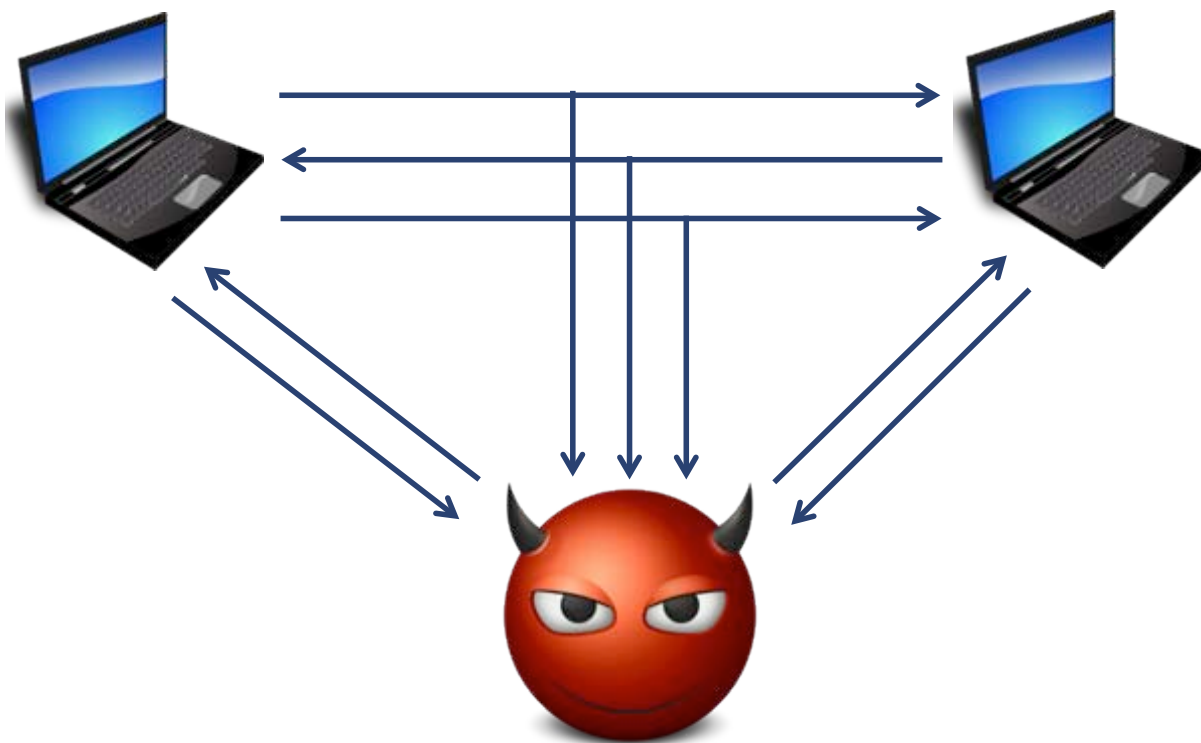


BEYOND POST-QUANTUM CRYPTOGRAPHY

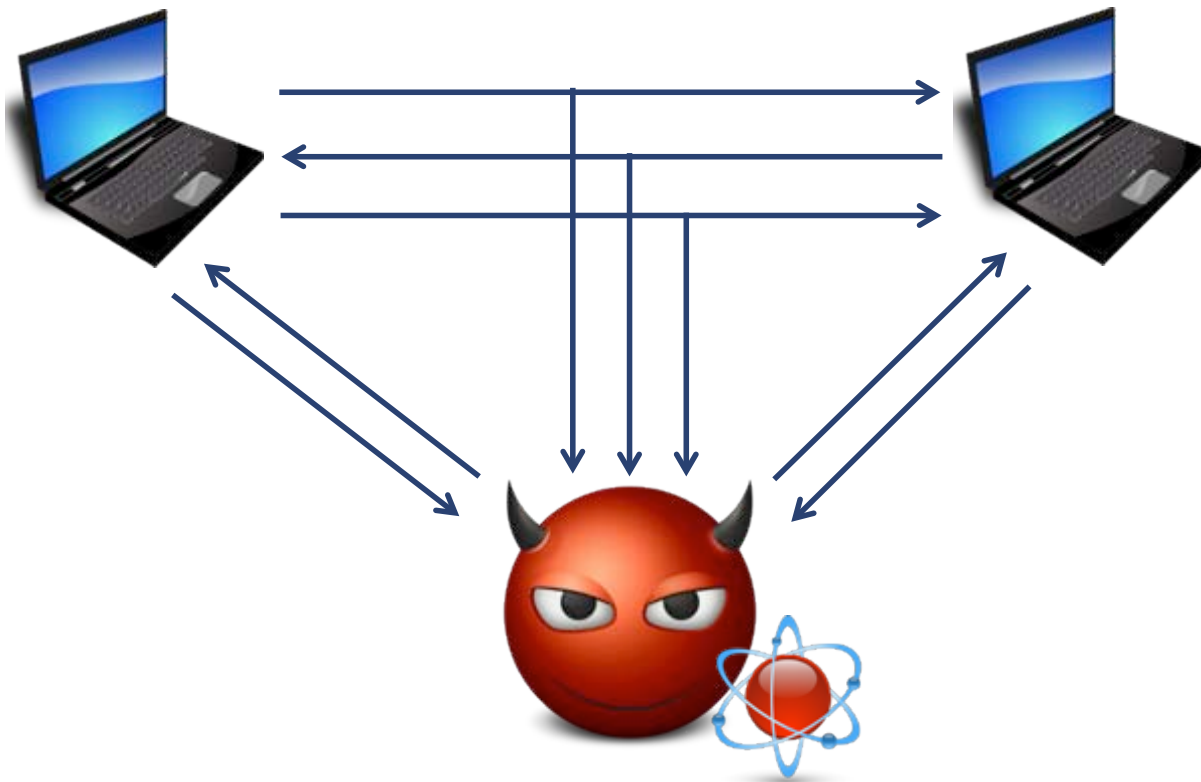
Mark Zhandry – Stanford University

Joint work with Dan Boneh

Classical Cryptography



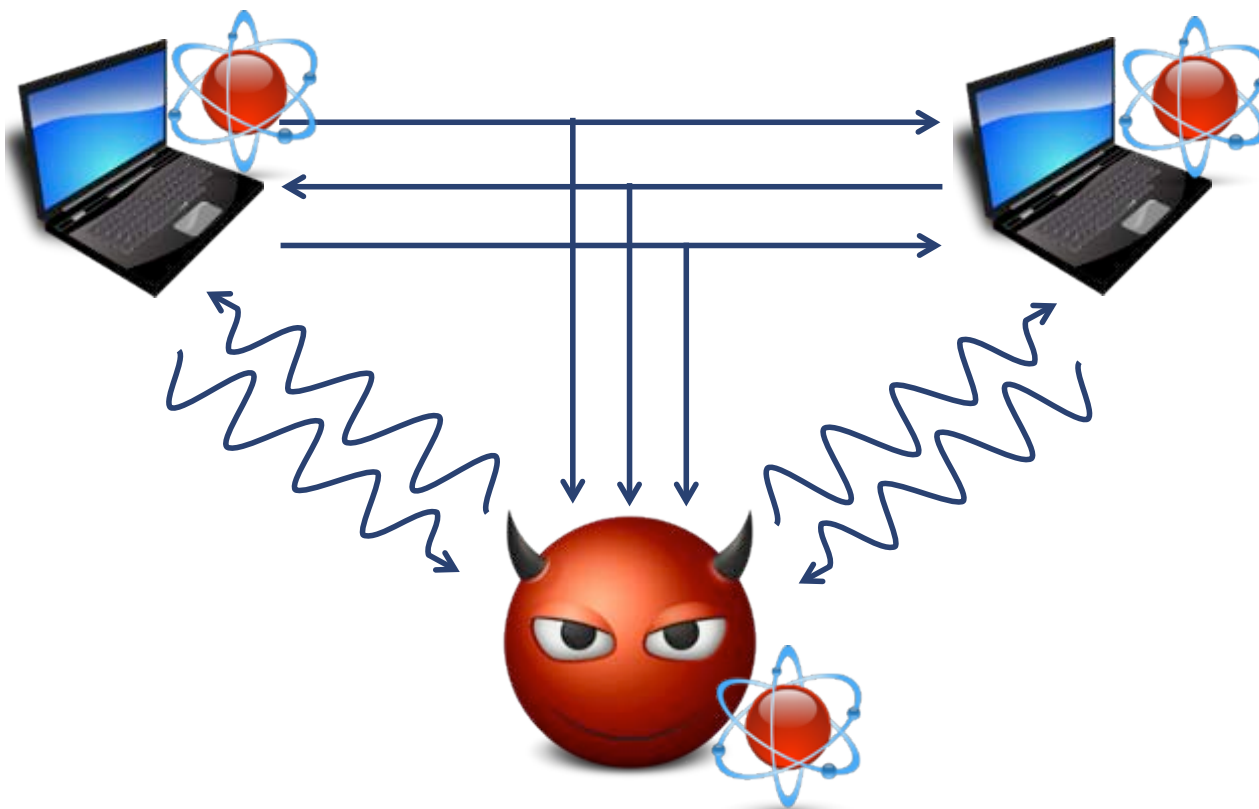
Post-Quantum Cryptography



All communication stays classical

Beyond Post-Quantum Cryptography

Eventually, all computers will be quantum

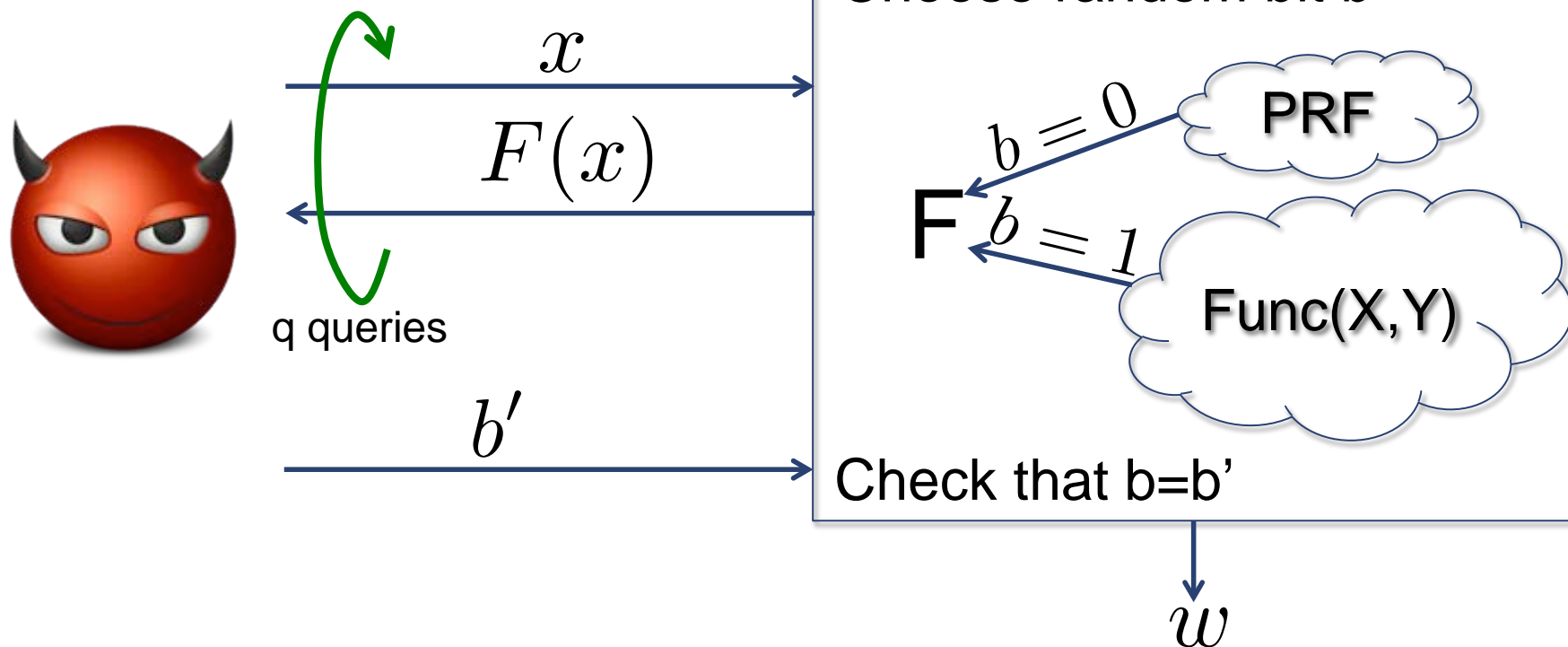


Adversary may use quantum interactions
→ need new security definitions

Example: Pseudorandom Functions

[GGM'84]

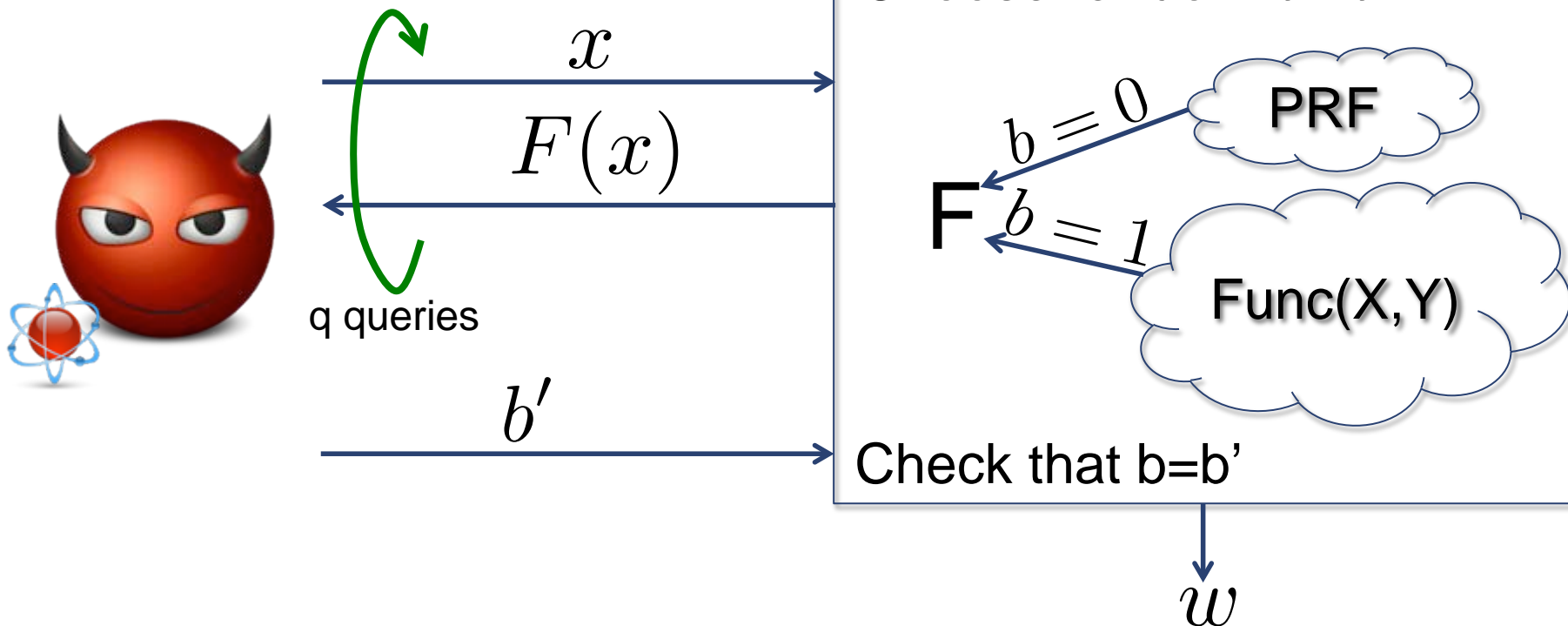
Classical security:



PRF is secure if $\left| \Pr[w = 1] - \frac{1}{2} \right| < \text{negl}$

Example: Pseudorandom Functions

Post-quantum security:

