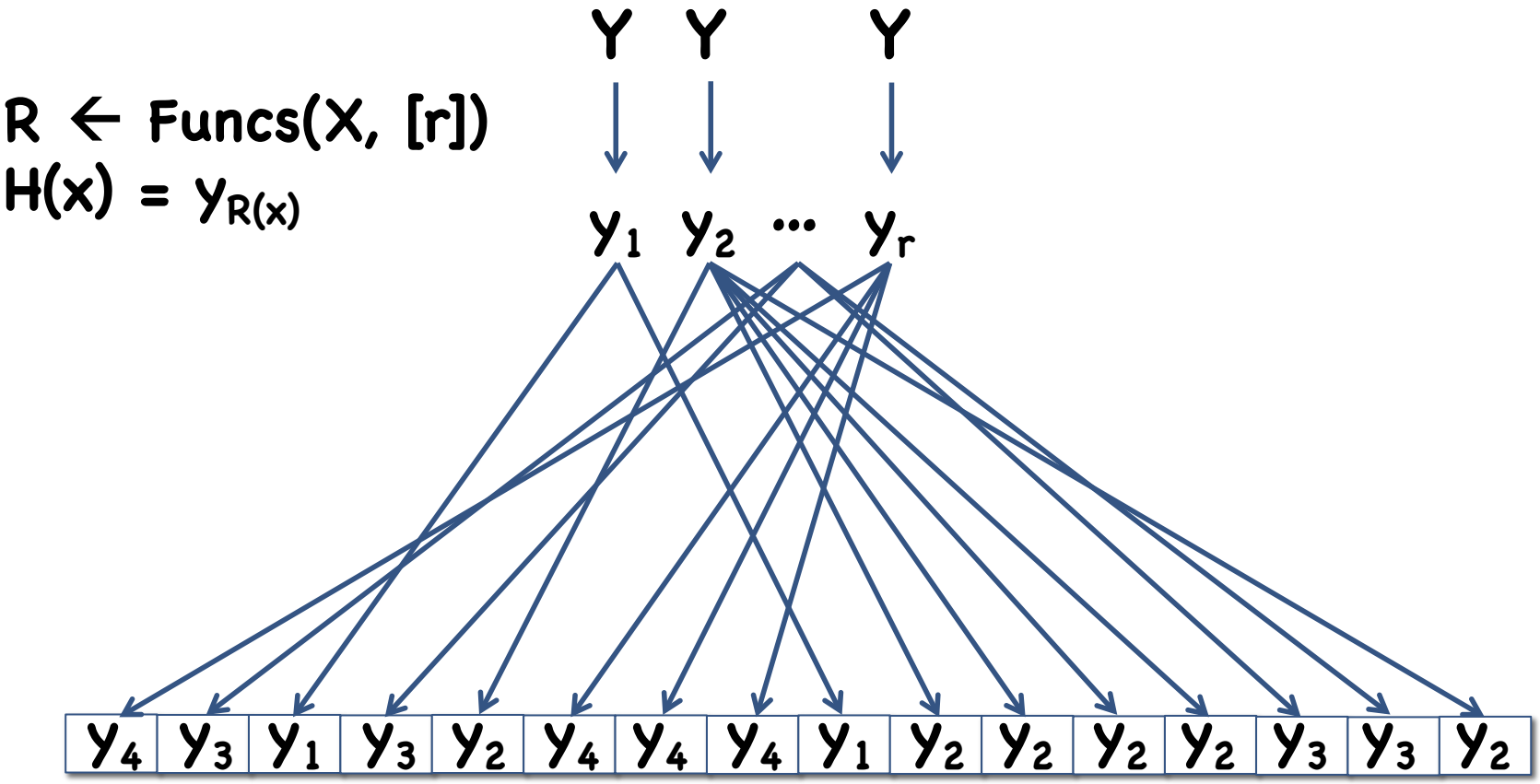
Result: PRG distinguisher

Impossible by assumption ⇒ PRF distinguisher impossible

Quantum Query Results

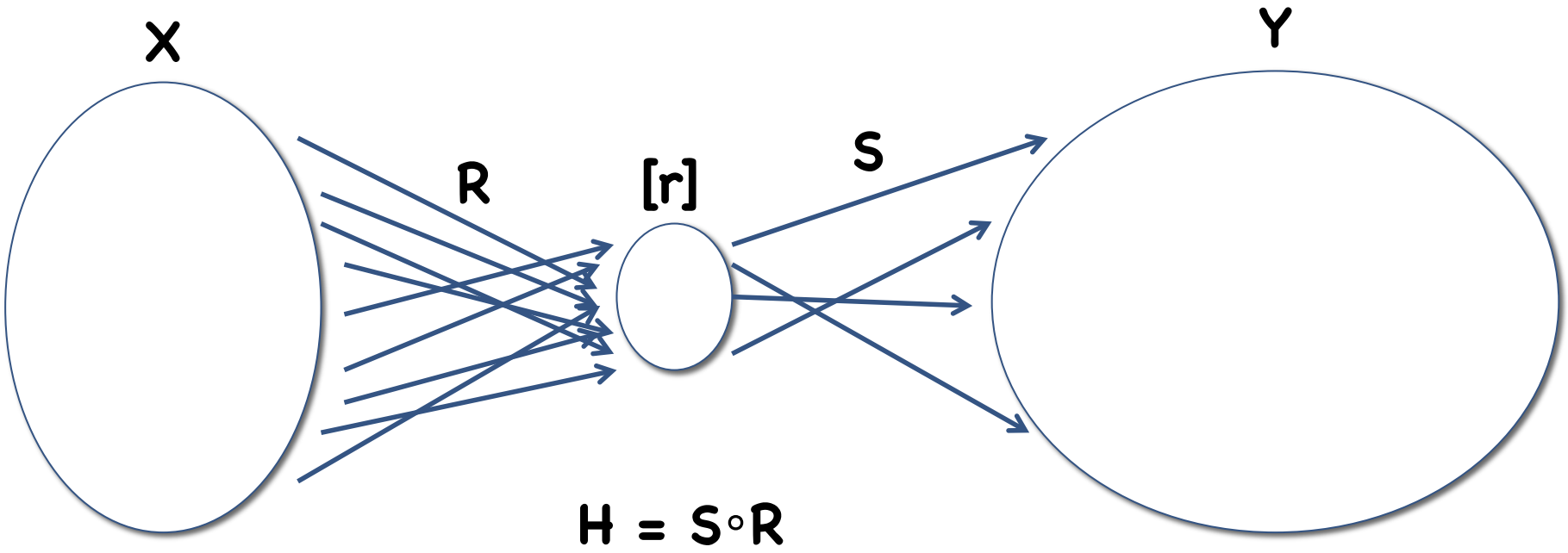
Quantum Collision Finding

Recall small-range distributions when **D** is uniform:



Quantum Collision Finding

Another view:



Theorem: H is indistinguishable from random by any q-query quantum algorithm, except with advantage $O(q^3/r)$

