COS433/Math 473: Cryptography

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Spring 2020

Announcements

HW6 Due SUNDAY
HW7 Due April 30th

Project 3 will be combined with HW 8, due on Dean's date

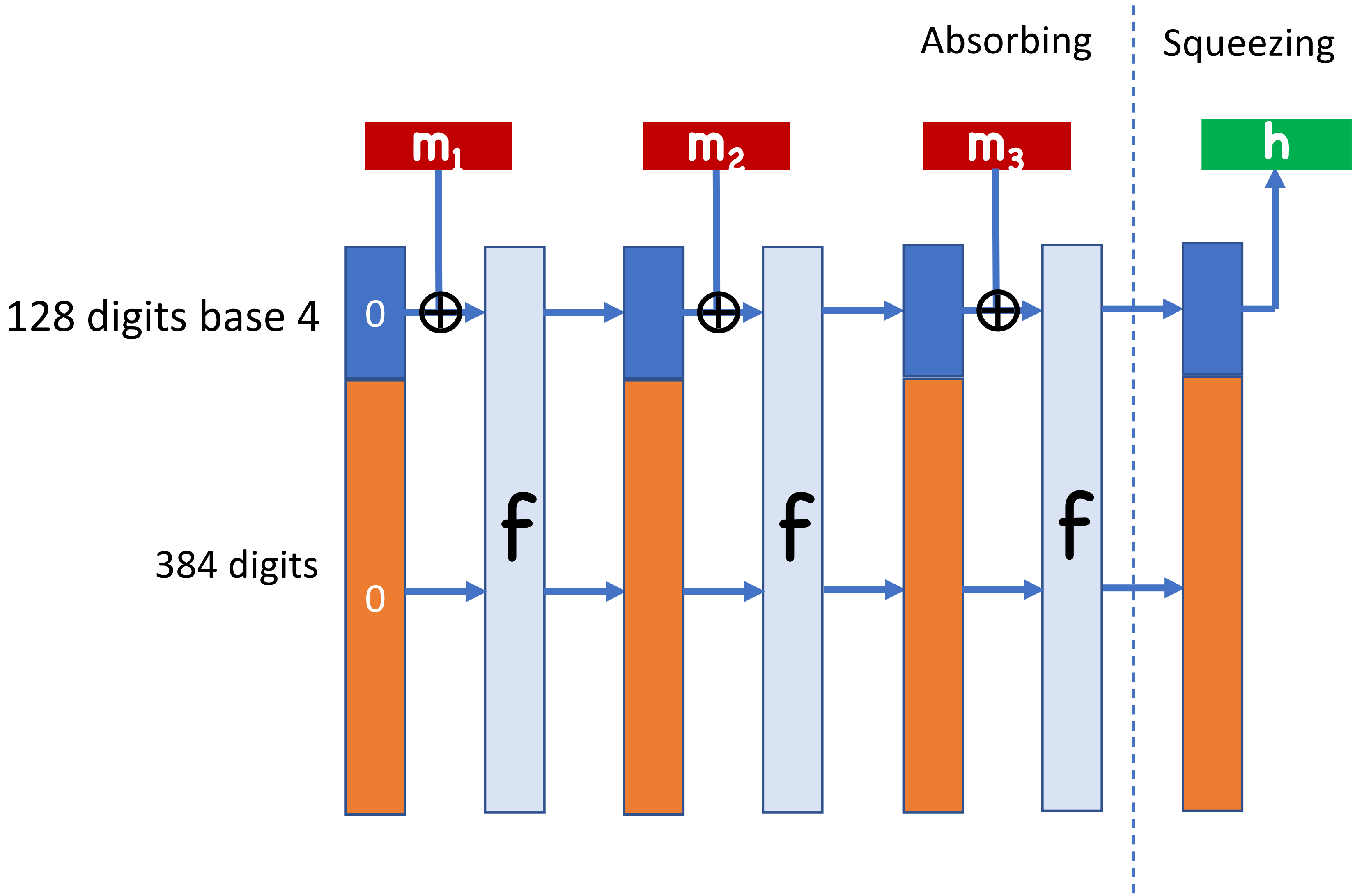
Project 2 Debrief

Motivation: Cryptocurrencies

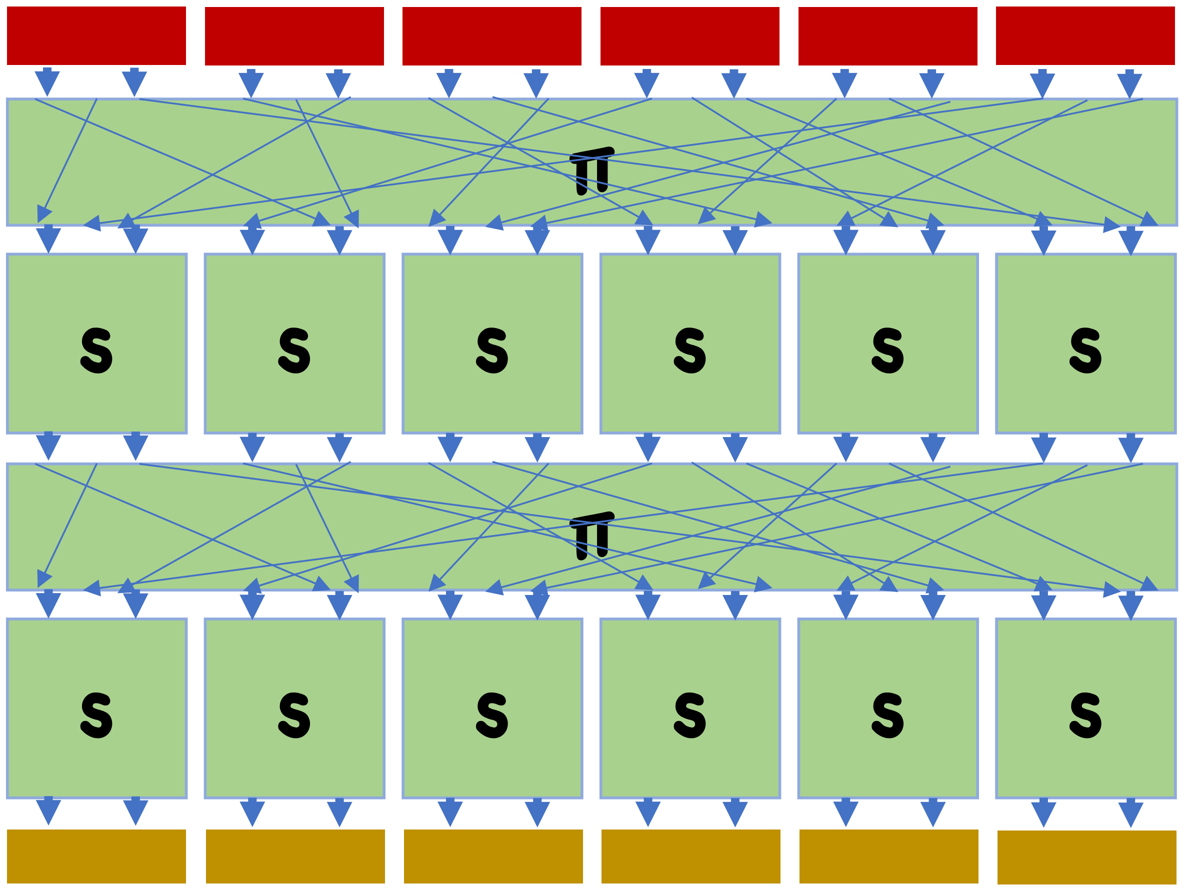
IOTA cryptocurrency used P-CURL hash function

- Sponge construction with SPN network
- S-box had bad differentials
- Let to collision-finding attacks