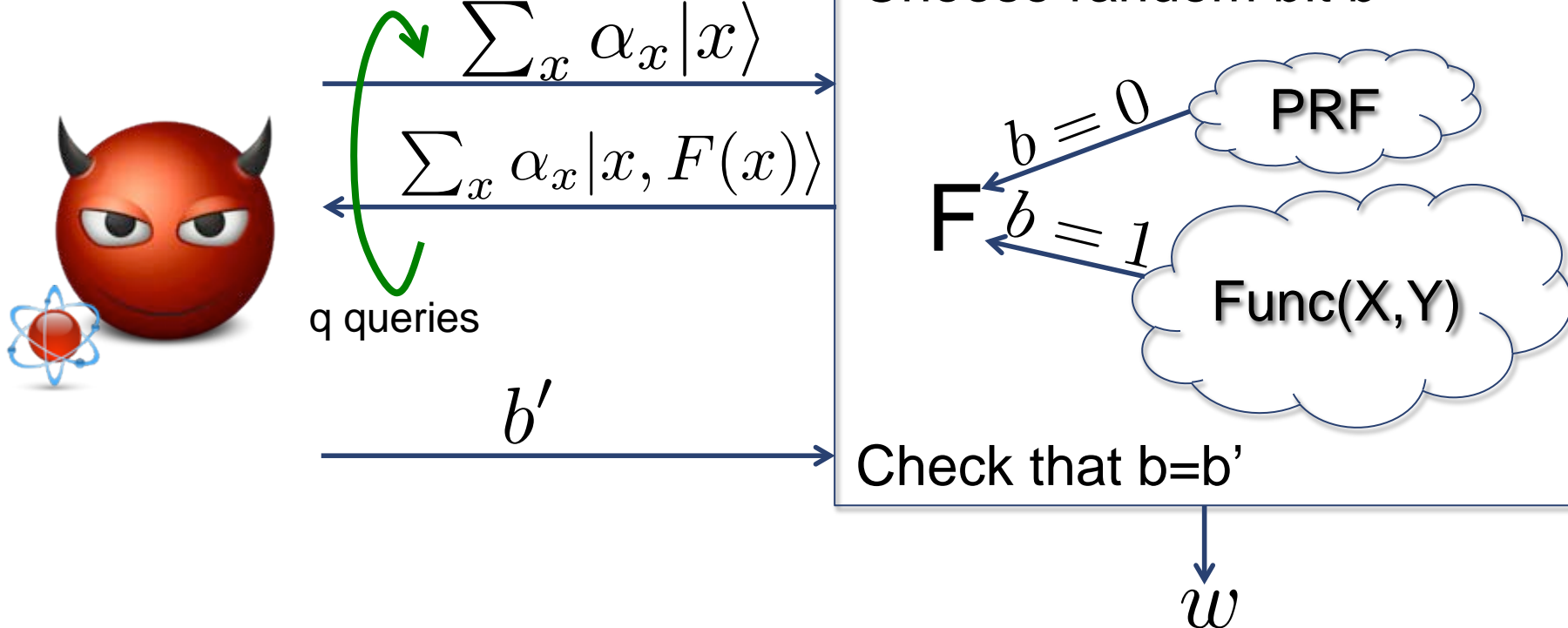


PRF is secure if $\left| \Pr[w = 1] - \frac{1}{2} \right| < \text{negl}$

Example: Pseudorandom Functions

[Aar'09]

Quantum security:



PRF is secure if $\left| \Pr[w = 1] - \frac{1}{2} \right| < \text{negl}$

Post-Quantum vs Full Quantum Security

In post-quantum setting, security games generally don't change, only adversary's computational power

→ Can often replace primitives with quantum-immune primitives and have classical proof carry through

For full quantum security, security game itself is quantum

→ Now, classical proofs often break down

→ Need new tools to prove security

Non-interactive Security Games

If no interaction, security game does not change

→ no difference between post-quantum and full quantum security

Examples:

- One-way functions
- Pseudorandom generators
- Collision-resistant hash functions

In these cases, classical proofs often do carry through

- Example:

quantum-secure OWFs → quantum-secure PRGs

This Talk

A First Step: The Quantum Random Oracle Model

[BDFLS^Z'11, ^{Zha}'12a]

Full Quantum Security:

- Quantum-secure PRFs (or quantum PRFs) [^{Zha}'12b]
- Quantum-secure MACs [B^Z'12]
- Quantum-secure Signatures and Encryption [B^Z'13]

Quantum Random Oracle Model

Quantum Random Oracle Model

[BDFLS^Z'11]

A first step towards full quantum security

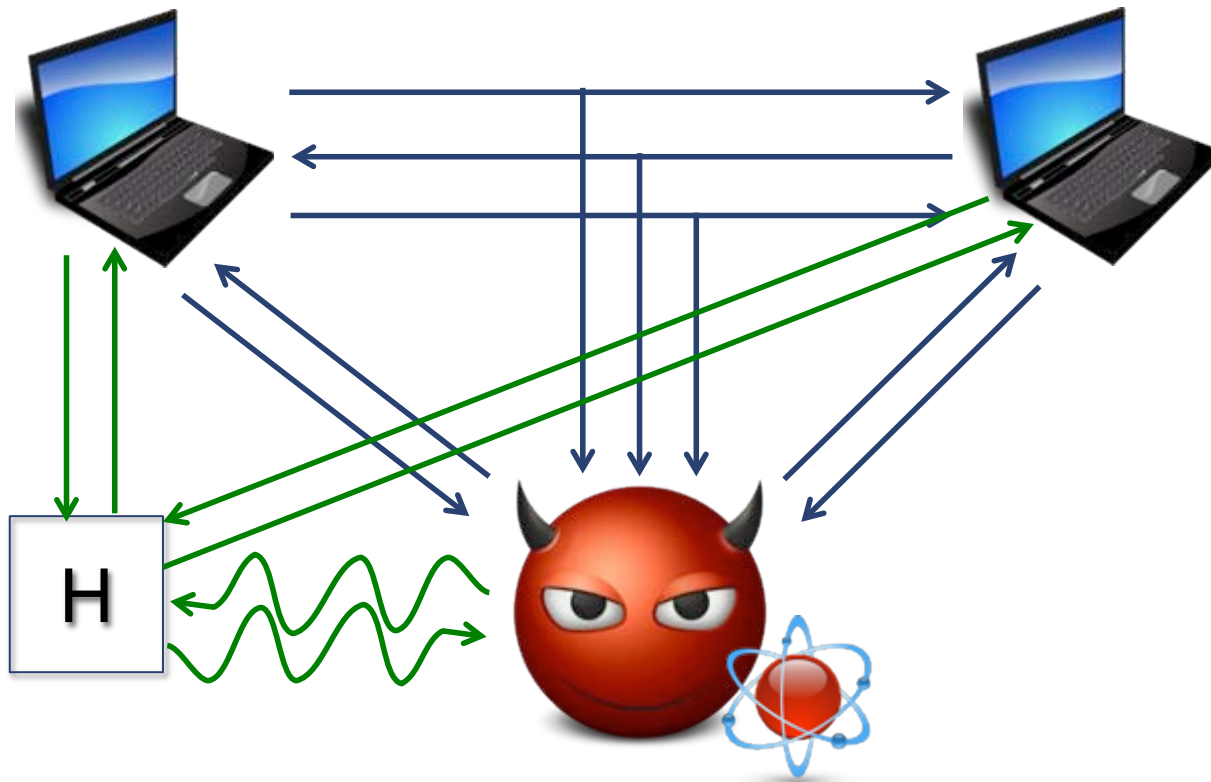
Honest parties still classical (i.e. post-quantum world)

Model hash function as a random oracle that accepts quantum queries

- Captures ability of adversary to evaluate hash function on superposition of inputs

All other interaction remains classical

Quantum Random Oracle Model



Quantum Random Oracle Model

Proven secure [BDFLS^Z'11, ^{Zha}'12a]

- Several signature schemes (inc. GPV)
- CPA-secure encryption
- GPV identity-based encryption

Not yet proven

- Signatures from identification protocols (Fiat-Shamir)
- CCA Encryption from weaker notions

Full Quantum Security

Quantum-secure PRFs:

- PRFs: building block for most of symmetric crypto
 - PRPs (e.g. Luby-Rankoff), encryption schemes, MACs

Quantum-secure MACs:

- PRF \rightarrow MAC
- Natural question: quantum PRF \rightarrow quantum-secure MAC?

Quantum-secure Signatures and Encryption

- From generic assumptions?
- Security of schemes in the literature?

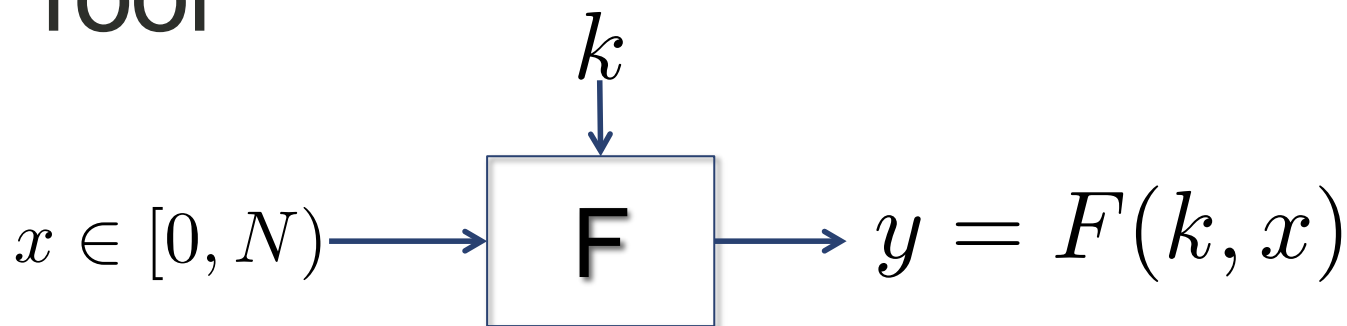
Quantum PRFs [Zha'12b]

Separation

PRF  Quantum PRF

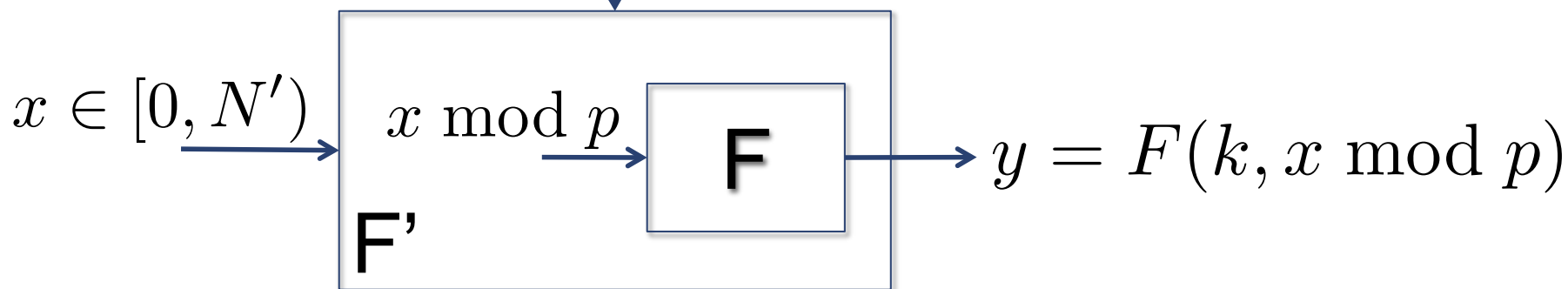
Theorem: If post-quantum PRFs exist, then there are post-quantum PRFs that are not quantum PRFs

Proof



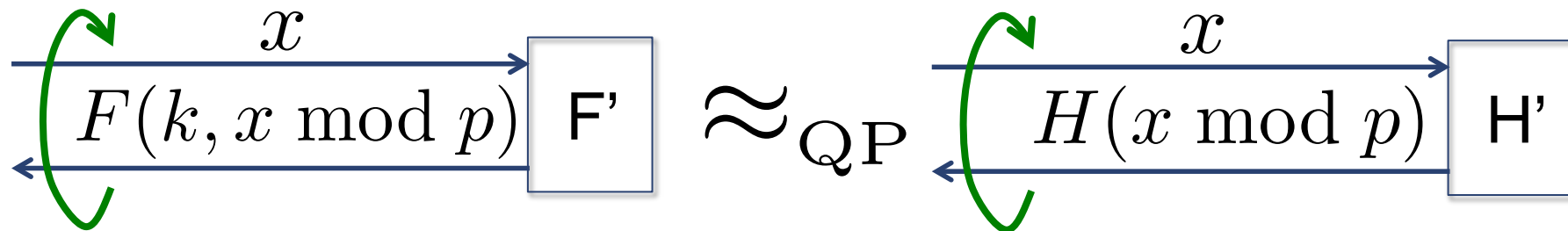
A large blue arrow points down to the following expression:

$$(k, p) \quad p \in \left[\frac{N}{2}, N \right), \text{ prime}$$

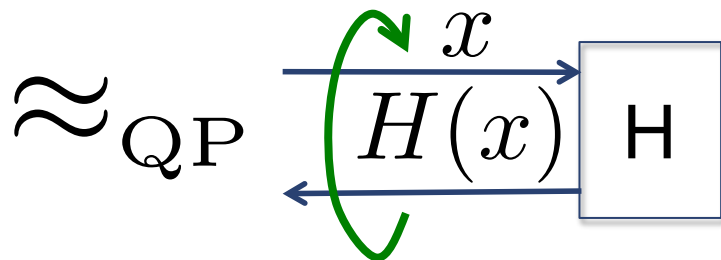


Proof

Lemma 1: If F is post-quantum secure, then so is F' .



As long as $x \bmod p \neq x' \bmod p$ for all queries $x \neq x'$, this looks like a random oracle



Probability this fails: $O(q^2(\log N)/N)$

Proof

Lemma 2: Either F or F' are not quantum secure.

$$F'(x+p) = F'(x) \quad \longleftarrow \quad \text{Periodic!}$$

Quantum queries can find p [BL'95]

Once we know p , easy to distinguish F' from random

How to Construct Quantum PRFs

Hope that classical PRFs work in quantum world:

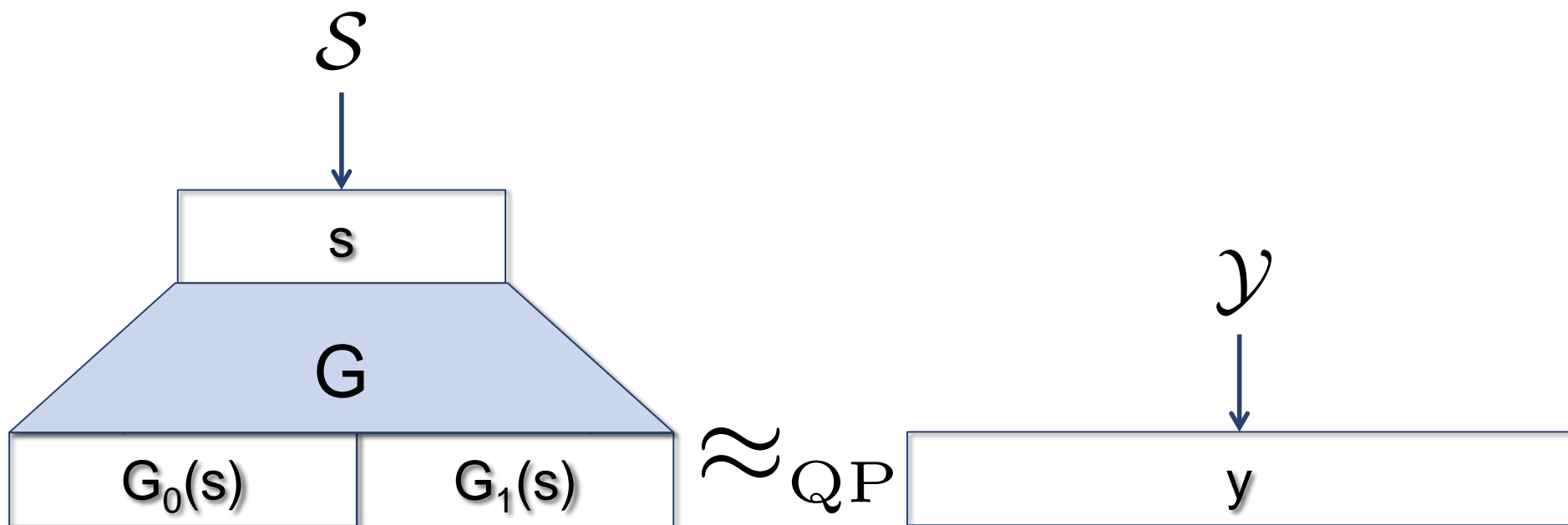
- From quantum-secure pseudorandom generators [GGM'84]
- From quantum-secure pseudorandom synthesizers [NR'95]
- Directly from lattices [BPR'11]