Corollary: If $|Y| >> |X|^2$, impossible to find collision in H unless $q \geq \Omega(r^{1/3})$

Quantum Collision Finding

Corollary: If $|Y| >> |X|^2$, impossible to find collision in H unless $q \geq O(r^{1/3})$

What about truly random functions with $|Y| << |X|^2$?

Theorem: $q \geq \Omega(r^{1/3})$ quantum queries are required to find collisions in a random function $R: X \rightarrow [r]$

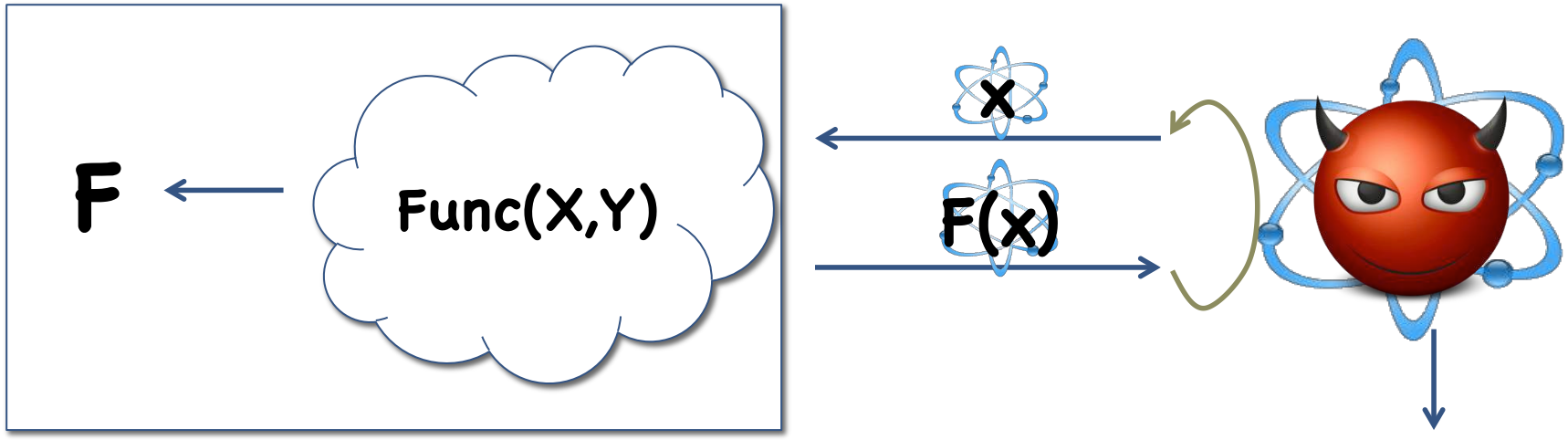
Previous $r^{1/3}$ lower bounds known for different settings