Project 2 Debrief

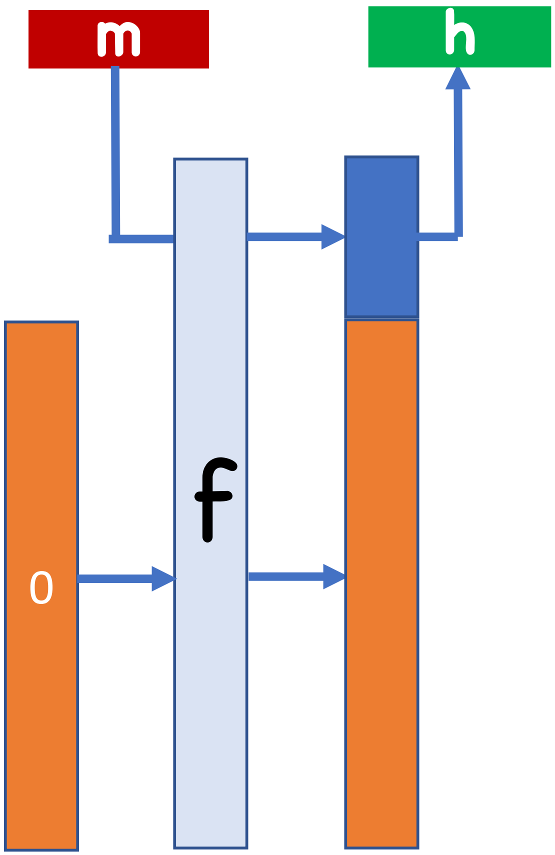


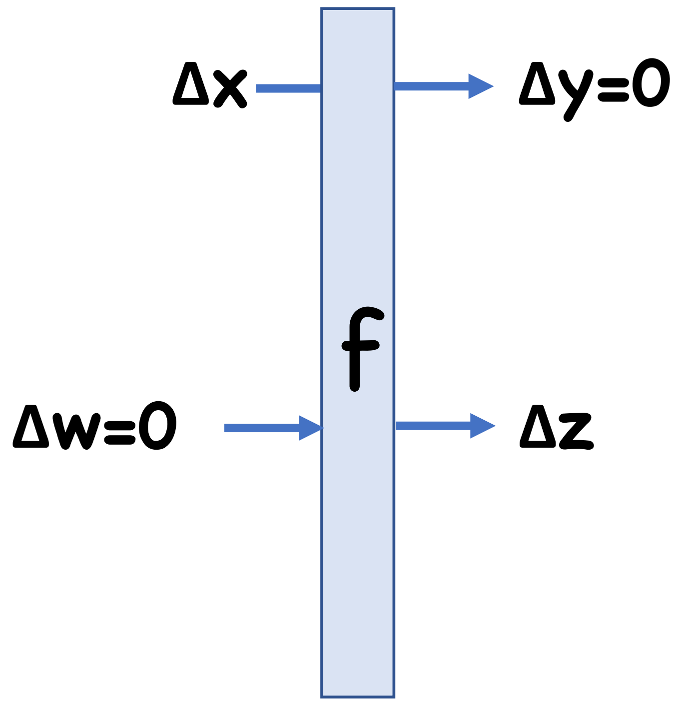
The Function **f**



Each wire is a base 4 number

Good Differentials for **f**?





If **((Δx,0) , (0,Δz))** is a differential for **f**, then **(Δx,0)** is a differential for **H**

Constructing Good Differentials

S-box differential has only 1 non-zero digit in both inputs and outputs

- Called “weight 1” differential

String together to get differential for overall SPN

Don’t care so much about exact differential, any sequence of weight 1 differentials will do

Attack Sketch:

Choose two random messages that differ in a single digit, hope that they are collision

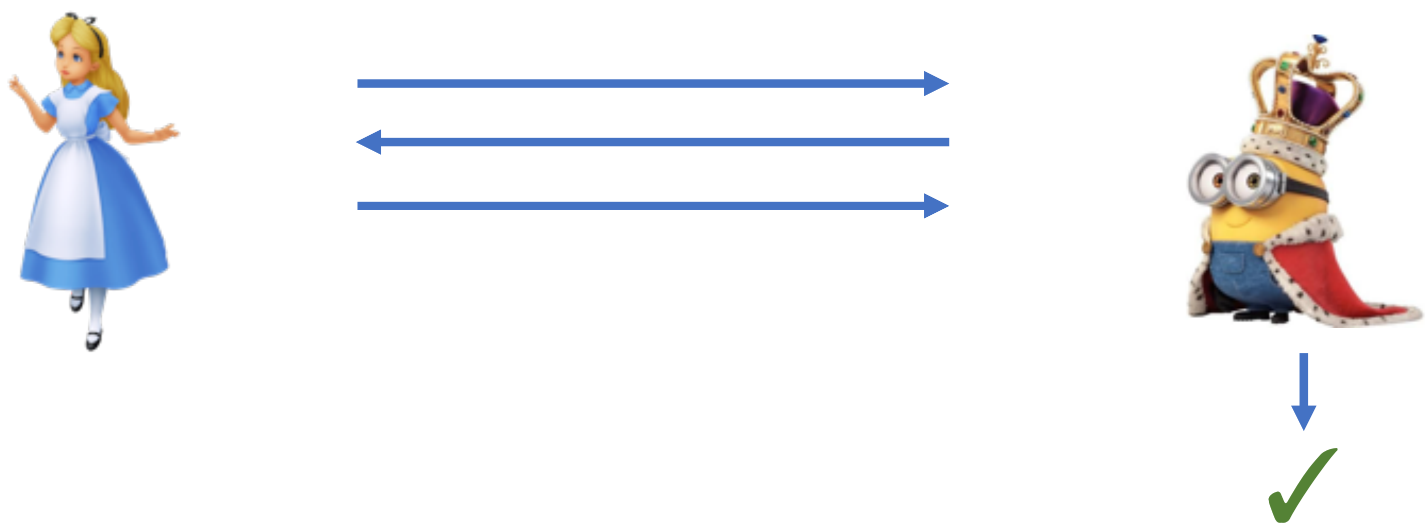
Probability of collision $\gtrsim \frac{3}{4} \times 2^{-20}$

- Prob $\geq 2^{-20}$ input differential gives weight 1 output differential
- Prob ¾ differing digit will be among first 128 digits