Classical proofs do not carry over into the quantum setting

→ Need new proof techniques

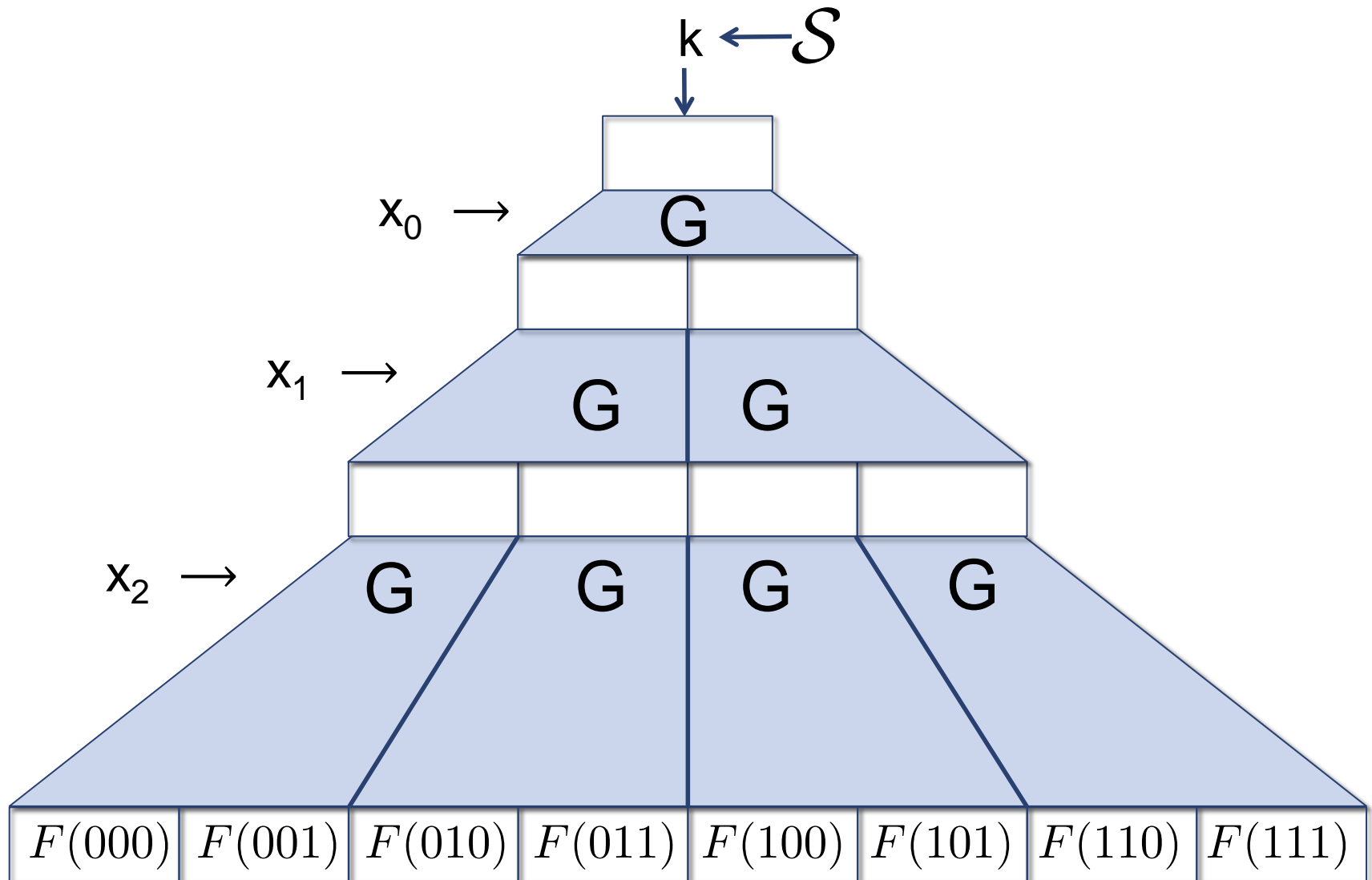
Example: GGM

Pseudorandom Generators



Indistinguishable for Quantum Machines

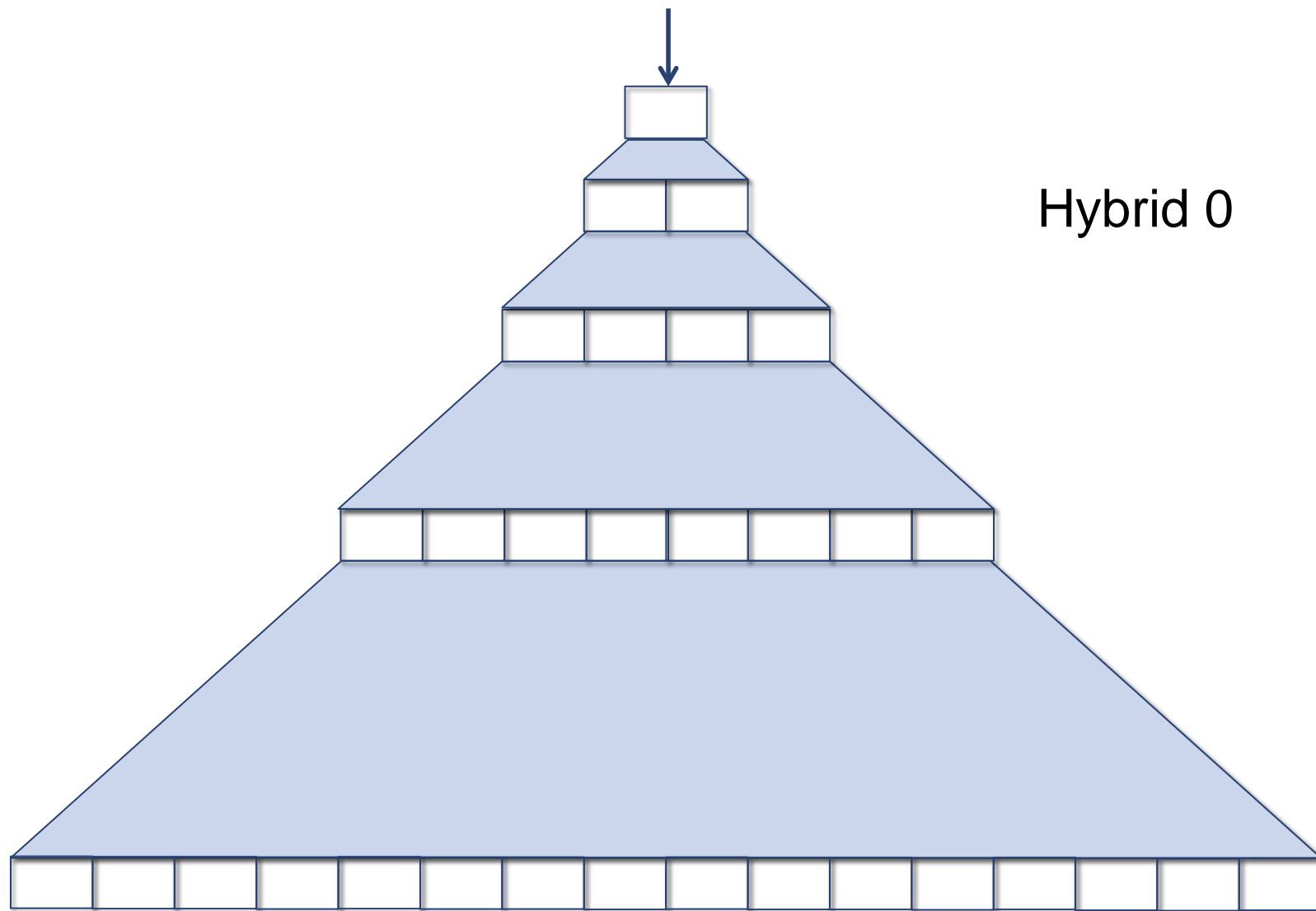
The GGM Construction



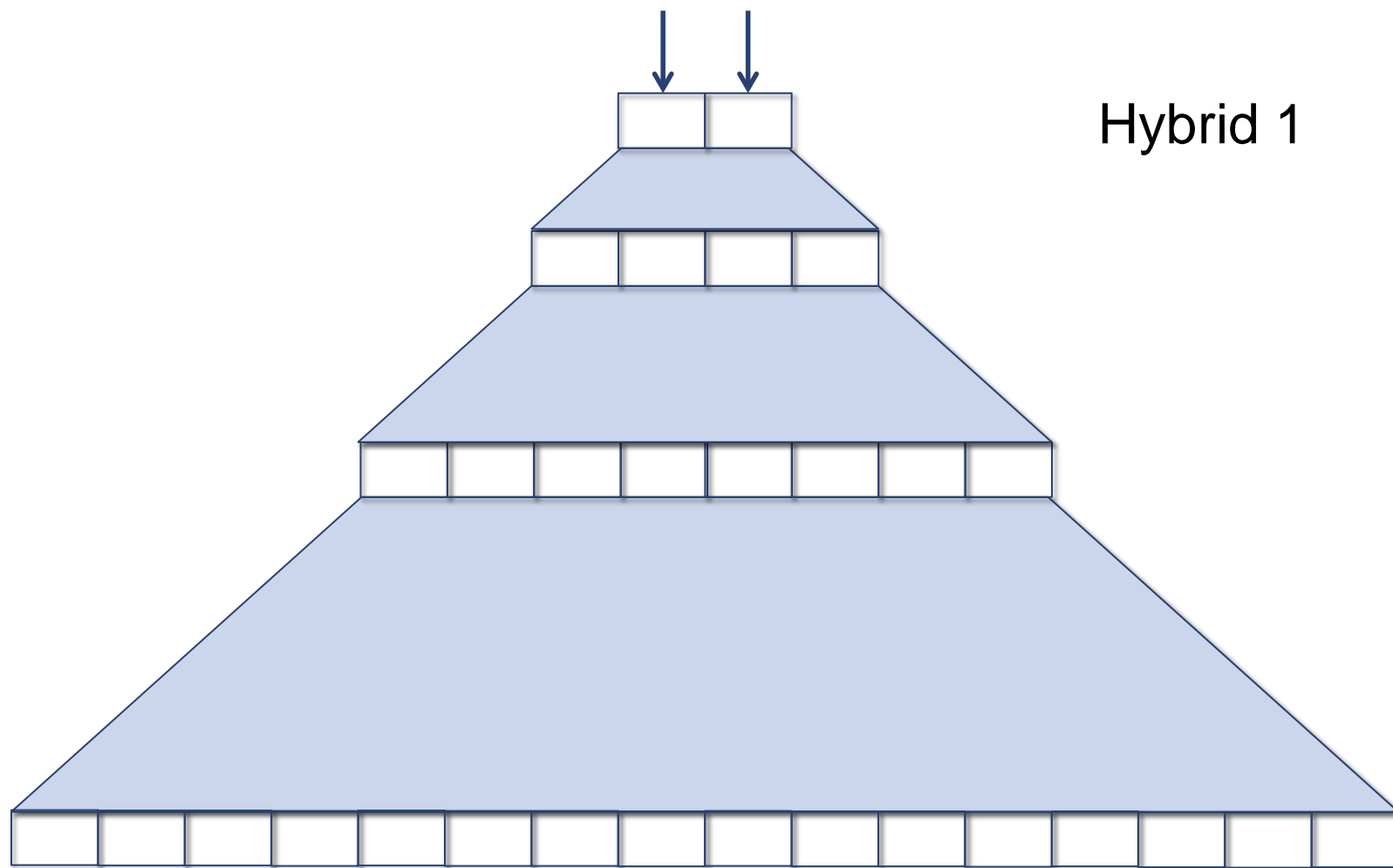
Original Security Proof

Step 1: Hybridize over levels of tree

Original Security Proof: Step 1

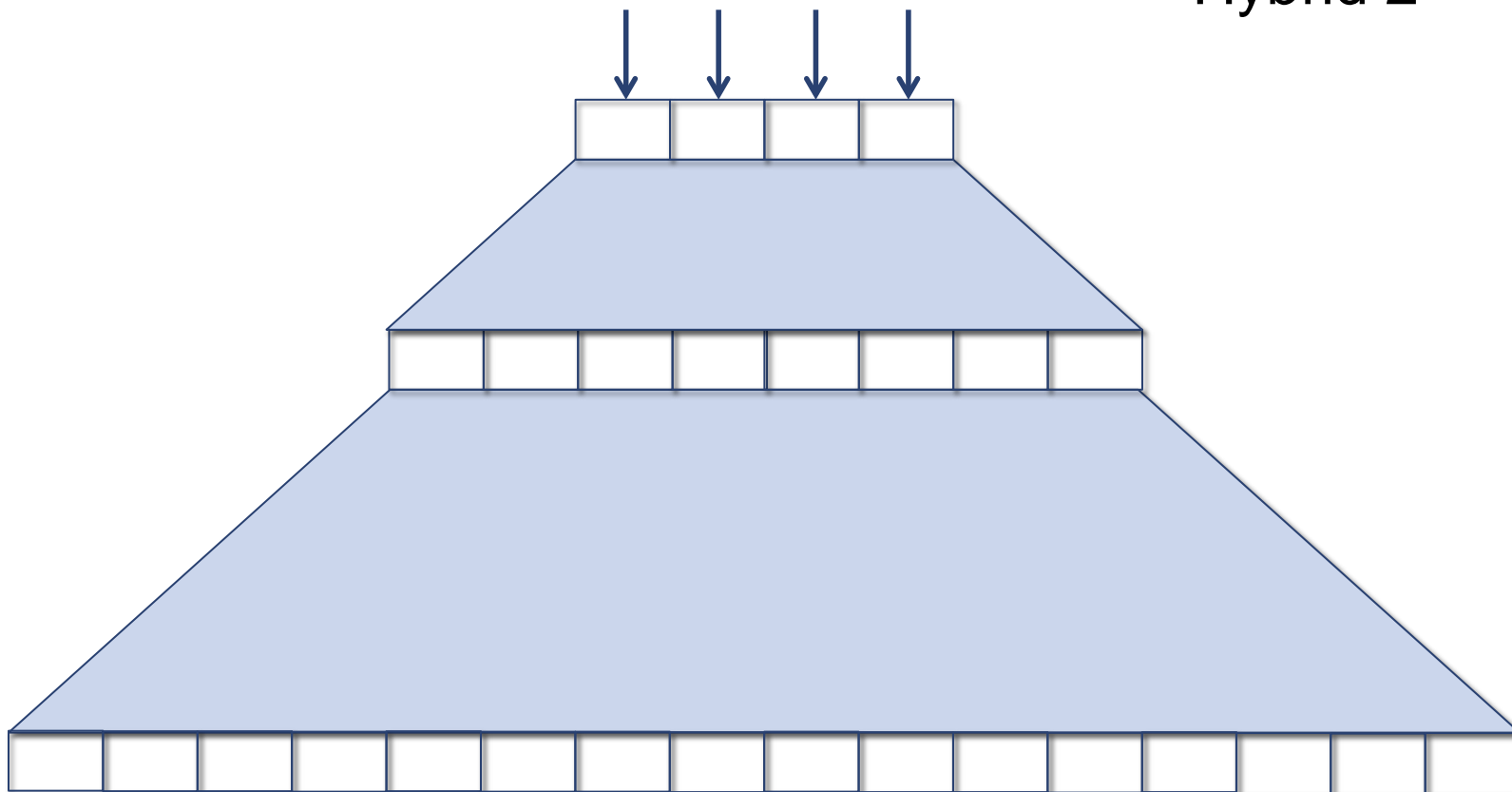


Original Security Proof: Step 1



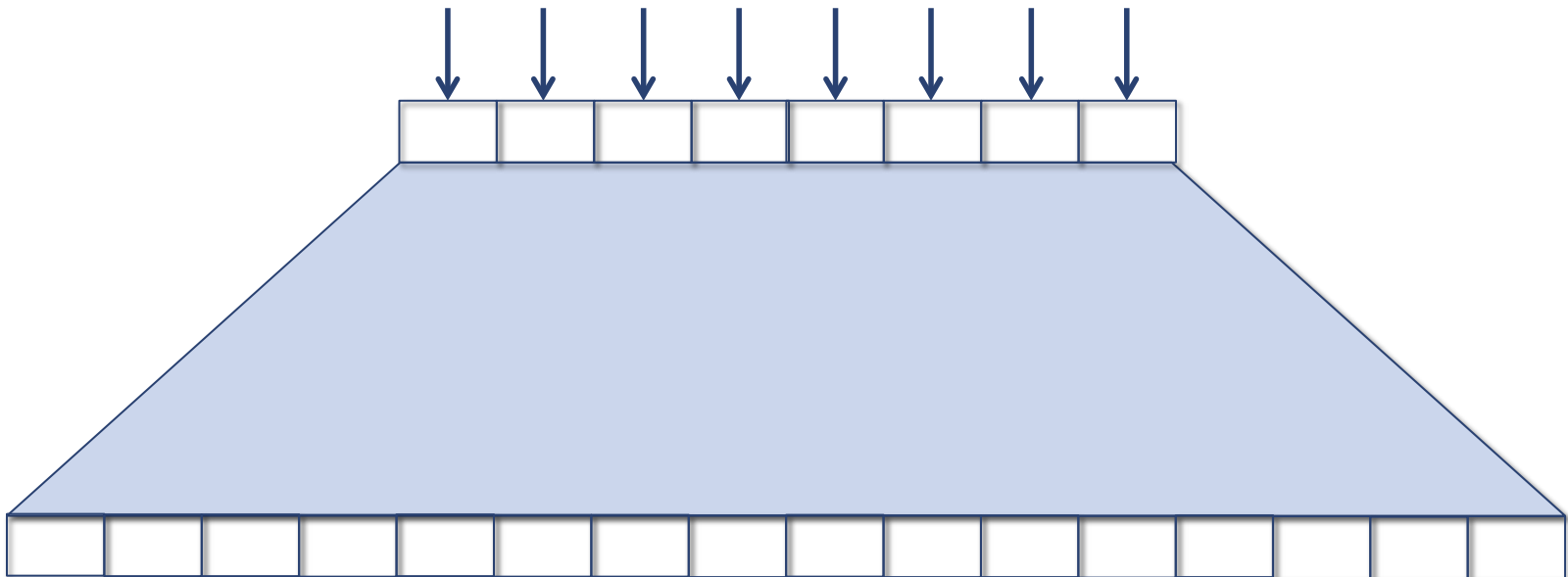
Original Security Proof: Step 1

Hybrid 2



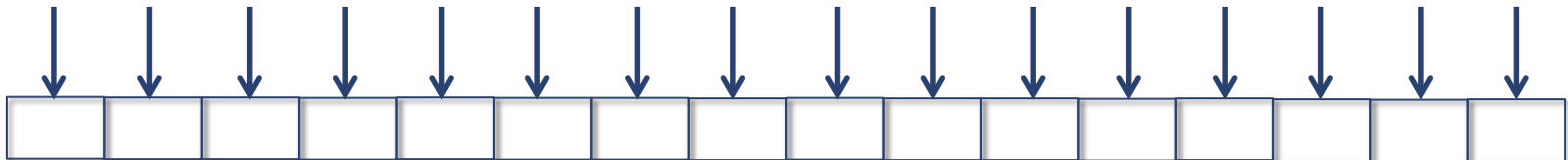
Original Security Proof: Step 1

Hybrid 3



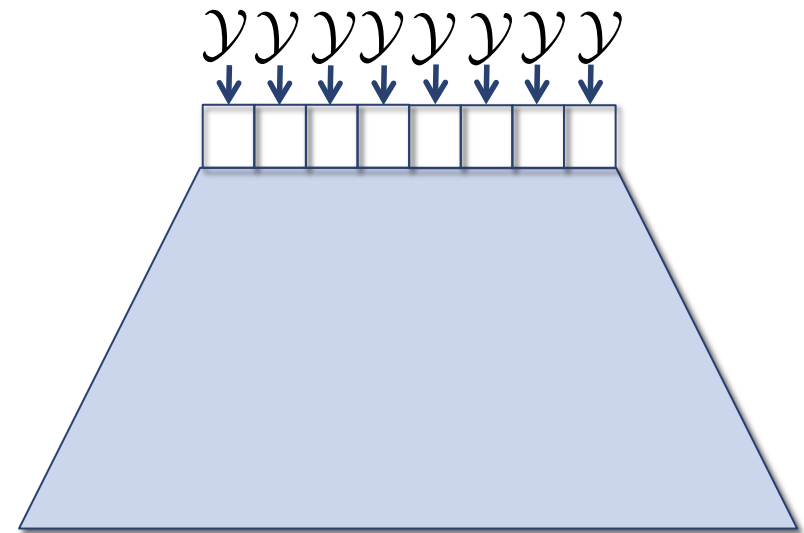
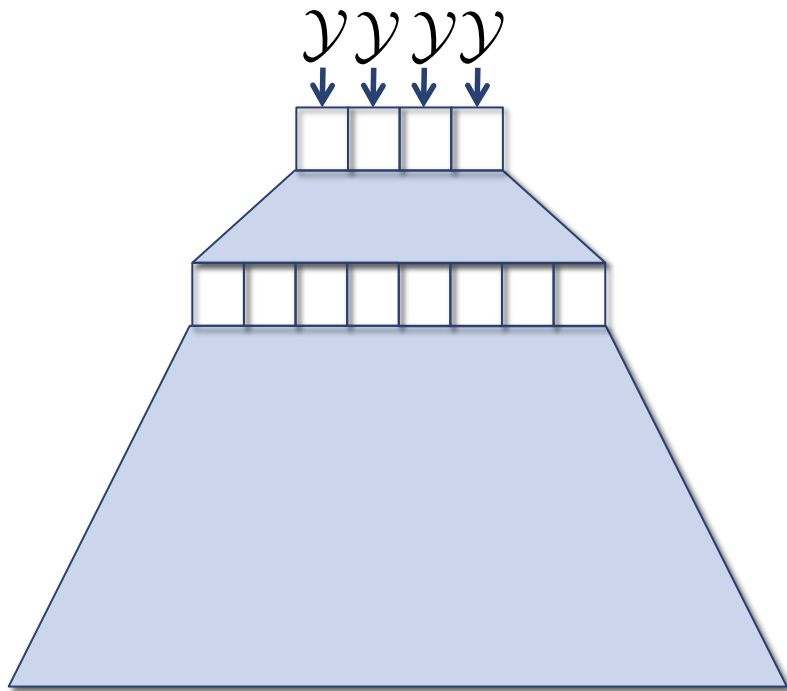
Original Security Proof: Step 1

Hybrid n



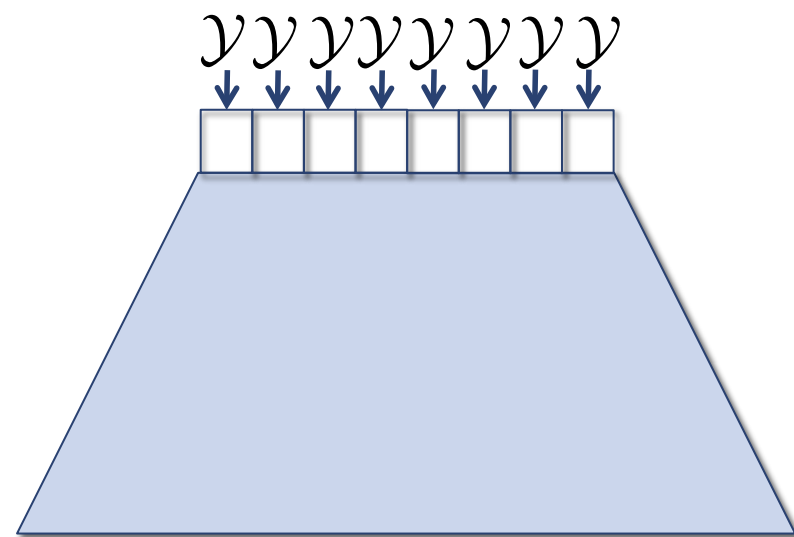
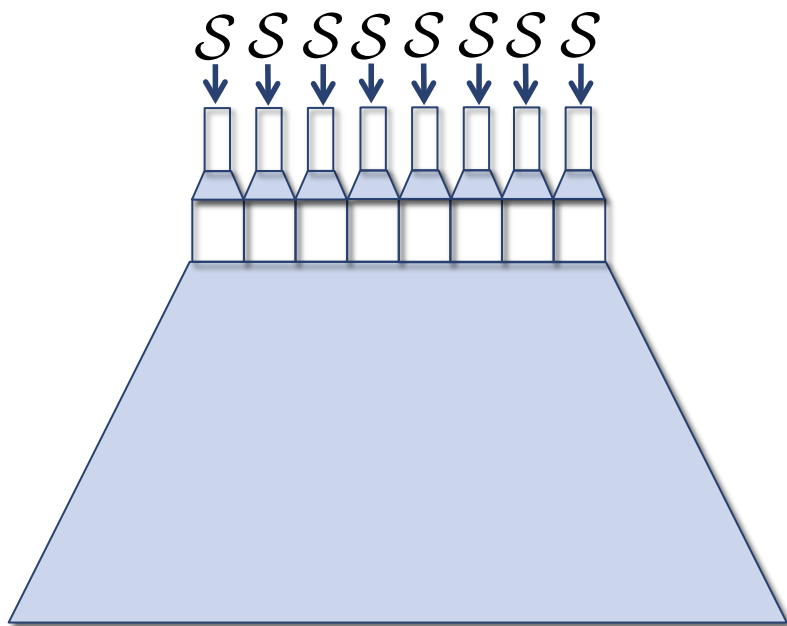