- E.g. k-to-1 functions [AS'01]
- All prior settings required $|Range| \geq |Domain|$
- Our works for all domain/range sizes

Bound is tight: [BHT'97] $q = O(r^{1/3})$

Quantum Oracle Interrogation

Using **q** queries, determine function at **k>q** points



$(x_1, F(x_1)), (x_2, F(x_2)), \ldots (x_k, F(x_k))$

Important for MAC, signature security

Quantum Oracle Interrogation

Classically: hard $\quad\quad\quad\quad\quad\quad\quad\quad$ **Adv = $1/|Y|^{k-q}$**

- Large outputs: **Adv = negl** even for **k=q+1**
- Small outputs: **Adv = negl** for **k = c q**

Quantum: not so fast

> **Theorem [vD'98]**: For **F:X→{0,1}**,
> **q** quantum queries ⇒ **k = 1.9q** points w.h.p

Also true for small ranges:

> **Theorem**: For **F:X→$\{0,1\}^2$**,
> **q** quantum queries ⇒ **k = 1.3q** points w.h.p

Question: What about large range sizes?

Quantum Oracle Interrogation

Theorem: For $F:X \to \{0,1\}^n$,
q quantum queries $\Rightarrow \Pr[k=q+1 \text{ points}] \leq (q+1)/2^n$

Highly non-trivial

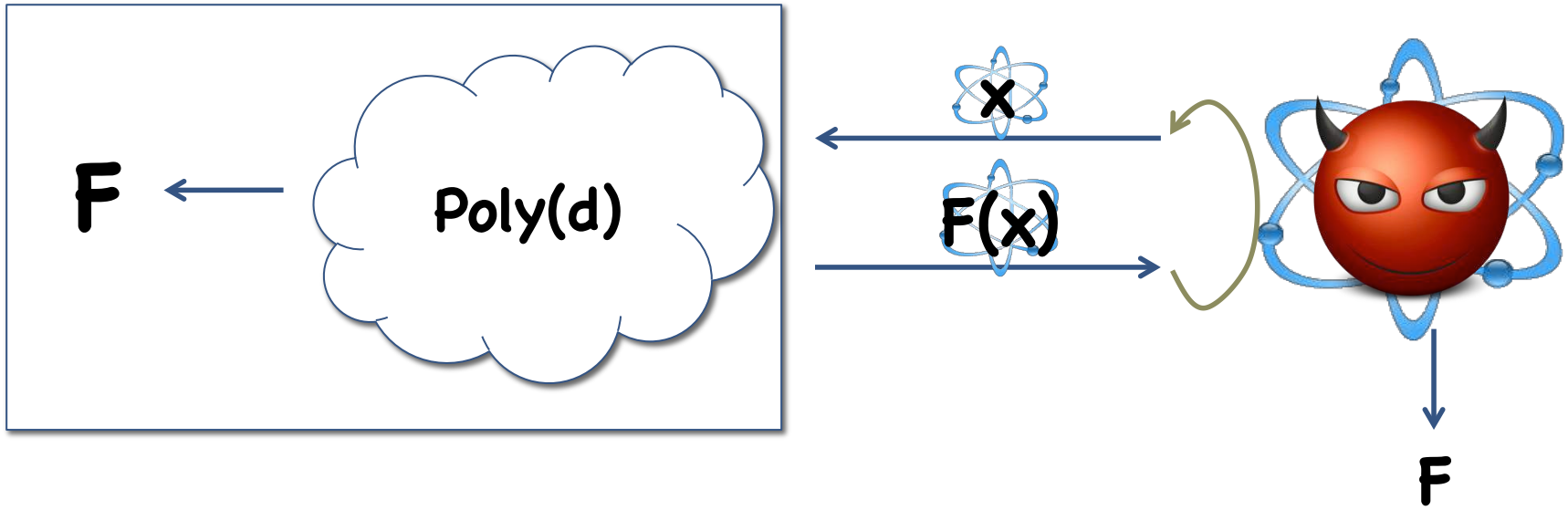
New quantum impossibility tool: **The Rank Method**

Therefore:

- Small range: $\Pr[q+1 \text{ points}]$ large
- Large range: $\Pr[q+1 \text{ points}]$ small

Quantum Polynomial Interpolation

Using **q** queries to a polynomial, determine polynomial



Classical: **q=d+1** → easy
q<d+1 → hard

Quantum: **q=d/2** → hard [KK'10]

Theorem: (quantum) **q=d** → easy

Theorem: (quantum) **q=(d+1)/2**, "large" **d** → hard

Conclusion

Studying full quantum security notions important

- Quantum computers seem inevitable
- Unclear what attacks are possible
- Strive for strongest definitions
- **Bonus:** quantum query complexity results

Future work: more advanced primitives

- Identity-based encryption
- Functional encryption
- Fully homomorphic encryption
- Other quantum query questions

Acknowledgements

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