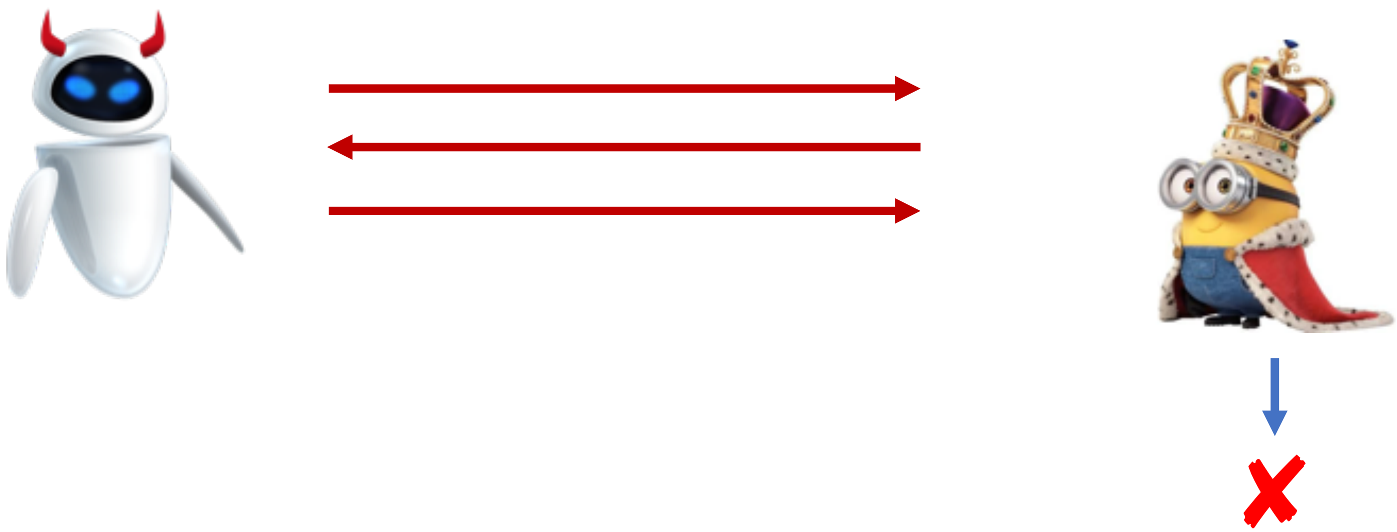
Previously on COS 433…

Identification Protocols

Identification

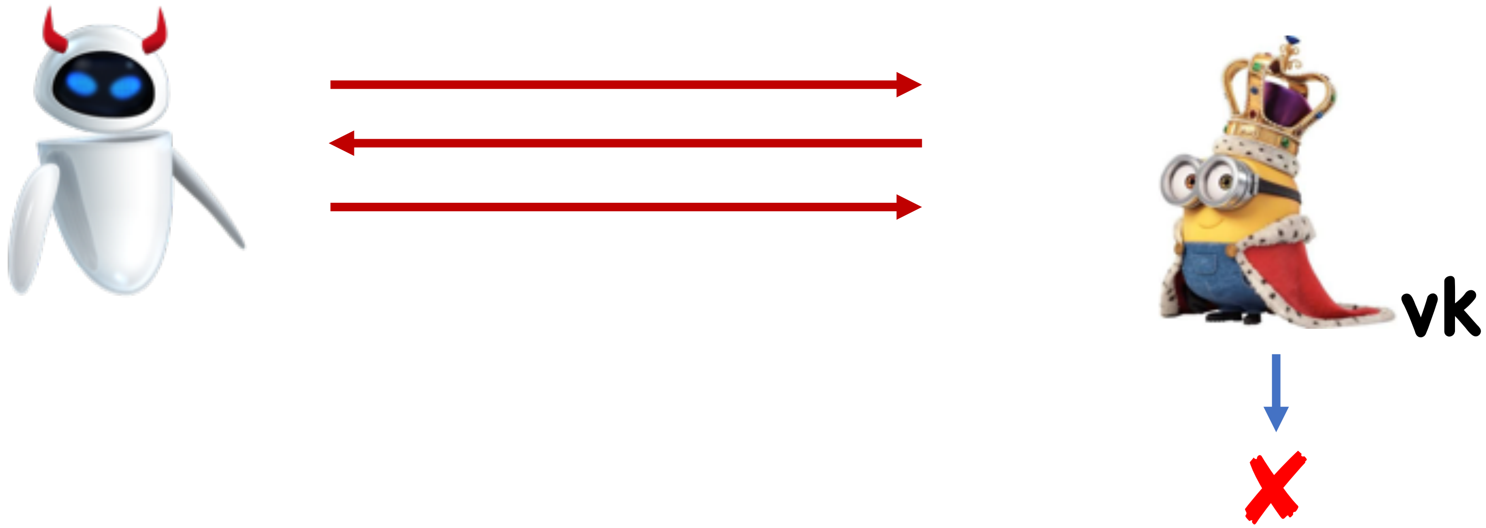


Identification



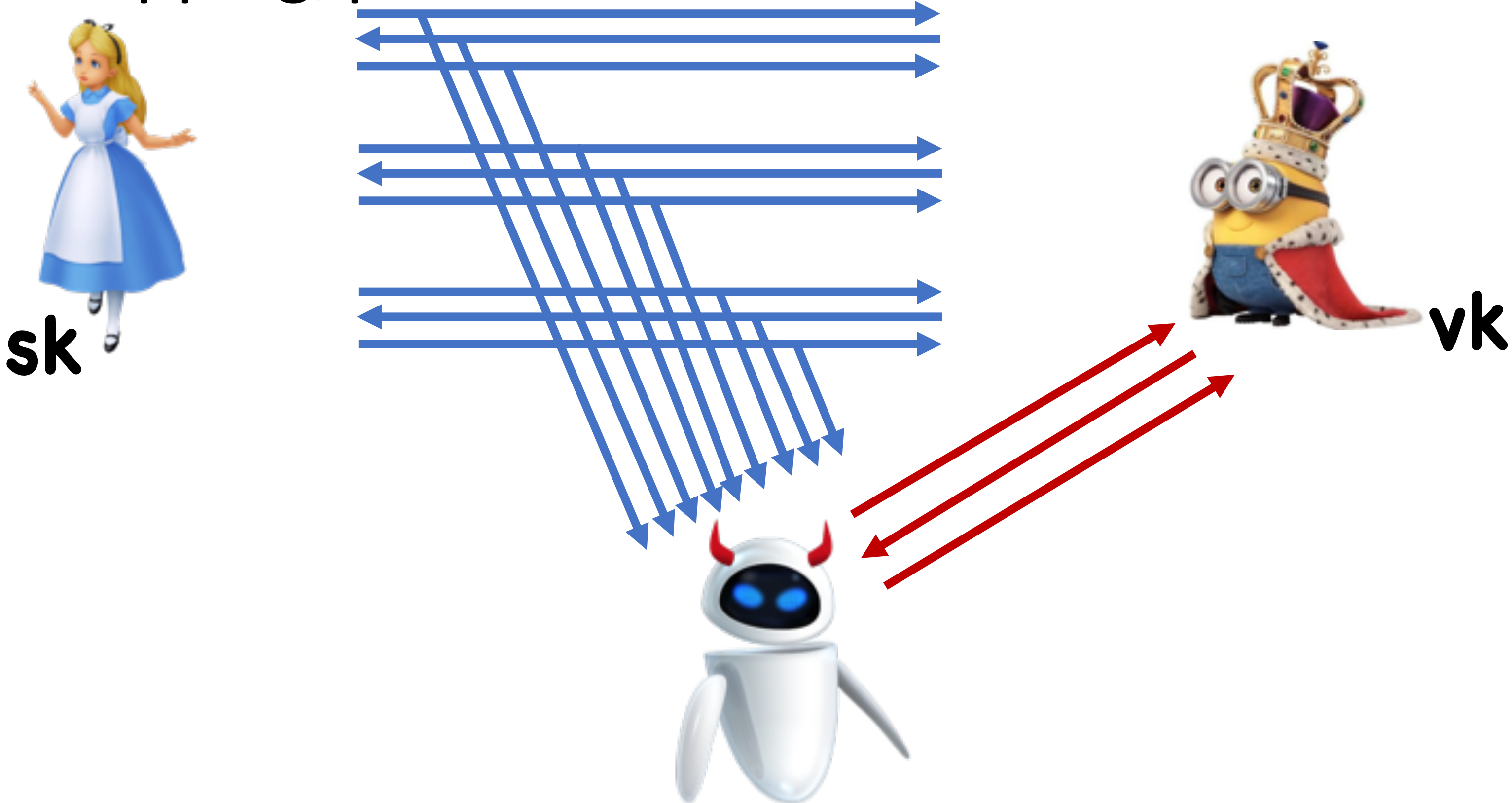
Types of Attacks

Direct Attack:



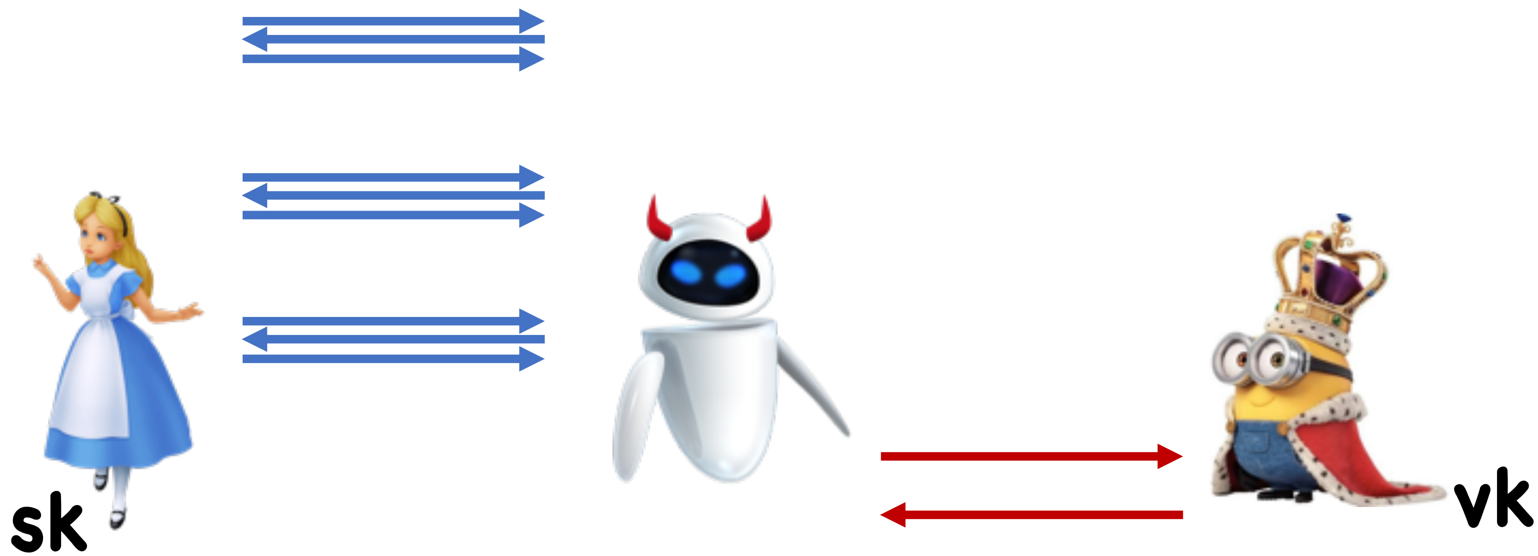
Types of Attacks

Eavesdropping/passive:



Types of Attacks

Man-in-the-Middle/Active:



Basic Password Protocol

Never ever (ever ever…) use

sk=pwd

sk →

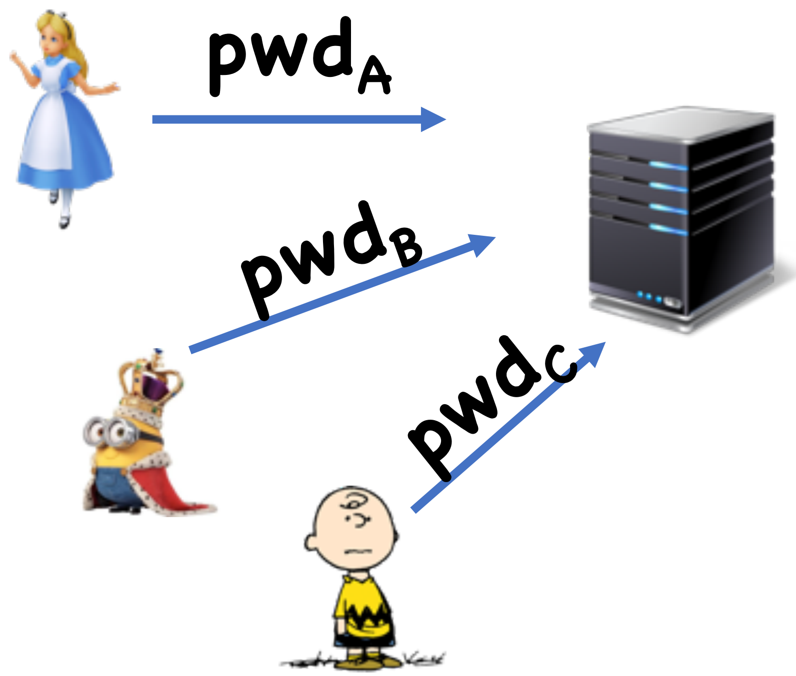
vk=pwd

sk == vk?

Salting

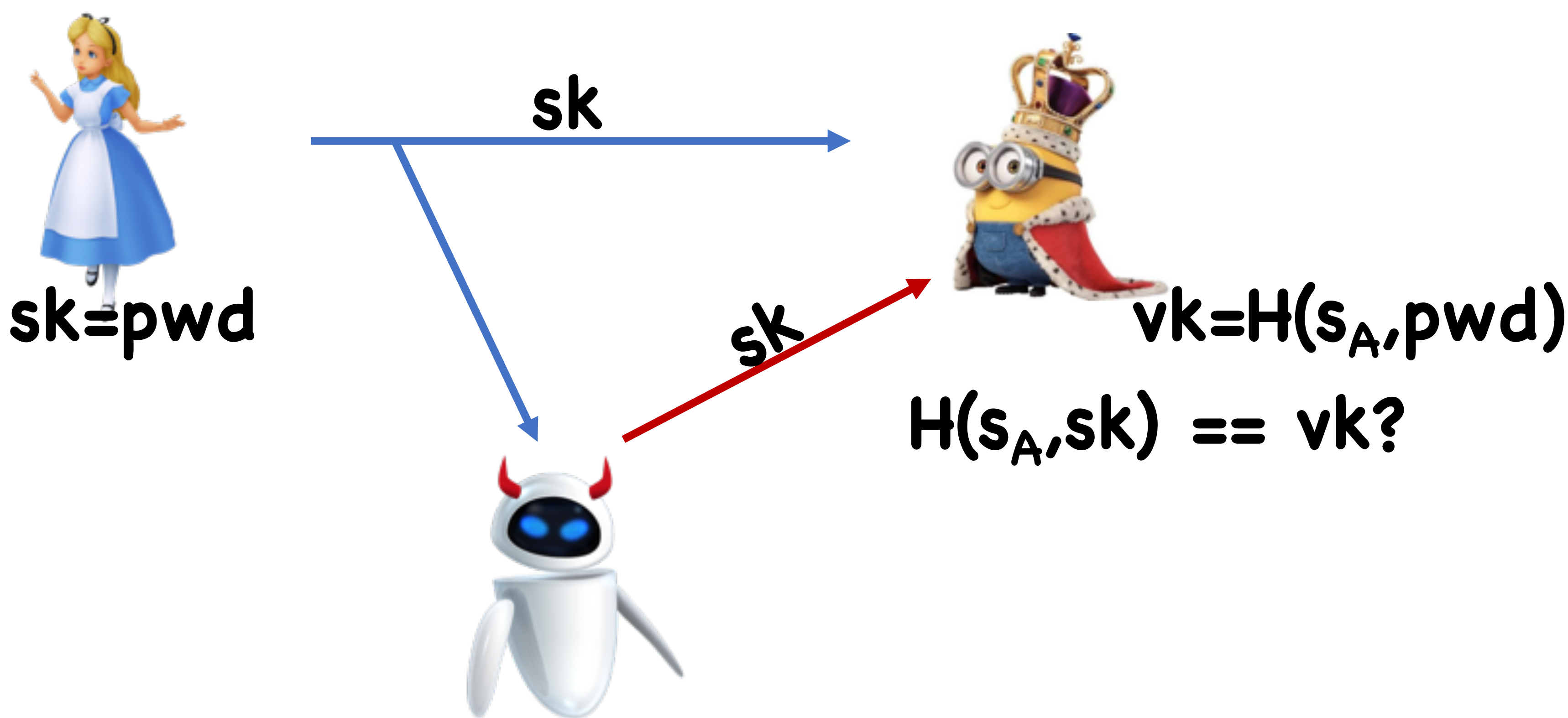
Let **H** be a hash function

s_i random



User	Salt	Pwd
Alice	s_A	$H(s_A,pwd_A)$
Bob	s_B	$H(s_B,pwd_B)$
Charlie	s_C	$H(s_C,pwd_C)$
…	…	…