Original Security Proof: Step 1

PRF distinguisher will distinguish two adjacent hybrids



Original Security Proof: Step 1

PRF distinguisher will distinguish two adjacent hybrids



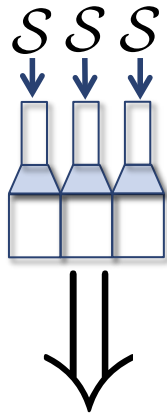
Original Security Proof

Step 1: Hybridize over levels of tree

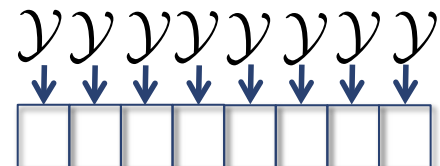
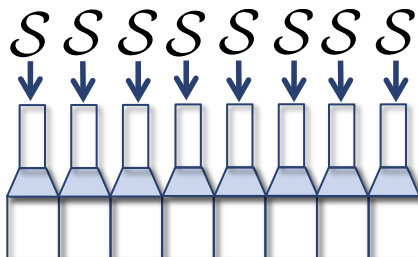
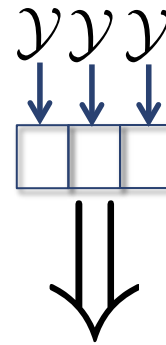


Step 2: Simulate hybrids using q samples

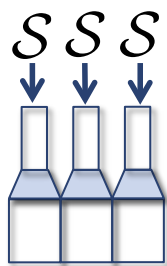
Original Security Proof: Step 2



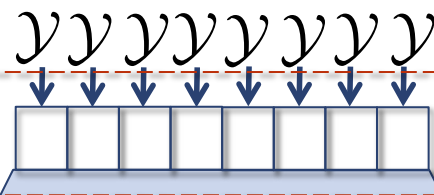
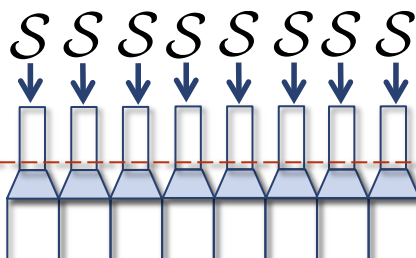
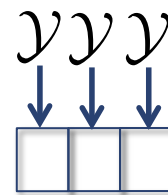
Simulate



Original Security Proof: Step 2



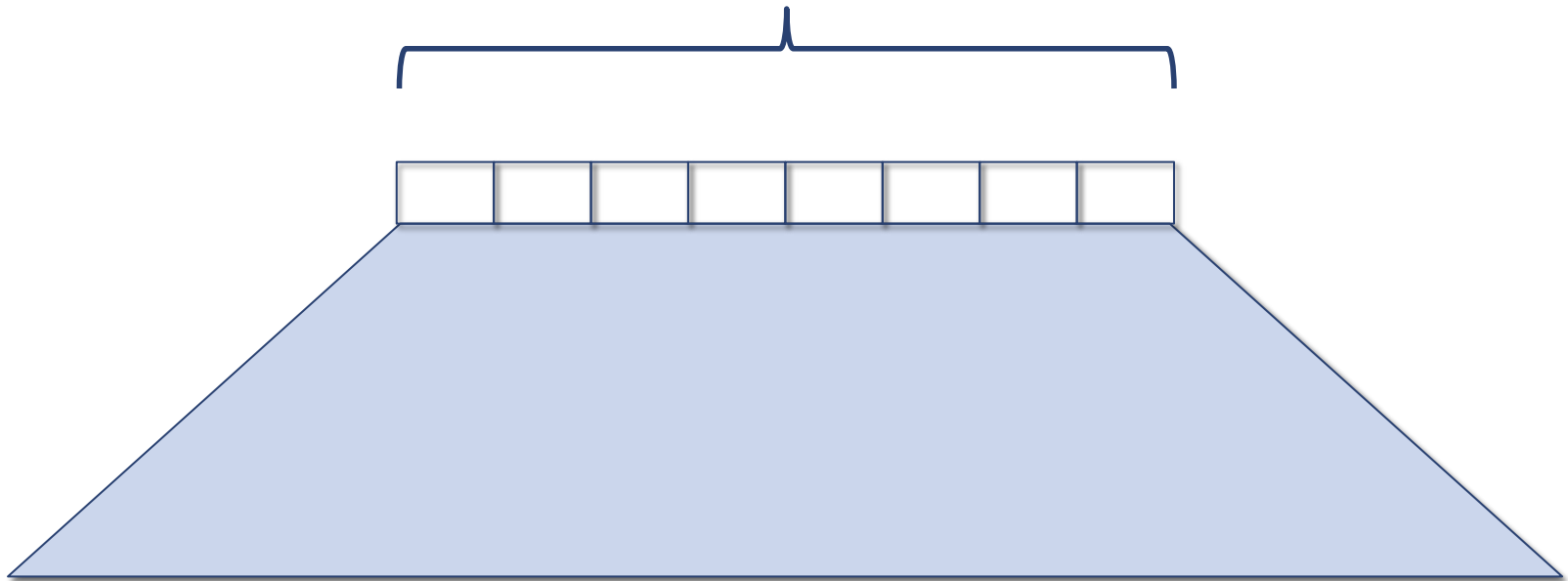
Simulate



Put samples here

Original Security Proof: Step 2

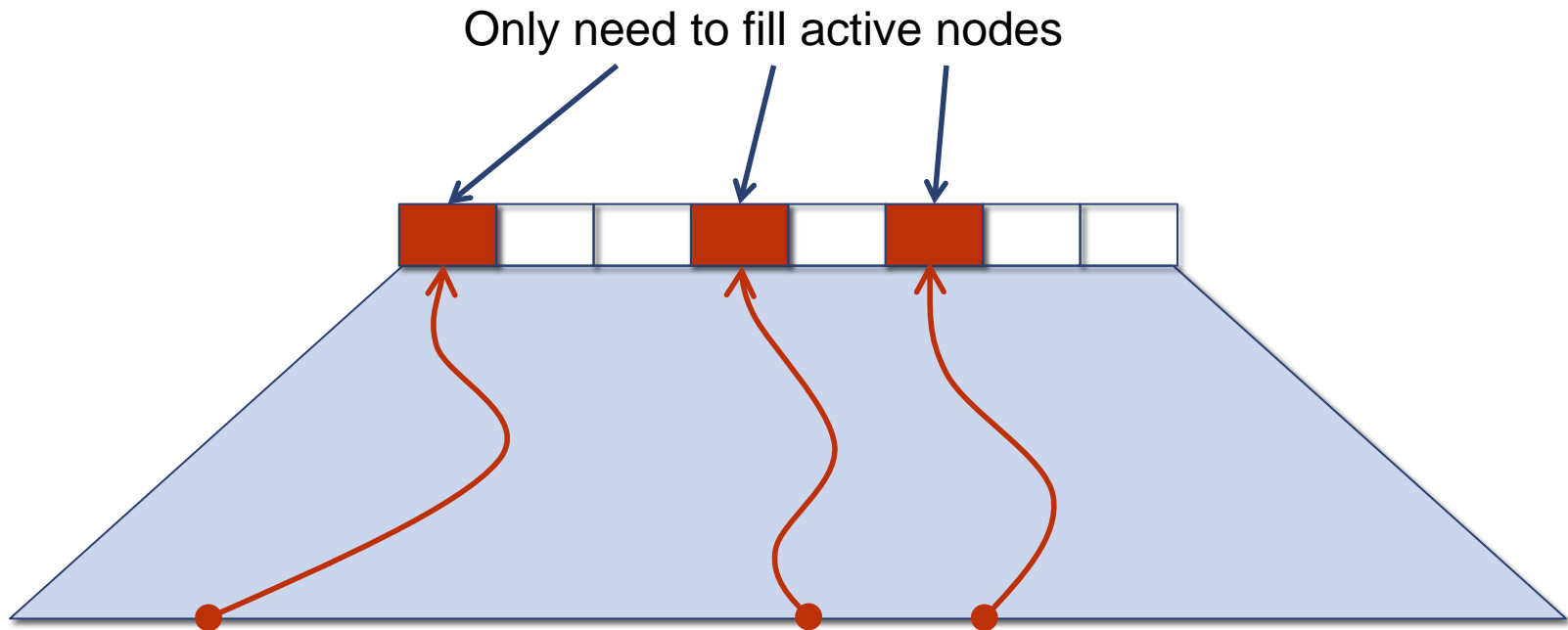
Rows are exponentially wide



Problem?

Original Security Proof: Step 2

Active node: value used to answer query



Adversary only queries polynomial number of points

Original Security Proof

Step 1: Hybridize over levels of tree

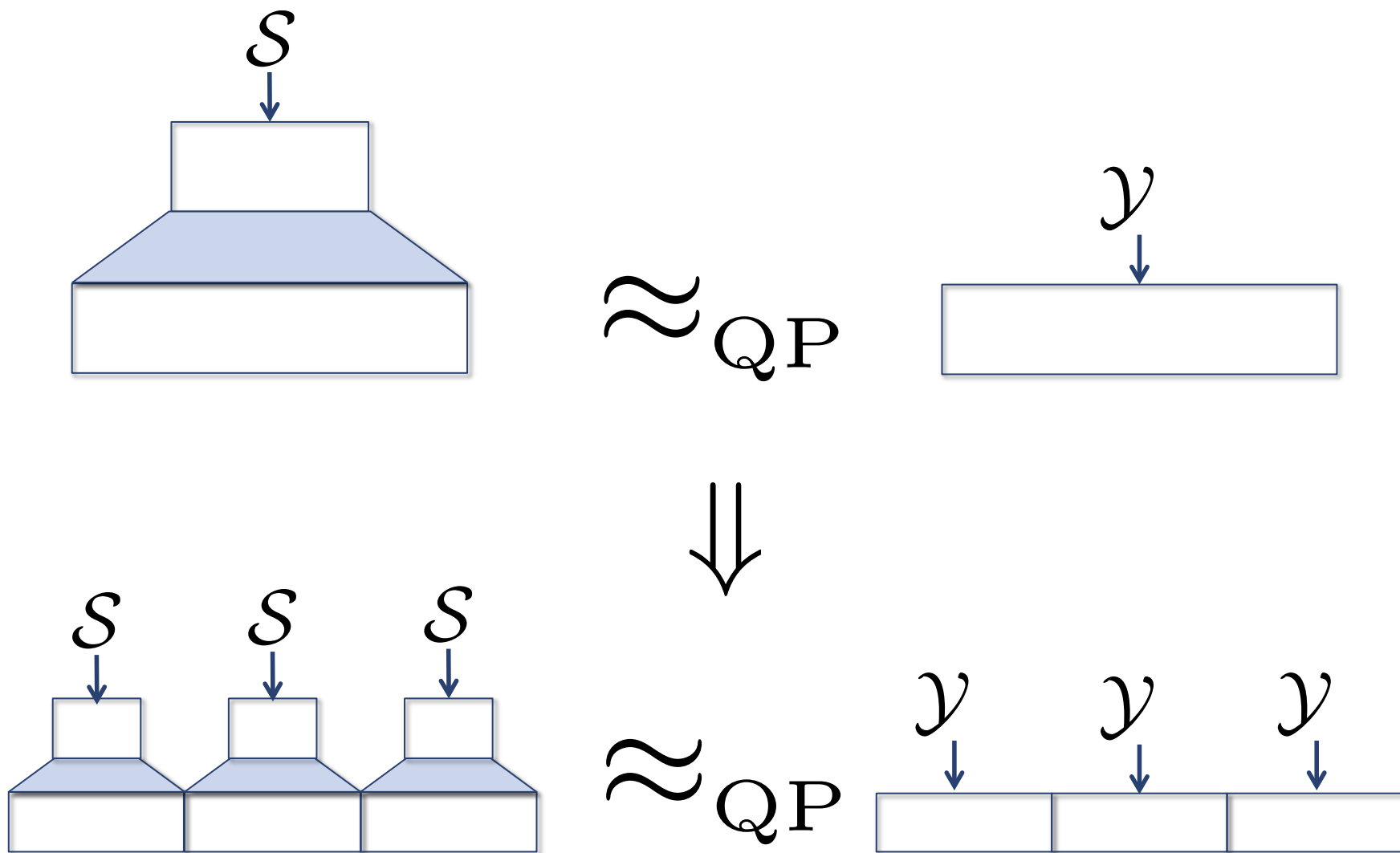


Step 2: Simulate hybrids using q samples



Step 3: Pseudorandomness of one PRG sample
implies pseudorandomness of q samples

Original Security Proof: Step 3



Original Security Proof

Step 1: Hybridize over levels of tree



Step 2: Simulate hybrids using q samples



Step 3: Pseudorandomness of one PRG sample
implies pseudorandomness of q samples



Quantum Security Proof Attempt

Step 1: Hybridize over levels of tree



Step 2: Simulate hybrids using q samples



Step 3: Quantum pseudorandomness of one PRG sample implies quantum pseudorandomness of q samples

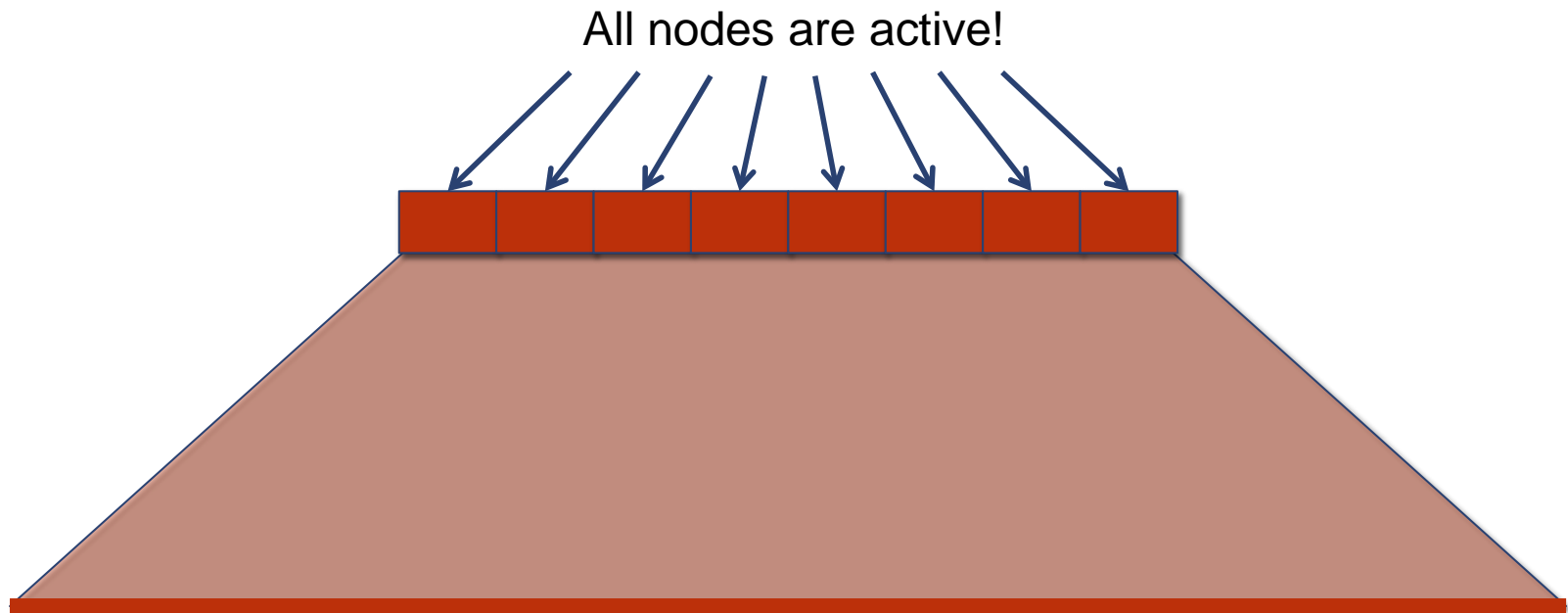


Difficulty Simulating Hybrids



Adversary can query on all exponentially-many inputs

Difficulty Simulating Hybrids



Exact simulation requires exponentially-many samples

Need new simulation technique

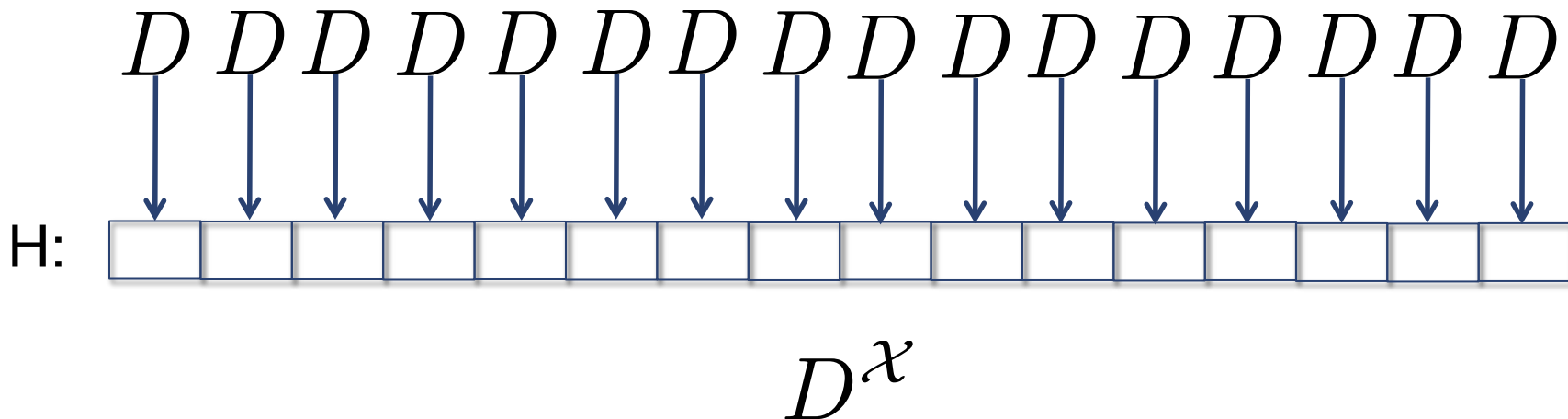
A Distribution to Simulate

Any distribution D on values induces a distribution on functions

For all $x \in \mathcal{X}$

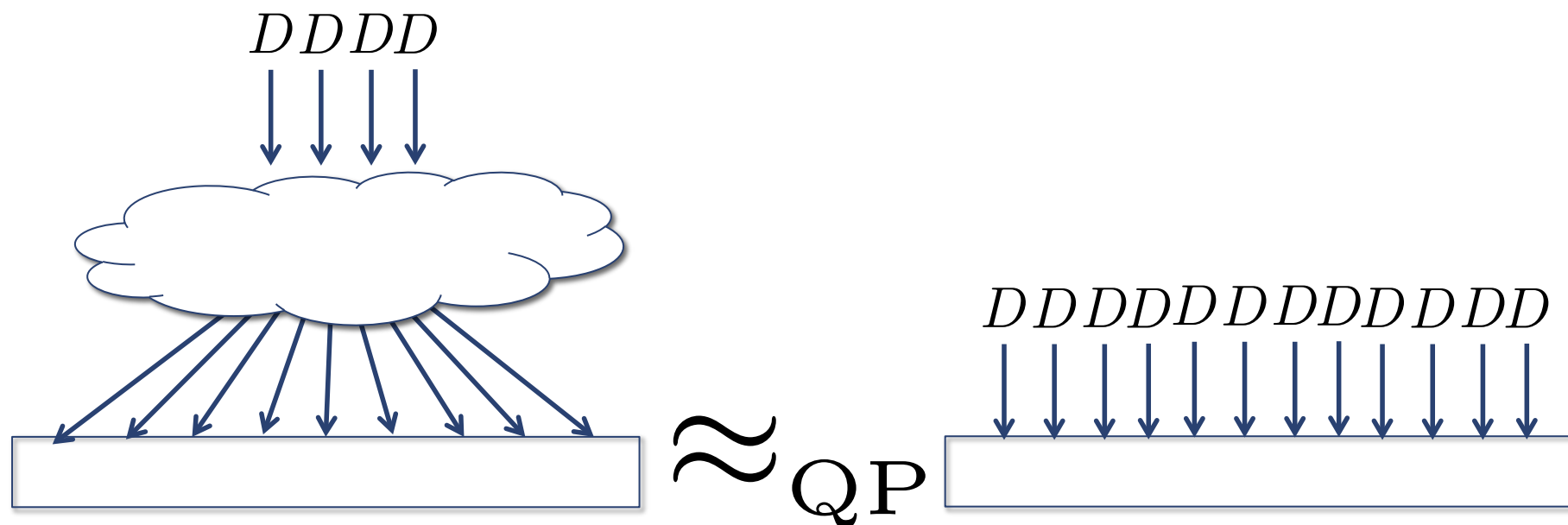
$$y_x \leftarrow D$$

$$H(x) = y_x$$

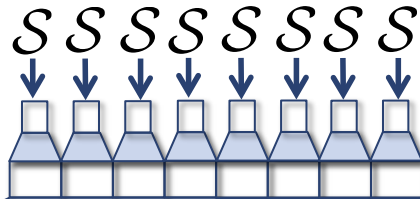


A Distribution to Simulate

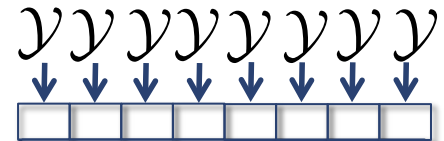
Suppose we could simulate D^X approximately using a polynomial number of samples from D :



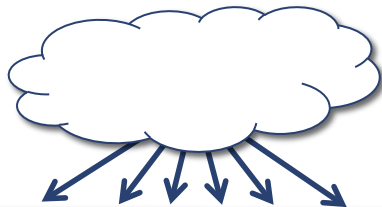
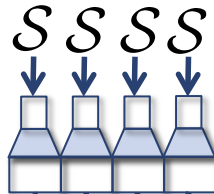
Fixing the GGM Proof



PRF distinguisher will distinguish two adjacent hybrids

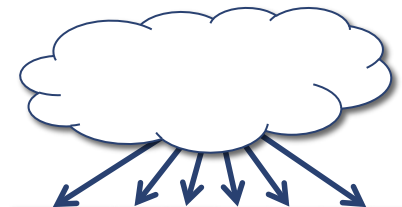
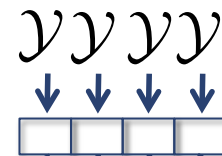


\approx_{QP}



\approx_{QP}

\approx_{QP}



Quantum Security Proof

Step 1: Hybridize over levels of tree



Step 2: Simulate hybrids **approximately** using **polynomially-many** samples



Step 3: Quantum pseudorandomness of one sample implies quantum pseudorandomness of **polynomially-many** samples



We have r samples:

-
- A diagram illustrating a process where four 'D' characters are input into a system. Above a light blue rectangular box, the letter 'D' is repeated four times. Below each 'D', a blue arrow points downwards into the box.

Diagram illustrating a 1D lattice system with 28 sites. The top row shows the initial state with 14 'D' particles (dark blue) and 14 empty sites (light blue). The bottom row shows the final state after 100 steps, with 14 'D' particles and 14 empty sites. Arrows indicate the movement of particles from the top row to the bottom row.