Security Against Eavesdropping



One-time Passwords

Let **F** be a PRF

$sk = (k, 0)$

$sk_0 = F(k, 0)$ →

$vk = (k, 0)$

$sk_0 == F(k, 0)?$

One-time Passwords

Let **F** be a PRF

$sk_1 = F(k,1)$

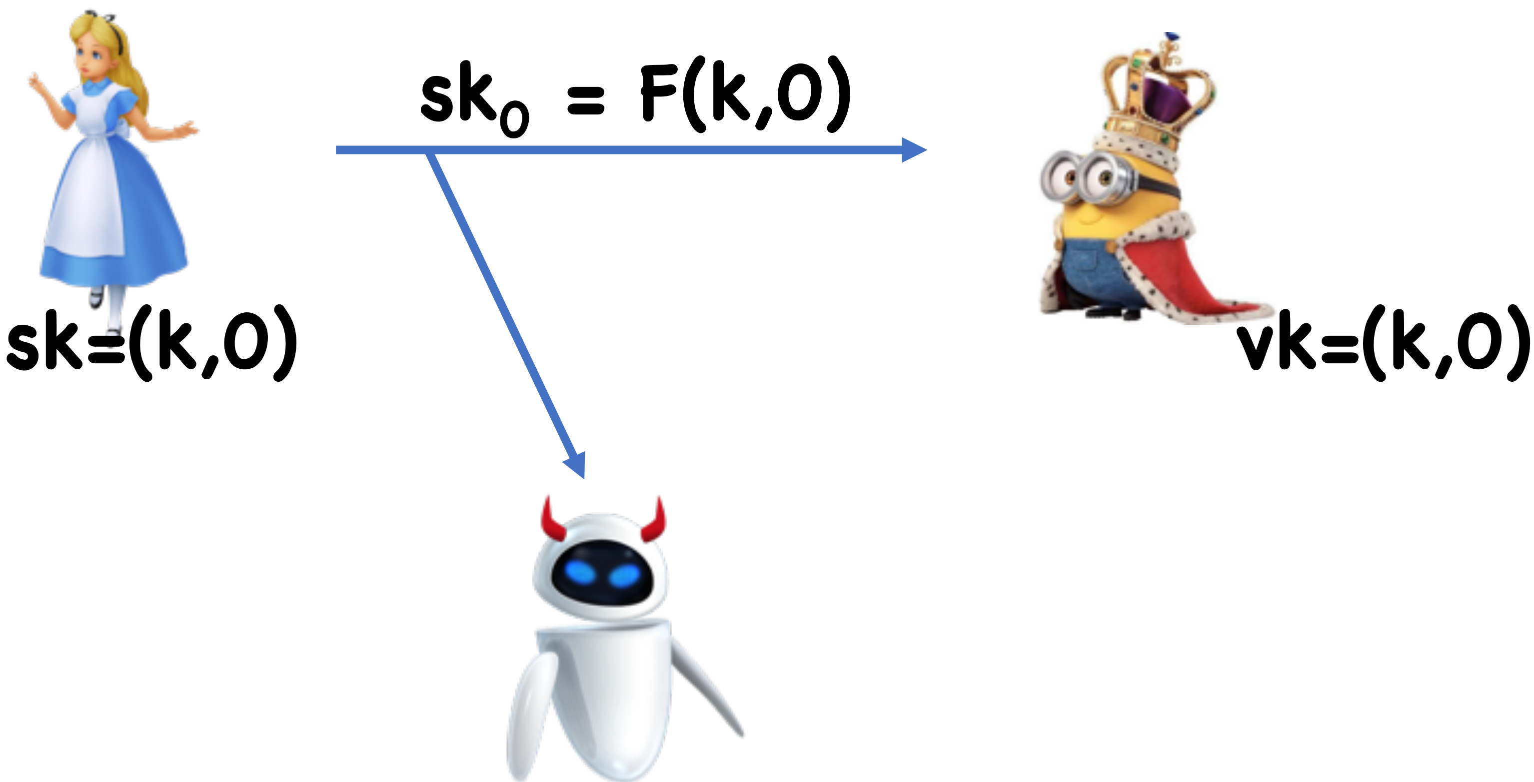
$sk=(k,1)$

$vk=(k,1)$

$sk_1 == F(k,1)?$

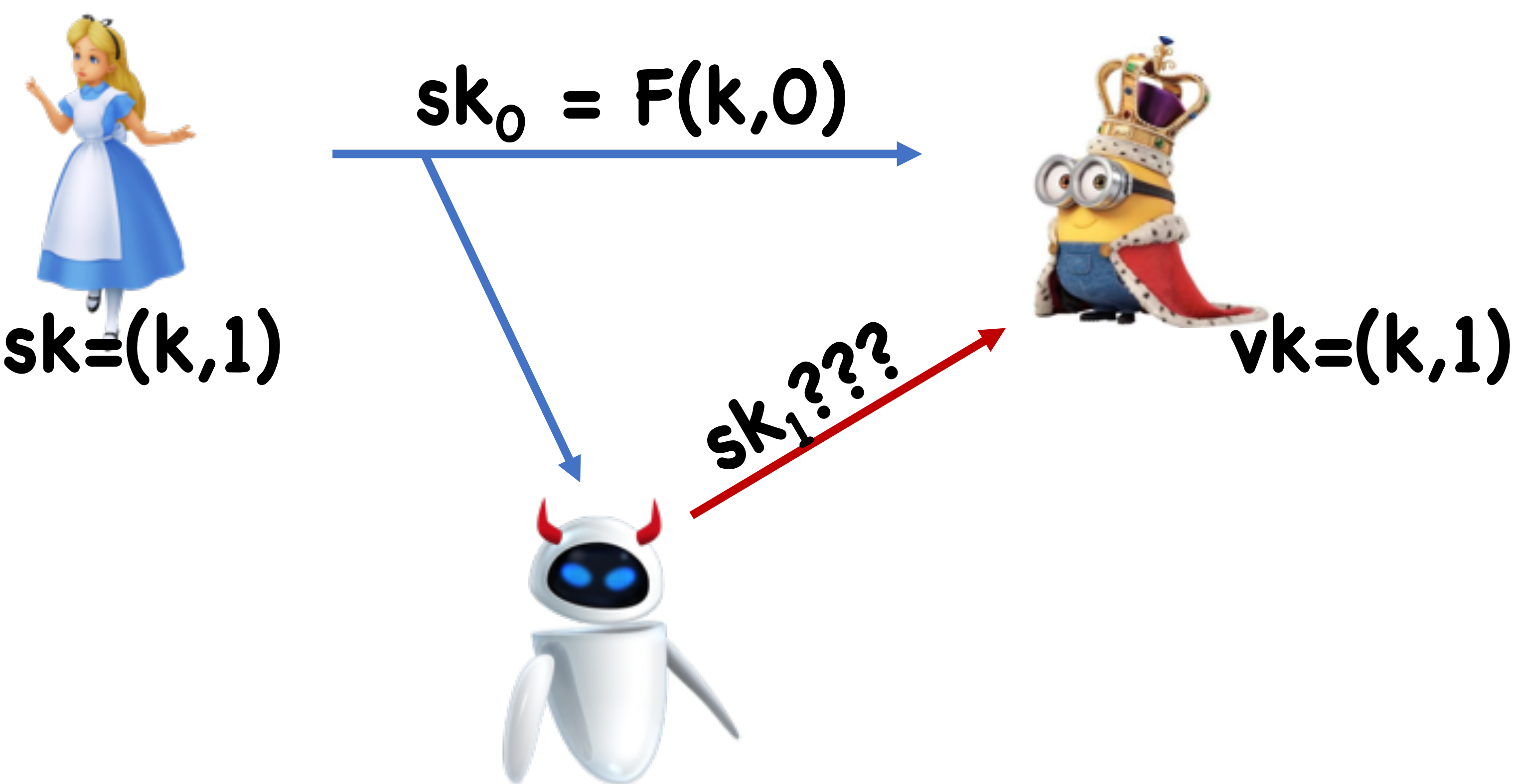
One-time Passwords

Let **F** be a PRF



One-time Passwords

Let **F** be a PRF



One-time Passwords

Advancing state:

- Time based (e.g. every minute, day, etc)
- User Action (button press)

Must allow for small variation in counter value

- Clocks may be off, user may accidentally press button

Stateless Schemes?