New Tool: Small Range Distributions

For each $i \in [1, r]$

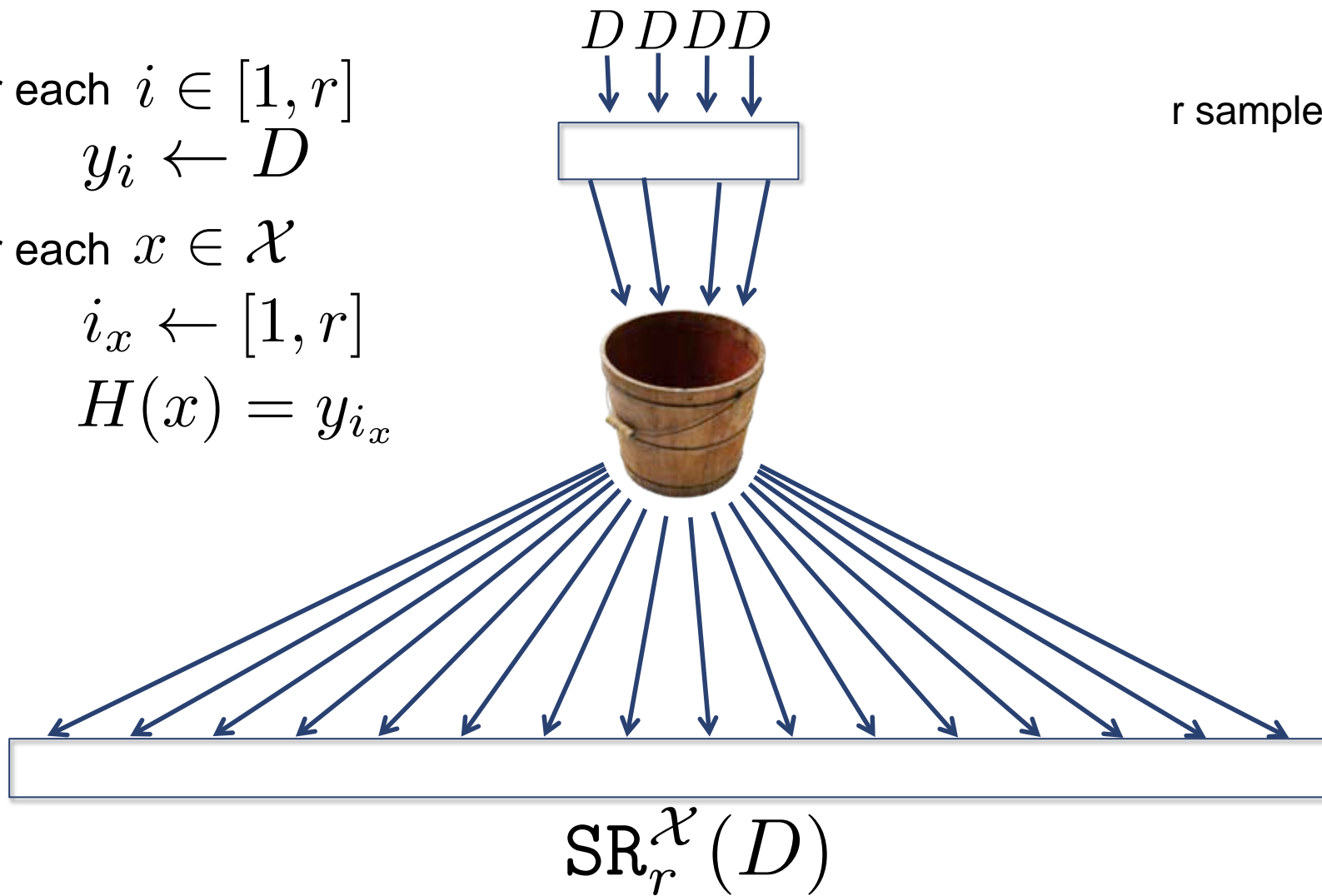
$$y_i \leftarrow D$$

For each $x \in \mathcal{X}$

$$i_x \leftarrow [1, r]$$

$$H(x) = y_{i_x}$$

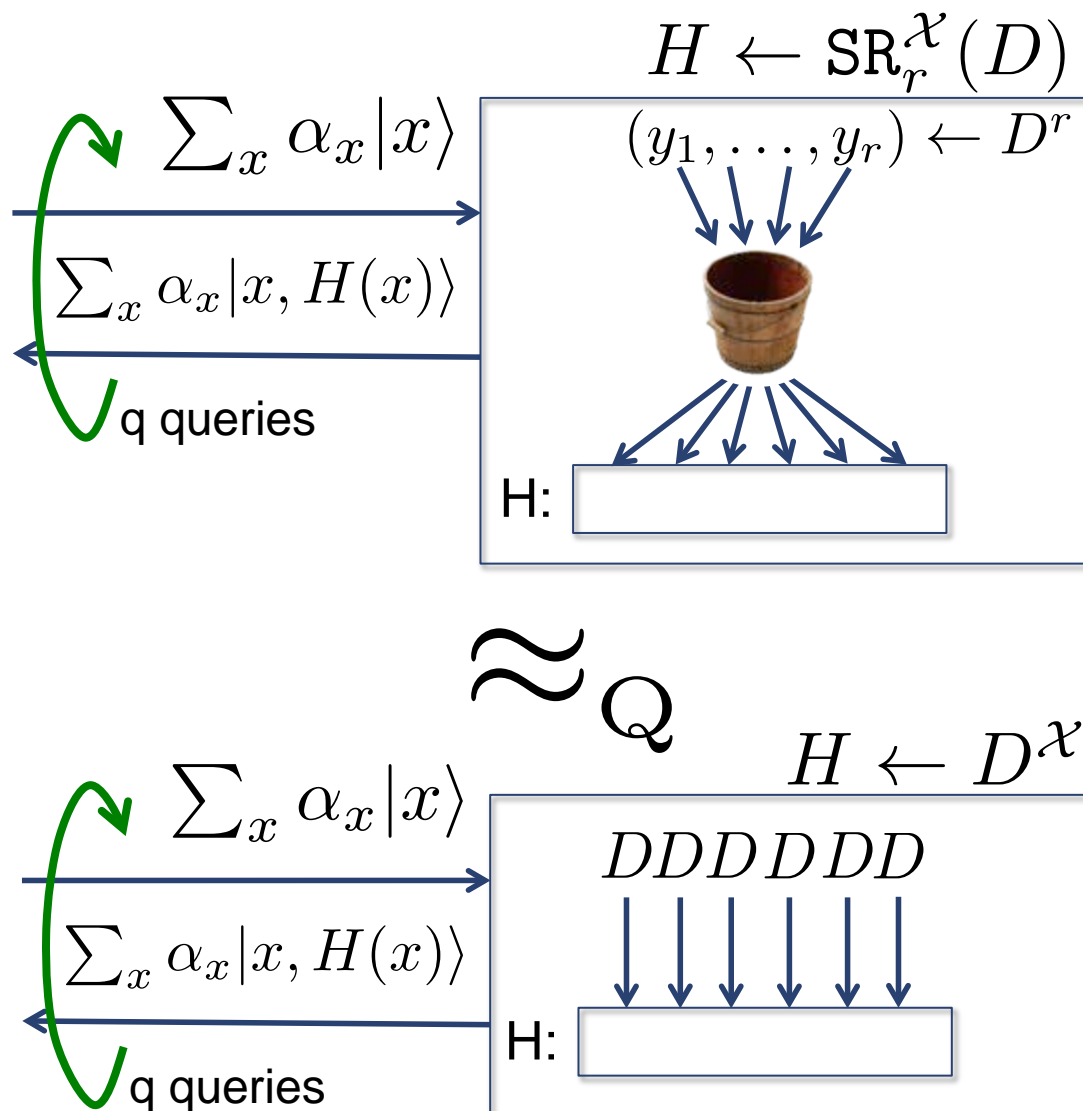
r samples of D



Technical Theorem

Theorem: $\text{SR}_r^{\mathcal{X}}(D)$ is indistinguishable from $D^{\mathcal{X}}$ by any q -query quantum algorithm, except with probability $O(q^3/r)$

Not negligible, but good enough for our purposes



Proving the Technical Theorem

Let $p(1/r) = \Pr[A^{SR_r^{\mathcal{X}}(D)}() = 1]$

Observation: $SR_{\infty}^{\mathcal{X}}(D) = D^{\mathcal{X}}$

Goal: bound $|p(1/r) - p(0)|$

First, we'll need

Lemma: If A makes q quantum queries, then p is a polynomial in $1/r$ of degree at most $2q$

What does this buy us?

Polynomials!

Let $\lambda \in [0, 1]$ parameterize a family of oracle distributions E_λ

Let A be an oracle algorithm, $p(\lambda) = \Pr[A^{E_\lambda}() = 1]$

$$0 \leq p(\lambda) \leq 1 \forall \lambda \in [0, 1]$$

What if $p(\lambda)$ is a polynomial of degree d ?

Markov inequality:

$$|p'(\lambda)| \leq d^2 \forall \lambda \in [0, 1]$$

Therefore, $|p(\lambda) - p(0)| \leq d^2 \lambda$

Proving the Technical Theorem

Idea: let $E_\lambda = \text{SR}_{1/\lambda}^x(D)$

→ $p(\lambda)$ has degree $2q$

$$\begin{aligned} & \left| \Pr[A^{\text{SR}_r^x(D)}() = 1] - \Pr[A^{D^x}() = 1] \right| \\ &= \left| \Pr[A^{E_{1/r}}() = 1] - \Pr[A^{E_0}() = 1] \right| \\ &= |p(1/r) - p(0)| \leq (2q)^2 / r \quad ? \end{aligned}$$

Problem: E_λ only a distribution for $\lambda = 1/r$ (integer r)

→ $0 \leq p(\lambda) \leq 1$ only for $\lambda = 1/r$

→ Need replacement for Markov inequality

Replacement for Markov Inequality

Lemma: If $0 \leq p(1/r) \leq 1 \forall r \in \mathbb{Z}^+$
and p is a degree- d polynomial in $1/r$, then

$$|p(\lambda) - p(0)| < (\pi^2/6)d^3\lambda$$

for all λ in $[0,1]$

Proving the Technical Theorem

If $p(1/r) = \Pr[A^{SR_r^x(D)}() = 1]$, then p satisfies the revised Markov inequality with $d=2q$

$$\begin{aligned} & \left| \Pr[A^{SR_r^x(D)}() = 1] - \Pr[A^{D^x}() = 1] \right| \\ &= |p(1/r) - p(0)| < (\pi^2/6)(2q)^3/r \quad \checkmark \end{aligned}$$

One Final Step

Recall definition of SR distribution:

For each $i \in [1, r]$

$$y_i \leftarrow D$$

For each $x \in \mathcal{X}$

$$i_x \leftarrow [1, r]$$

$$H(x) = y_{i_x}$$

How do we pick the i_x ?

- Let R be a drawn from $(2q)$ -wise indep. function family
- $i_x = R(x)$

Theorem: $(2q)$ -wise independent functions look like random functions to any q -query quantum algorithm

Quantum GGM

Step 1: Hybridize over levels of tree



Step 2: Simulate hybrids **approximately** using **small range distributions** and **polynomially-many** samples



Step 3: Quantum pseudorandomness of one sample implies quantum pseudorandomness of **polynomially-many** samples



Our PRF Results

Separation: PRFs \neq quantum PRFs

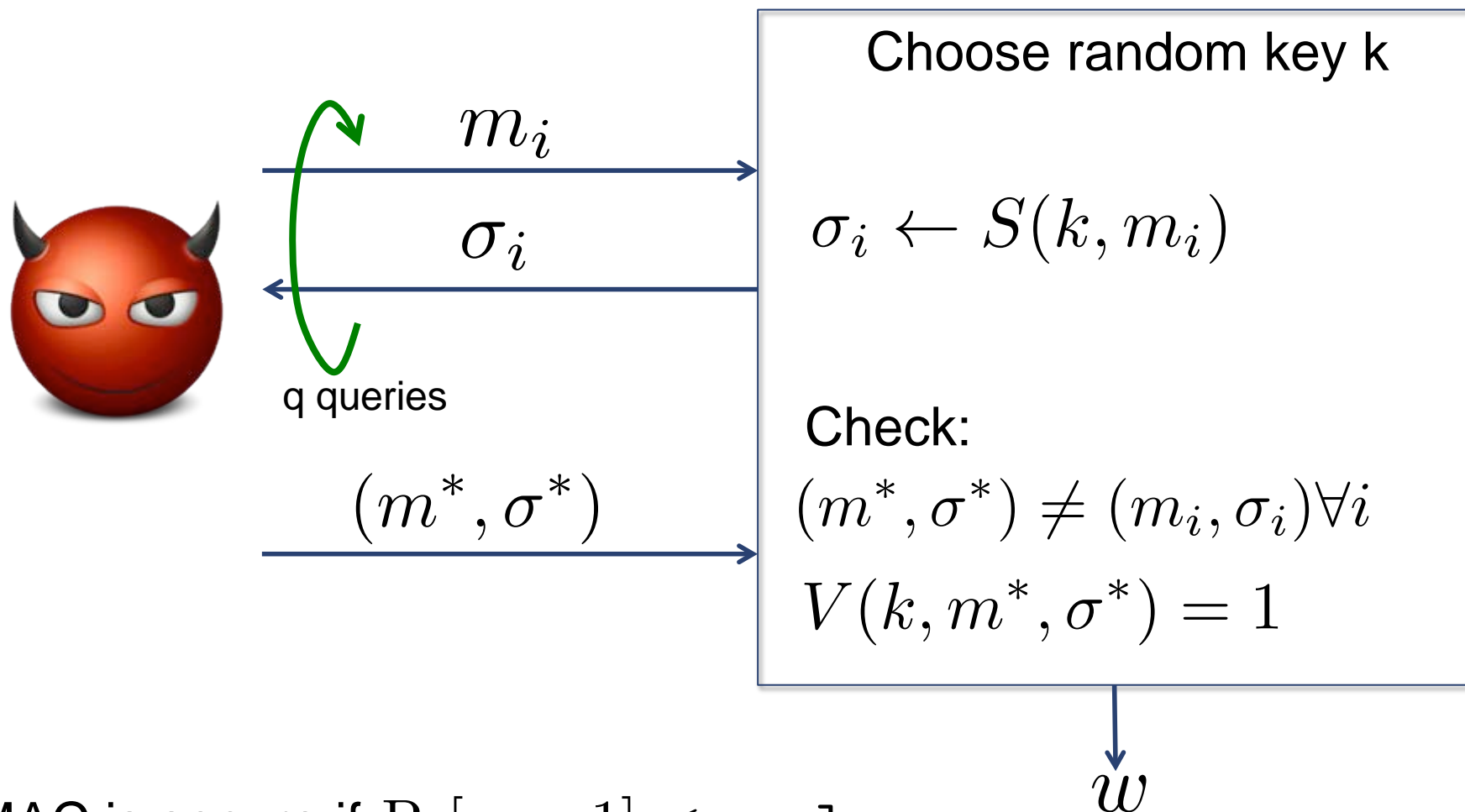
New tool: small-range distributions

Proofs of quantum security for some classical PRF constructions:

- From quantum-secure pseudorandom generators [GGM'84]
- From quantum-secure pseudorandom synthesizers [NR'95]
- Directly from lattices [BPR'11]

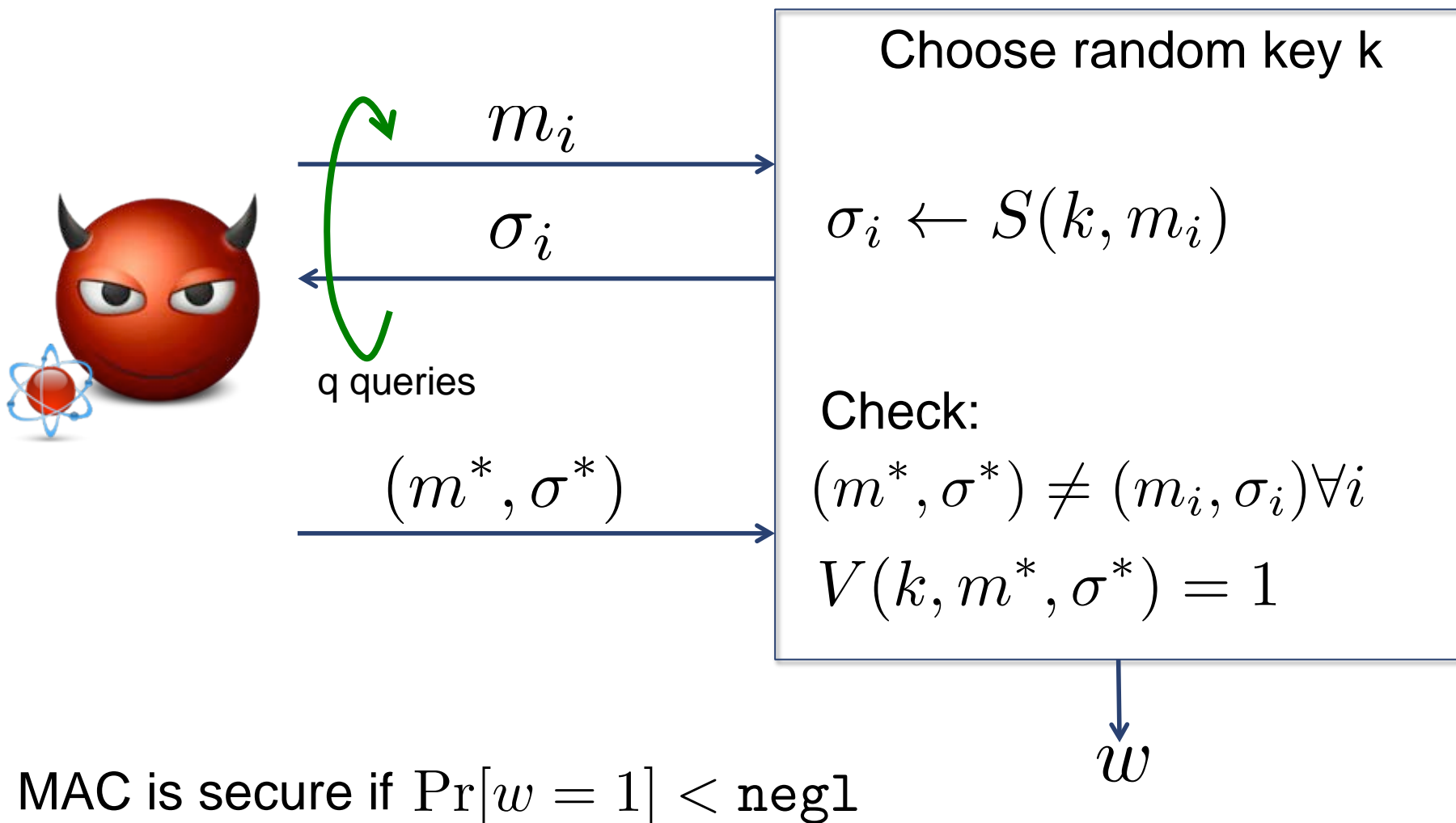
Quantum-secure MACs _[BZ'12]

Classical Security

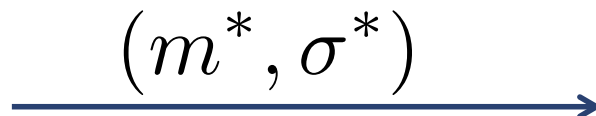
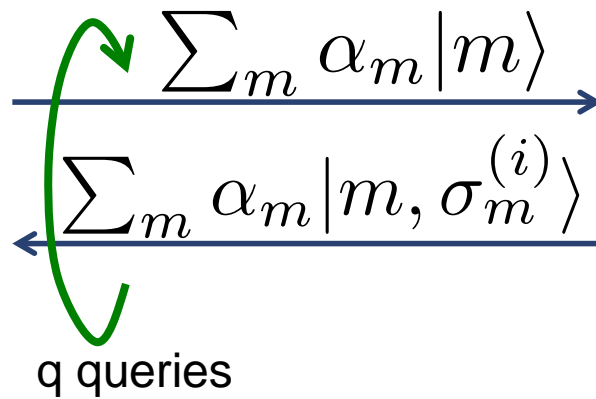


MAC is secure if $\Pr[w = 1] < \text{negl}$

Post-Quantum Security



Quantum Security?



Choose random key k

Pick random r_i

$$\sigma_m^{(i)} = S(k, m; r_i)$$

Check:

Too restrictive

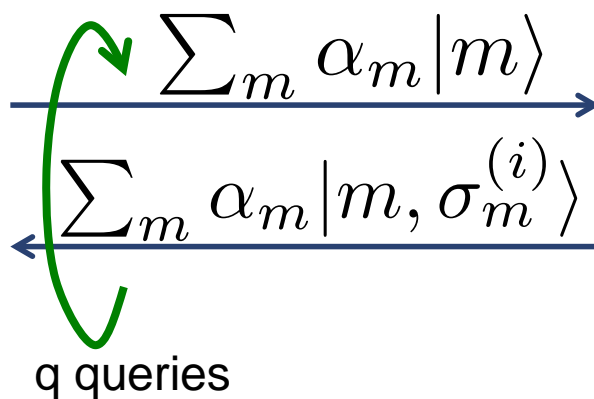
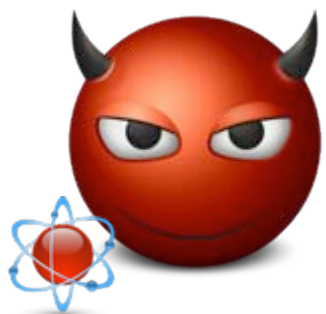
~~$$(m^*, \sigma^*) \neq (m, \sigma_m^{(i)}) \forall i, m$$~~

$$V(k, m^*, \sigma^*) = 1$$

w

MAC is secure if $\Pr[w = 1] < \text{negl}$

Quantum Security



$$(\mathbf{m}, \boldsymbol{\sigma}) \in \mathcal{M}^{q+1} \times \mathcal{S}^{q+1}$$

Choose random key k

Pick random r_i

$$\sigma_m^{(i)} = S(k, m; r_i)$$

Check:

$$(m_j, \sigma_j) \neq (m_k, \sigma_k)$$

$$V(k, m_j, \sigma_j) = 1$$

w

MAC is secure if $\Pr[w = 1] < \text{negl}$

Separation

MAC \neq Quantum-secure MAC

Theorem: If post-quantum PRFs exist, then there are post-quantum MACs that are not quantum-secure MACs

Carries over immediately from PRF separation

Also have natural examples where underlying PRF is quantum-secure (Carter-Wegman MAC)

A Simple Classical MAC

Let F be a classically secure PRF

F is also a classically-secure MAC:

$$S(k,m) = F(k,m)$$

$$V(k,m,\sigma) = F(k,m) == \sigma?$$

Security: Replace F with random oracle

→ Adversary can't tell difference

→ Forgeries correspond to input/output pairs of oracle

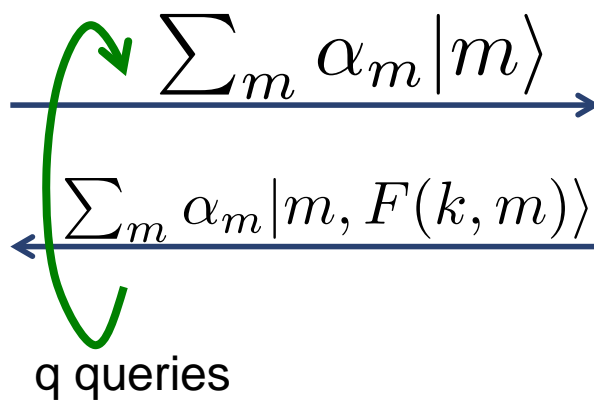
→ Impossible to generate new pairs

A Simple Quantum-secure MAC?

Let F be a quantum-secure PRF

Is F also a quantum-secure MAC?

Security of PRF as a MAC



The adversary outputs a tuple $(\mathbf{m}, \boldsymbol{\sigma}) \in \mathcal{M}^{q+1} \times \mathcal{S}^{q+1}$.

Choose random key k

Check:

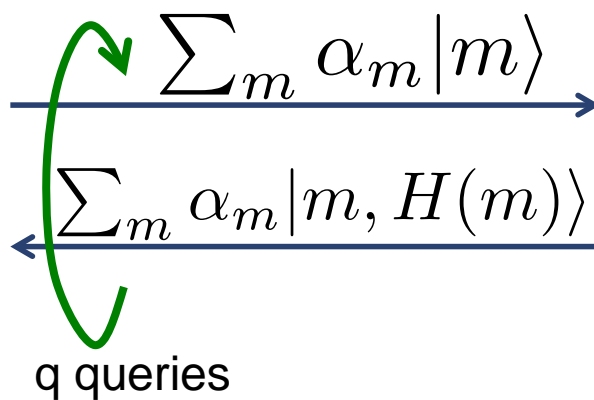
$$(m_j, \sigma_j) \neq (m_k, \sigma_k)$$

$$\sigma_j = F(k, m_j)$$

w

Adversary wins with prob ε

Security of PRF as a MAC



$$(\mathbf{m}, \boldsymbol{\sigma}) \in \mathcal{M}^{q+1} \times \mathcal{S}^{q+1}$$

Choose random oracle H

Check:

$$(m_j, \sigma_j) \neq (m_k, \sigma_k)$$

$$\sigma_j = H(m_j)$$

w

Adversary wins with prob ϵ -negl

Quantum Oracle Interrogation

Allowed q quantum queries to random oracle H

Goal: produce $q+1$ input/output pairs

Classical queries: can't do better than $1/|Y|$

→ Hard if H outputs super-logarithmically many bits

Quantum queries?

→ get to “see” entire oracle with a single query

Single-Bit Outputs

Bad news: If $|Y|=2$ (i.e. single bit output), the oracle interrogation problem is easy.

Theorem_[vD'98]: There is an algorithm that makes q quantum queries to any oracle $H:X \rightarrow \{0,1\}$ and produces $1.99q$ input/output pairs, with probability $1 - \text{negl}(q)$

Are we in trouble?

Arbitrary Output Size

We exactly characterize the difficulty of the oracle interrogation problem:

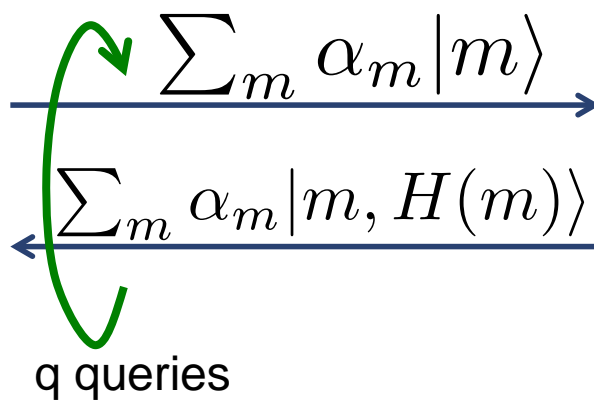
Theorem: Any quantum algorithm making q quantum queries to an oracle $H:X \rightarrow Y$ solves the oracle interrogation problem with probability at most $1 - (1 - |Y|^{-1})^{q+1}$.

Moreover, there is a quantum algorithm exactly matching this bound.

Two cases:

- $\log |Y| \leq (\log q)/2$: probability is negligibly close to 1 \rightarrow Easy
- $\log |Y| = \omega(\log q)$: probability is negligible \rightarrow Hard ✓

Security of PRF as a MAC



$$(\mathbf{m}, \boldsymbol{\sigma}) \in \mathcal{M}^{q+1} \times \mathcal{S}^{q+1}$$

Must be negligible
 $\rightarrow \epsilon$ is negligible

Adversary wins with prob ϵ -negl

Choose random oracle H

Check:

$$(m_j, \sigma_j) \neq (m_k, \sigma_k)$$

$$\sigma_j = H(m_j)$$

w

The Rank Method

Fix q , let $|\psi_H\rangle$ be final state (before measurement) of quantum algorithm after q queries to H

$\{|\psi_H\rangle : H \in \mathcal{H}\}$ spans some subspace of the overall Hilbert space

Let $\text{Rank} = \text{Dim Span}\{|\psi_H\rangle : H \in \mathcal{H}\}$

Lemma: For any goal, the probability of success is at most Rank times the probability of success of the best 0-query algorithm

Applying the Rank Method

Goal: output $k=(q+1)$ input/output pairs

Best 0-query algorithm: pick k arbitrary distinct inputs, guess outputs

$$\text{Success prob: } (|Y|^{-1})^k = |Y|^{-(q+1)}$$

Only need to bound the rank of any q -query algorithm

The Rank Method

Lemma: The rank of any algorithm that makes q queries to an oracle $H: X \rightarrow Y$ is at most

$$\sum_{r=0}^q \binom{|X|}{r} (|Y| - 1)^r$$

Exact



Applying the Rank Method

Prob success of any q -query algorithm

$\leq \text{Rank} * \text{best success prob of 0-query algs}$

$$\leq \frac{\sum_{r=0}^q \binom{|\mathcal{X}|}{r} (|\mathcal{Y}| - 1)^r}{|\mathcal{Y}|^{q+1}} \approx \frac{\binom{|\mathcal{X}|}{q}}{|\mathcal{Y}|}$$



Too big!

Applying the Rank Method

Observation: for any $(q+1)$ inputs, knowing H at other points does not help determine H at these points

→ Might as well only query on superpositions of $(q+1)$ points

$$\frac{\sum_{r=0}^q \binom{|\mathcal{X}|}{r} (|\mathcal{Y}| - 1)^r}{|\mathcal{Y}|^{q+1}}$$



$$\frac{\sum_{r=0}^q \binom{q+1}{r} (|\mathcal{Y}| - 1)^r}{|\mathcal{Y}|^{q+1}} = 1 - \left(1 - \frac{1}{|\mathcal{Y}|}\right)^{q+1} \quad \checkmark$$

Our MAC Results

Exact characterization of success probability for quantum oracle interrogation

- Developed new general tool: Rank method

Quantum-secure MACs:

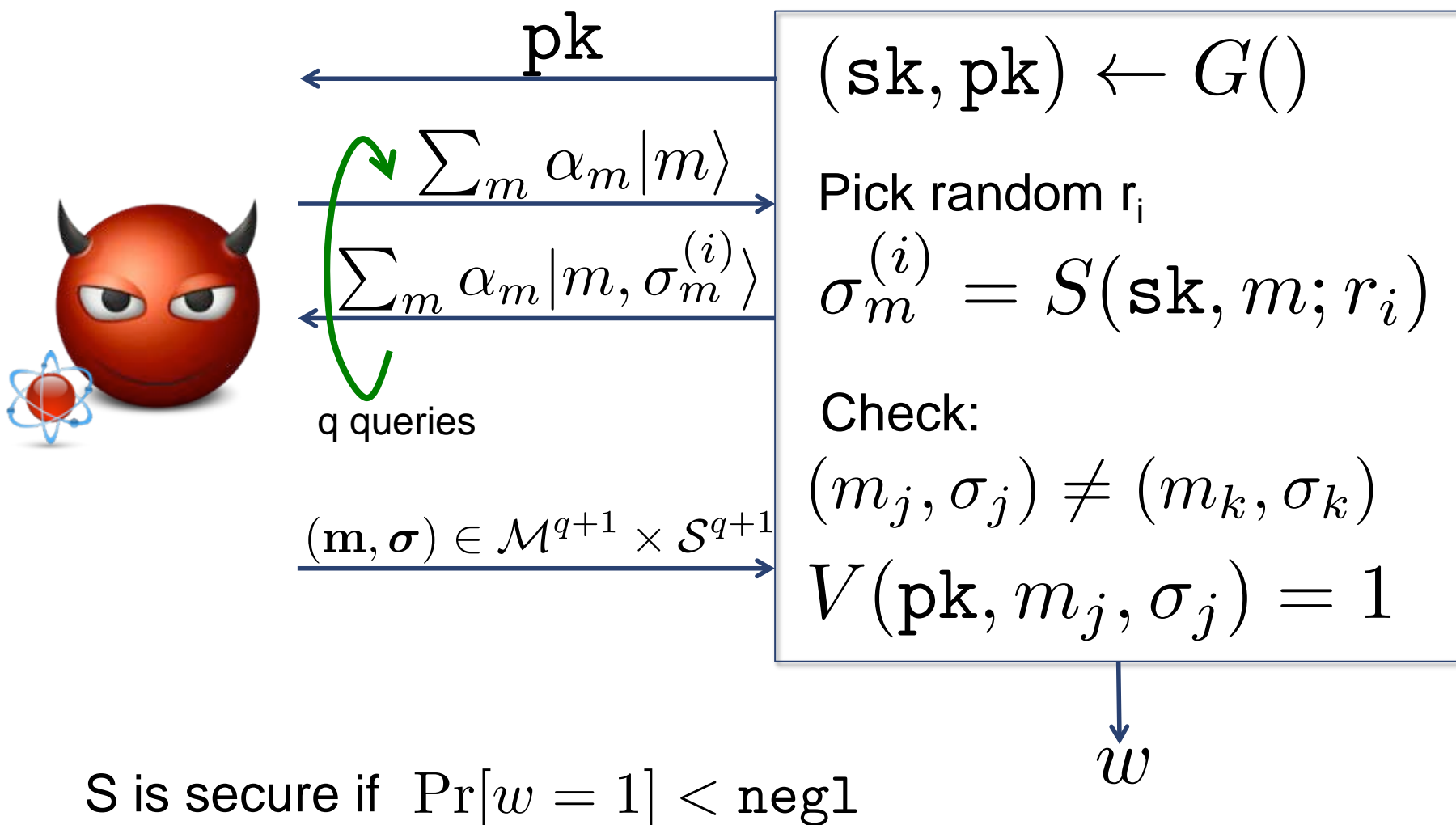
- Quantum-secure PRFs are quantum-secure MACs
- A variant of Carter-Wegman is quantum-secure

One-time quantum-secure MACs:

- Pairwise independence is not enough
- 4-wise independence is

Quantum-Secure Signatures [BZ'13]

Quantum Security



Separation

Sig \neq Quantum-secure Sig

Theorem: If post-quantum signatures exist, then there are post-quantum signatures that are not quantum-secure signatures

Building Quantum-secure Signatures

Hope that existing constructions can be proven secure:

- Lattice schemes [ABB'10,CHKP'10]
- Generic constructions (Lamport, Merkle)
- RO schemes [GPV'08]

Compilers to boost security?

One-time QROM Conversion

Let (G, S, V) be a classically secure signature scheme


Construct new QROM scheme (G, S', V') where:

$$\begin{aligned} S'(\mathbf{sk}, m) &= S(\mathbf{sk}, H(m)) \\ V'(\mathbf{pk}, m, \sigma) &= V(\mathbf{pk}, H(m), \sigma) \end{aligned}$$

Theorem: If (G, S, V) is one-time post-quantum secure, then (G, S', V') is one-time quantum secure in the quantum random oracle model.