So far, all schemes secure against eavesdropping are stateful

Easy theorem: any one-message stateless ID protocol is insecure if the adversary can eavesdrop

- Simply replay message

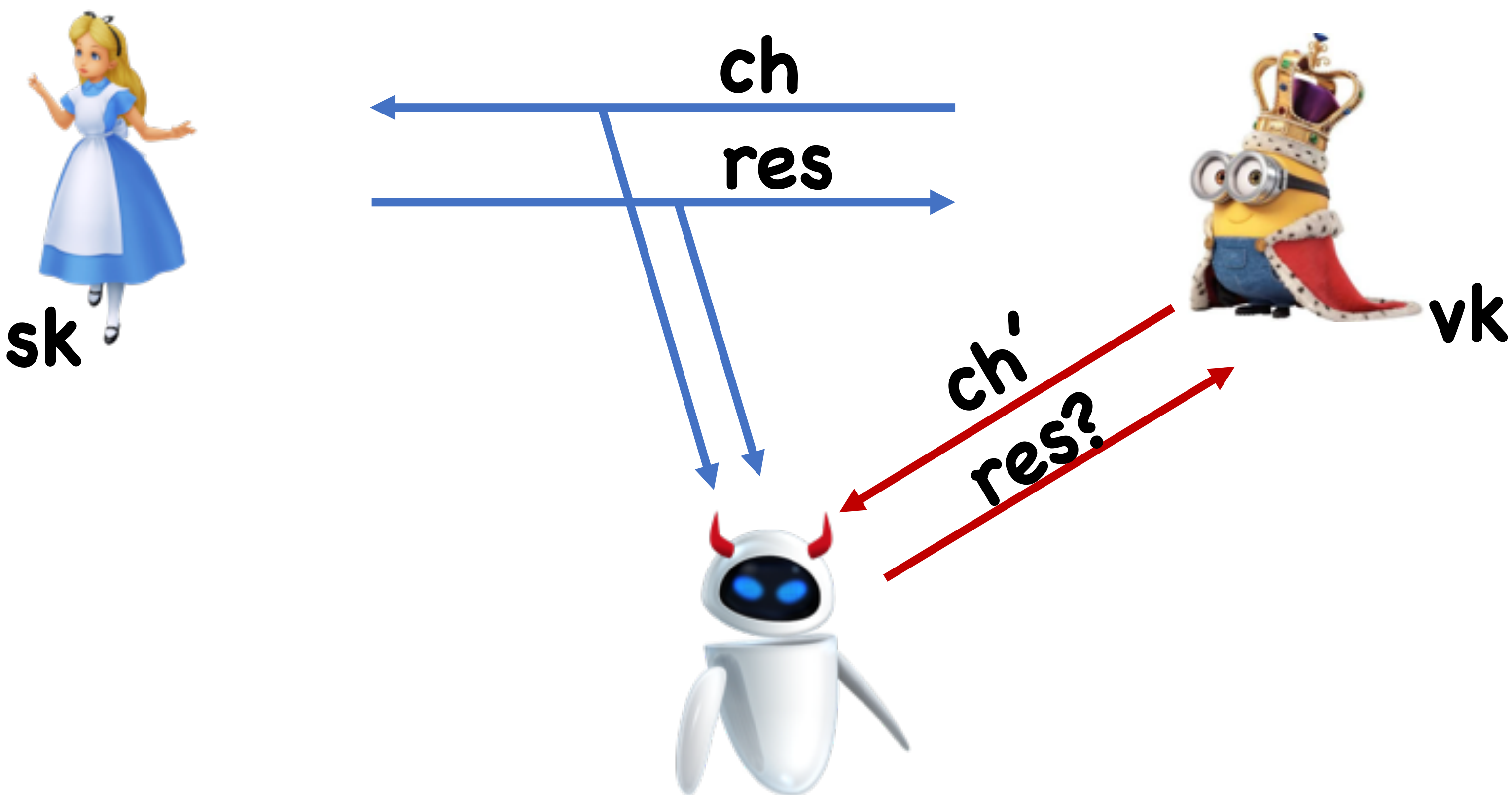
If want stateless scheme, instead want at least two messages