Proof Sketch

Start with a one-time adversary for S' :


$$\begin{array}{c} \xrightarrow{\sum_m \alpha_m |m\rangle} \\ \xleftarrow{\sum_m \alpha_m |m, S(\mathbf{sk}, H(m); r)\rangle} \end{array}$$

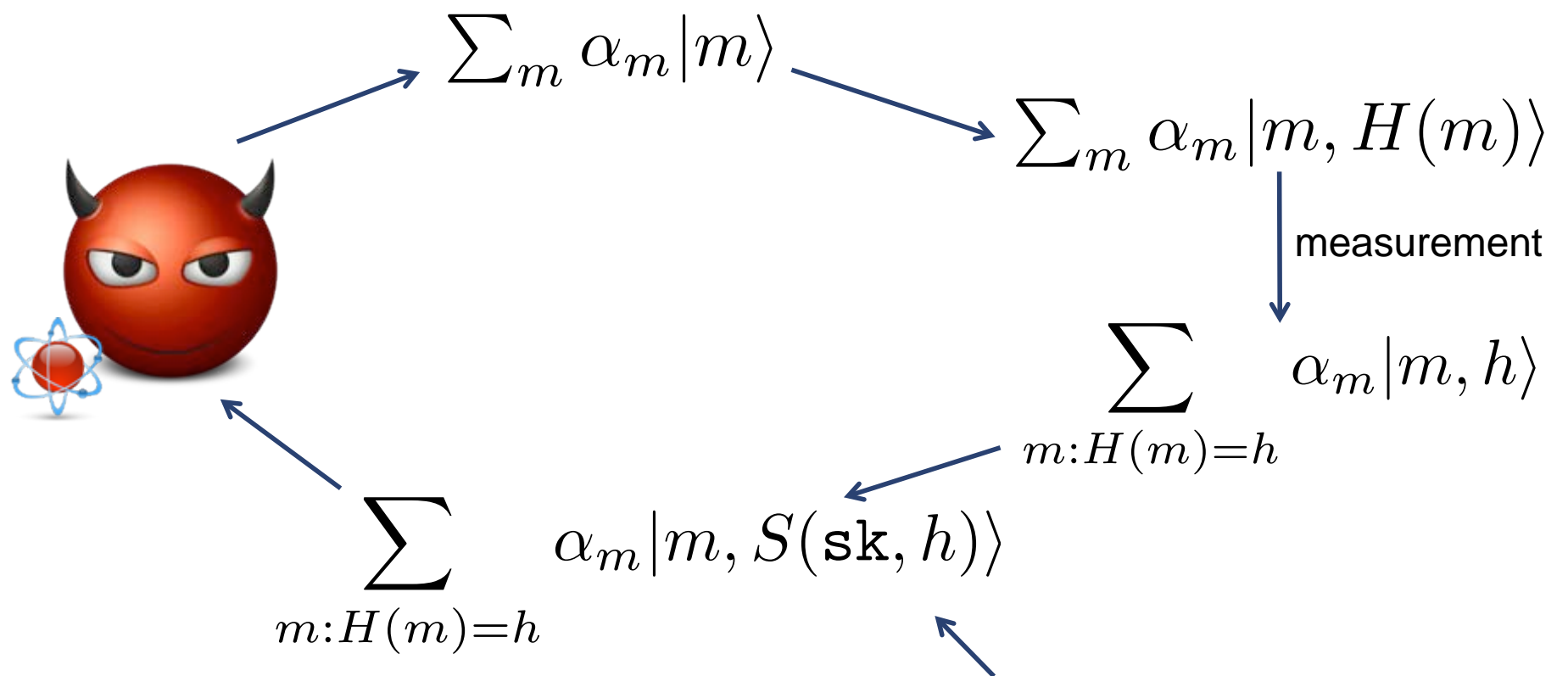
Step 1: Replace H with a SR distribution on t samples.

→ S only evaluated on t points

Problem: Adversary only generates 2 signatures!

Proof Sketch

Step 2: Sample $H(m)$



S only evaluated on 1 input!
 → One signature must be forgery

Measurement Lemma

$$A: |\psi_0\rangle \rightarrow |\psi_1\rangle \xrightarrow{\hspace{1.5cm}} |\psi_2\rangle \xrightarrow{\text{measurement}} x$$

$$A': |\psi_0\rangle \rightarrow |\psi_1\rangle \xrightarrow[\text{partial}]{\text{measurement}} |\psi'_1\rangle \rightarrow |\psi'_2\rangle \xrightarrow{\text{measurement}} x$$

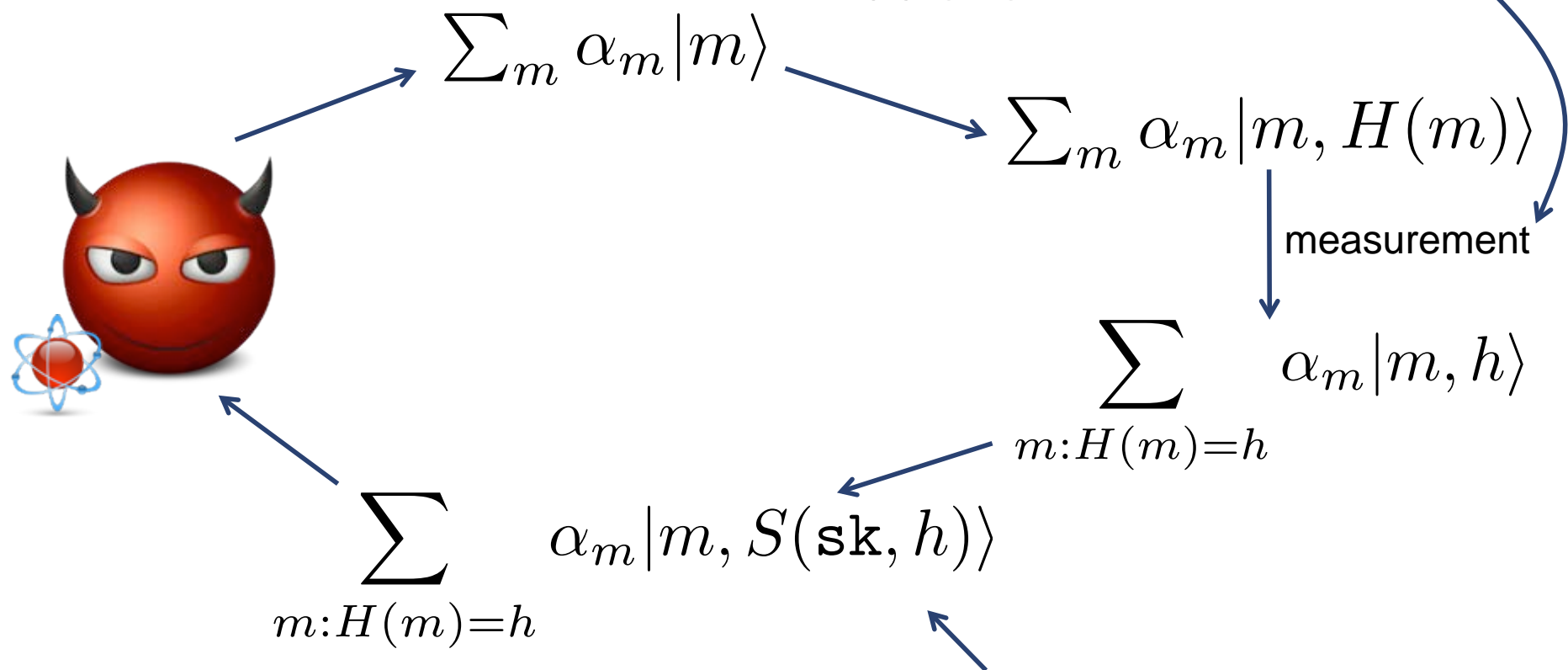
Results in one of k outcomes

Lemma: $\Pr[x \leftarrow A'] \geq \Pr[x \leftarrow A]/k$

Proof Sketch

Step 2: Sample $H(m)$

only reduces adversary's success probability by factor of t



S only evaluated on 1 input!
 \rightarrow One signature must be forgery

Generalizing to Many-time Security

Let \mathcal{R} be a pairwise independent function family.

$$S'(\mathbf{sk}, m) = r \xleftarrow{R} \{0, 1\}^\lambda, R \xleftarrow{R} \mathcal{R} \\ (r, S(\mathbf{sk}, H(m, r); R(m)))$$

Theorem: If (G, S, V) is classically secure, then (G, S', V') is quantum secure in the quantum random oracle model.

Our Signature Constructions

Two compilers:

- Post-quantum security \rightarrow Quantum security in the QROM
 - GPV probabilistic full domain hash
- Post-quantum security + chameleon hash \rightarrow Quantum security
 - CHKP'10 signatures
 - Modification to ABB'10 signatures

GPV in the QROM

Generalization of
Rank theorem

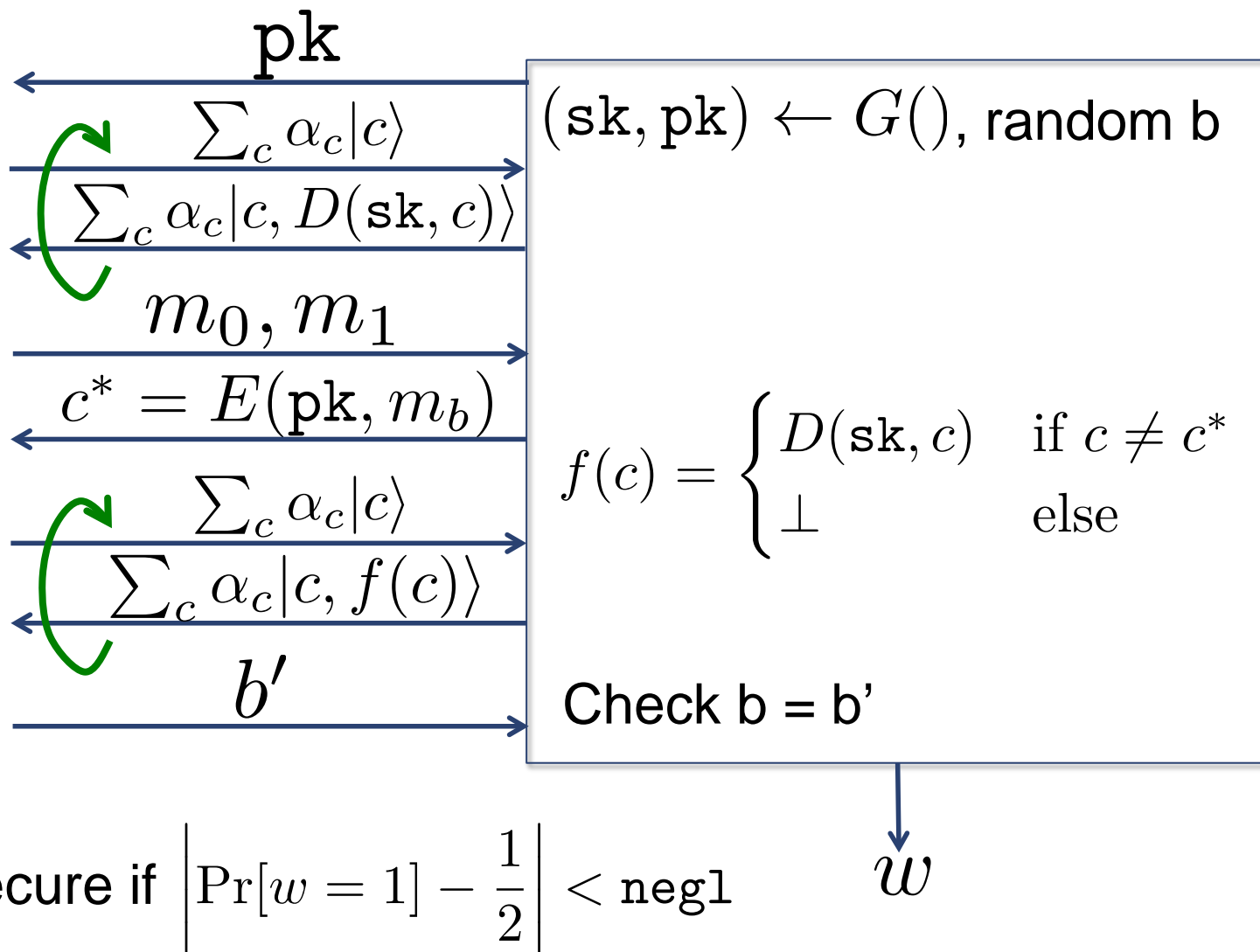


From generic assumptions:

- Lamport signatures + Merkle signatures
- From any hash function

Quantum-Secure Encryption [BZ'13]

Quantum Security



Encryption Results

Classical challenge is required

- Quantum challenge queries lead to unsatisfiable definitions

Separation:

- If classically secure encryption schemes exist, then there are classically secure encryption schemes that are not quantum-secure

Constructions:

- Symmetric CCA from quantum-secure PRFs
- Public Key CCA from LWE
 - Quantum selectively-secure IBE + generic conversion

Summary of Separation Results

Classical Security:

PRF
MAC
Sign
Enc

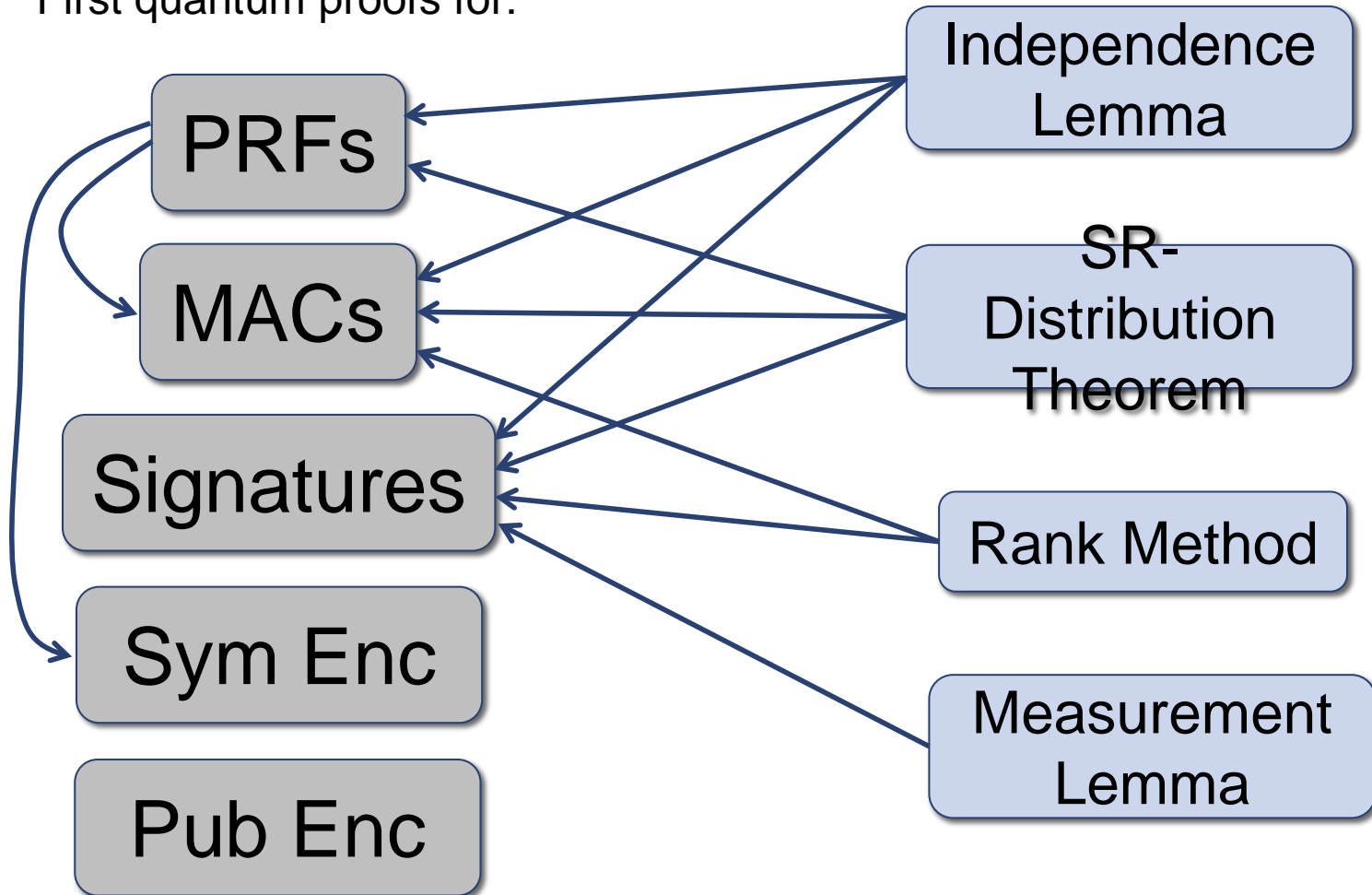
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Quantum Security:

PRF
MAC
Sign
Enc

Summary of Positive Results

First quantum proofs for:



Future Work

Many natural open questions:

- Quantum PRFs \Rightarrow Quantum PRPs (Luby-Rackoff)?
- 3-wise independence enough for 1-time MAC?
- Quantum-secure authenticated encryption \Rightarrow quantum-secure CCA?
- Signatures from one-way functions?

More complicated primitives?

- Adaptively secure (H)IBE?
- Functional encryption?

Thank you!