Today

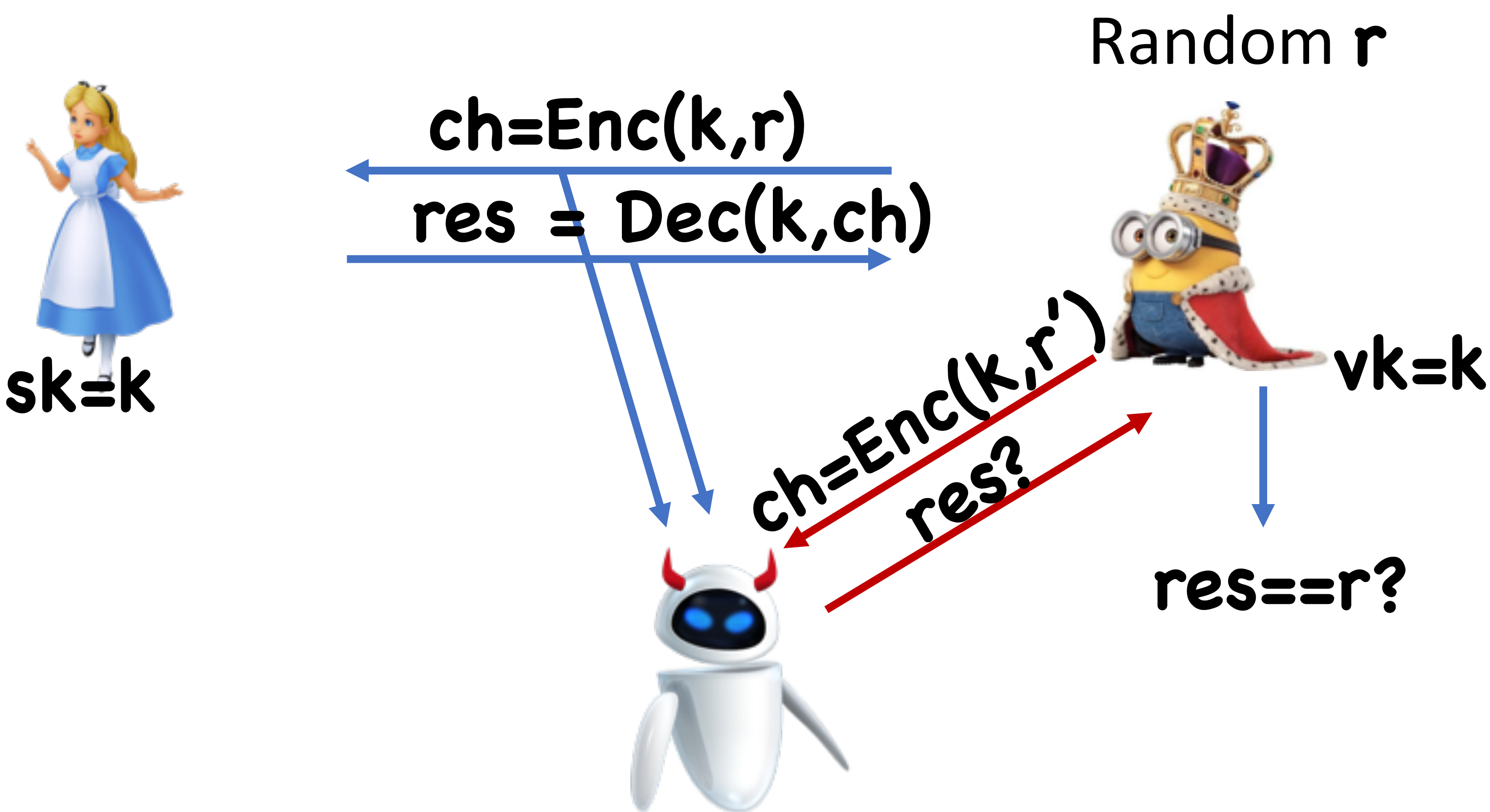
Challenge-Response authentication

Zero Knowledge

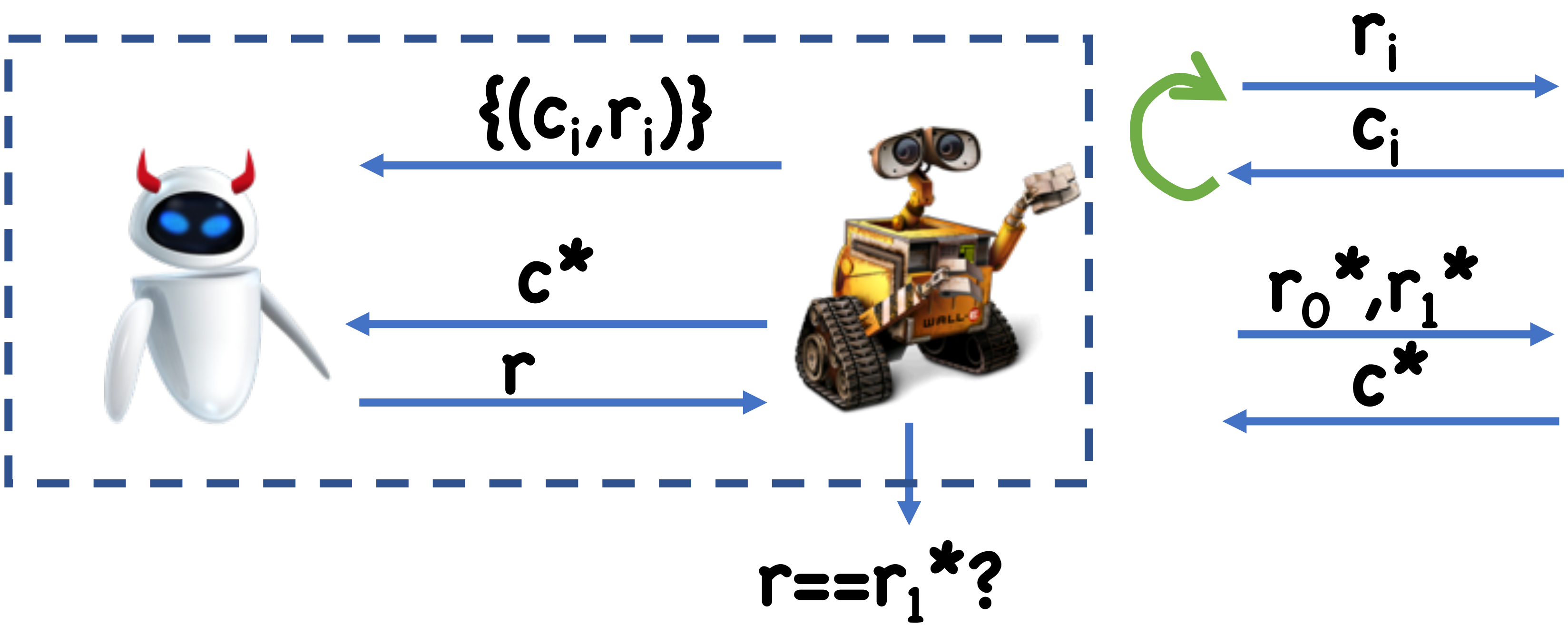
Challenge-Response



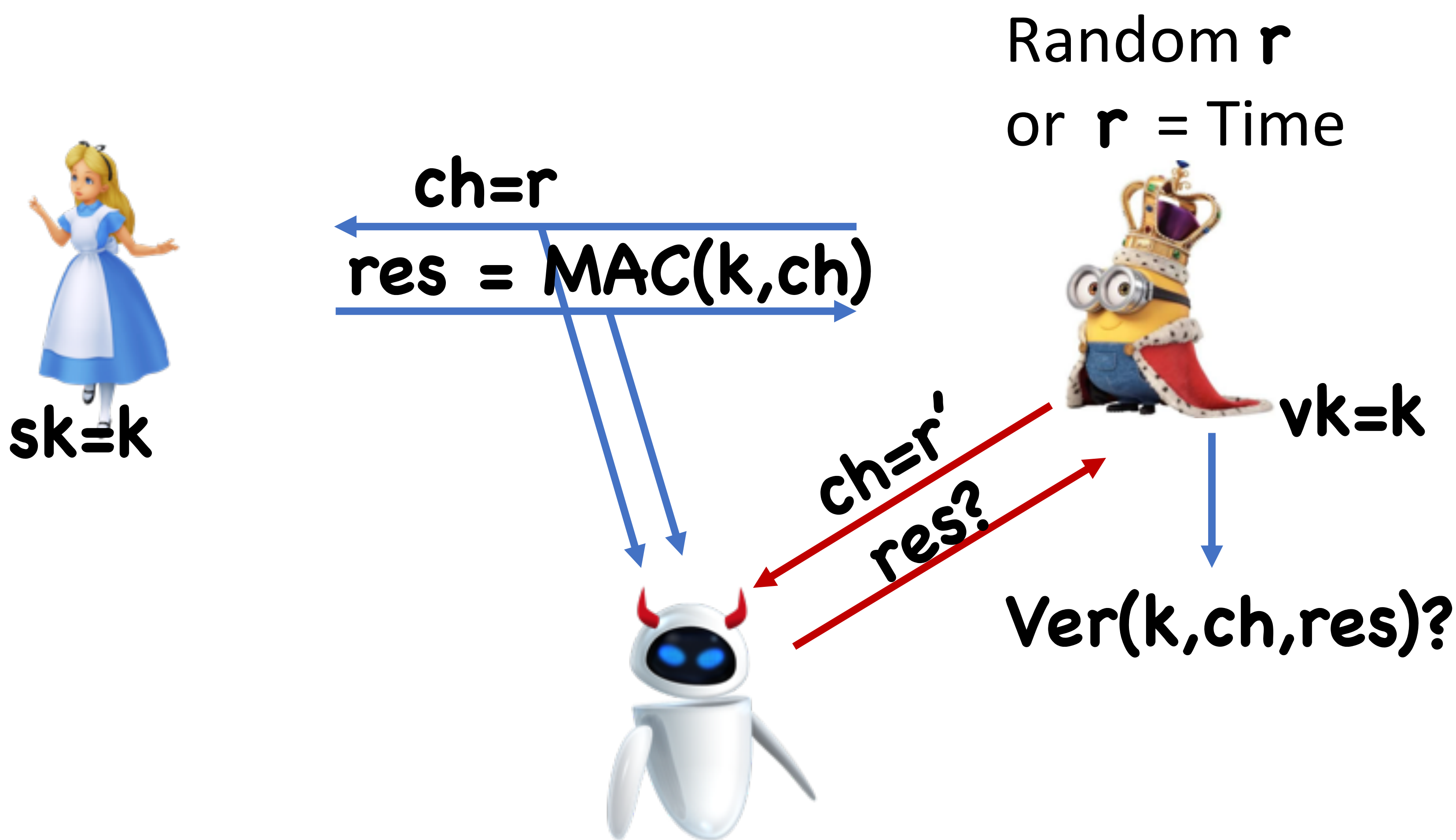
C-R Using Encryption



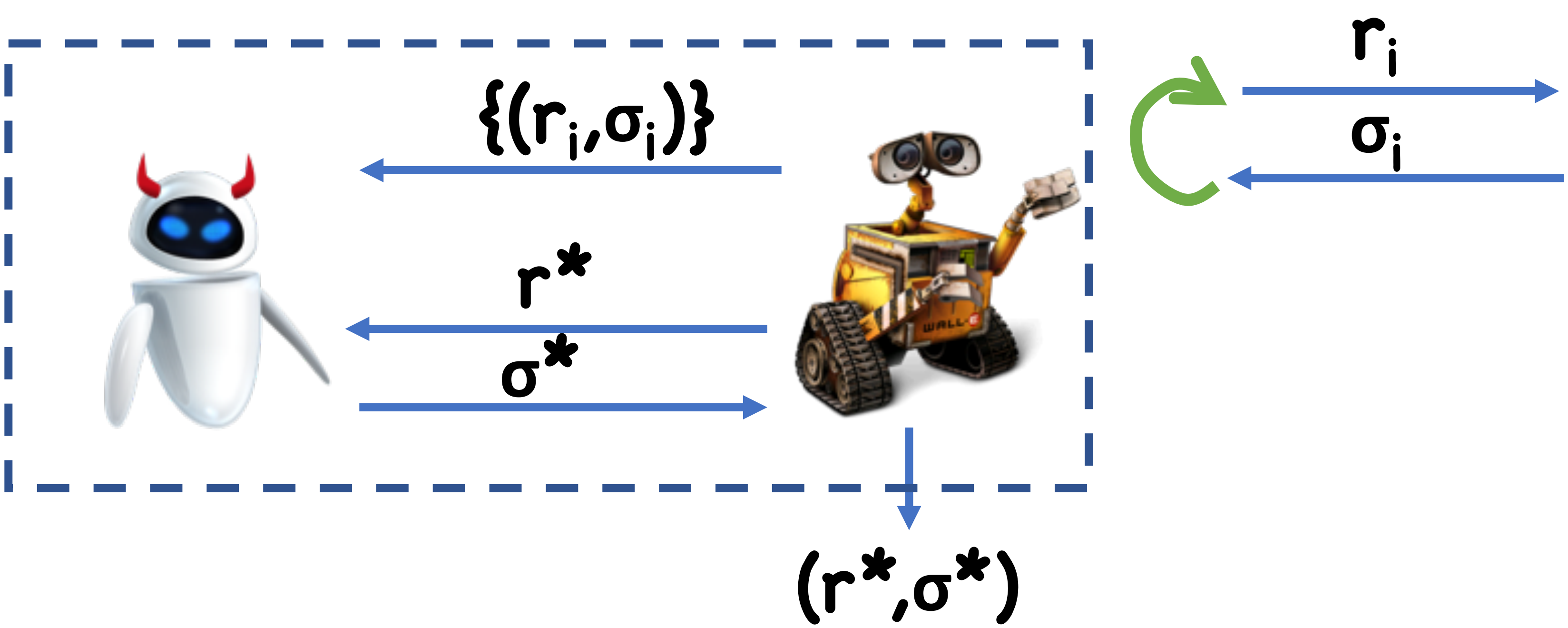
Theorem: If **(Enc,Dec)** is a CPA-secure secure SKE/PKE scheme, then the C-R protocol is a secret key/public key identification protocol secure against eavesdropping attacks



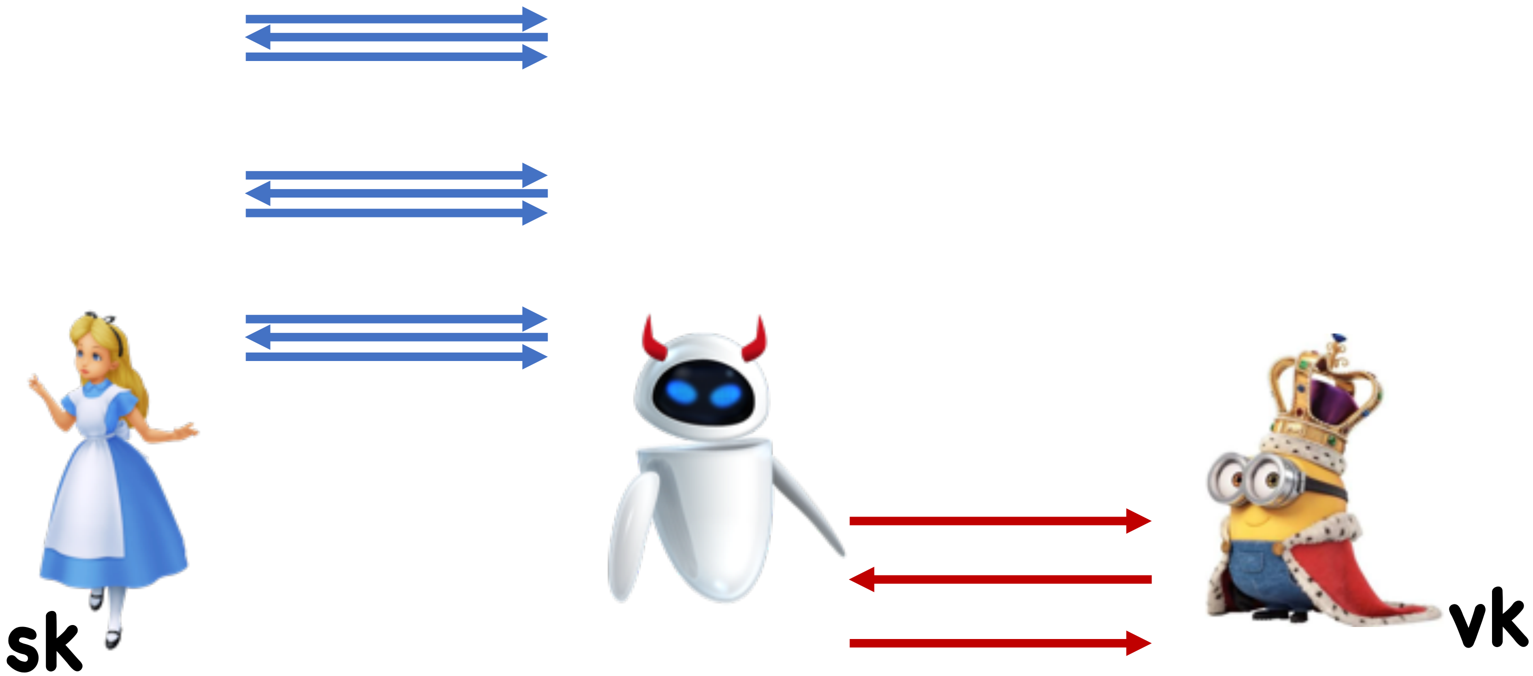
C-R Using MACs/Signatures



Theorem: If **(MAC,Ver)** is a CMA-secure secure MAC/Signature scheme, then the C-R protocol is a secret key/public key identification protocol secure against eavesdropping attacks



Active Attacks



Active Attacks

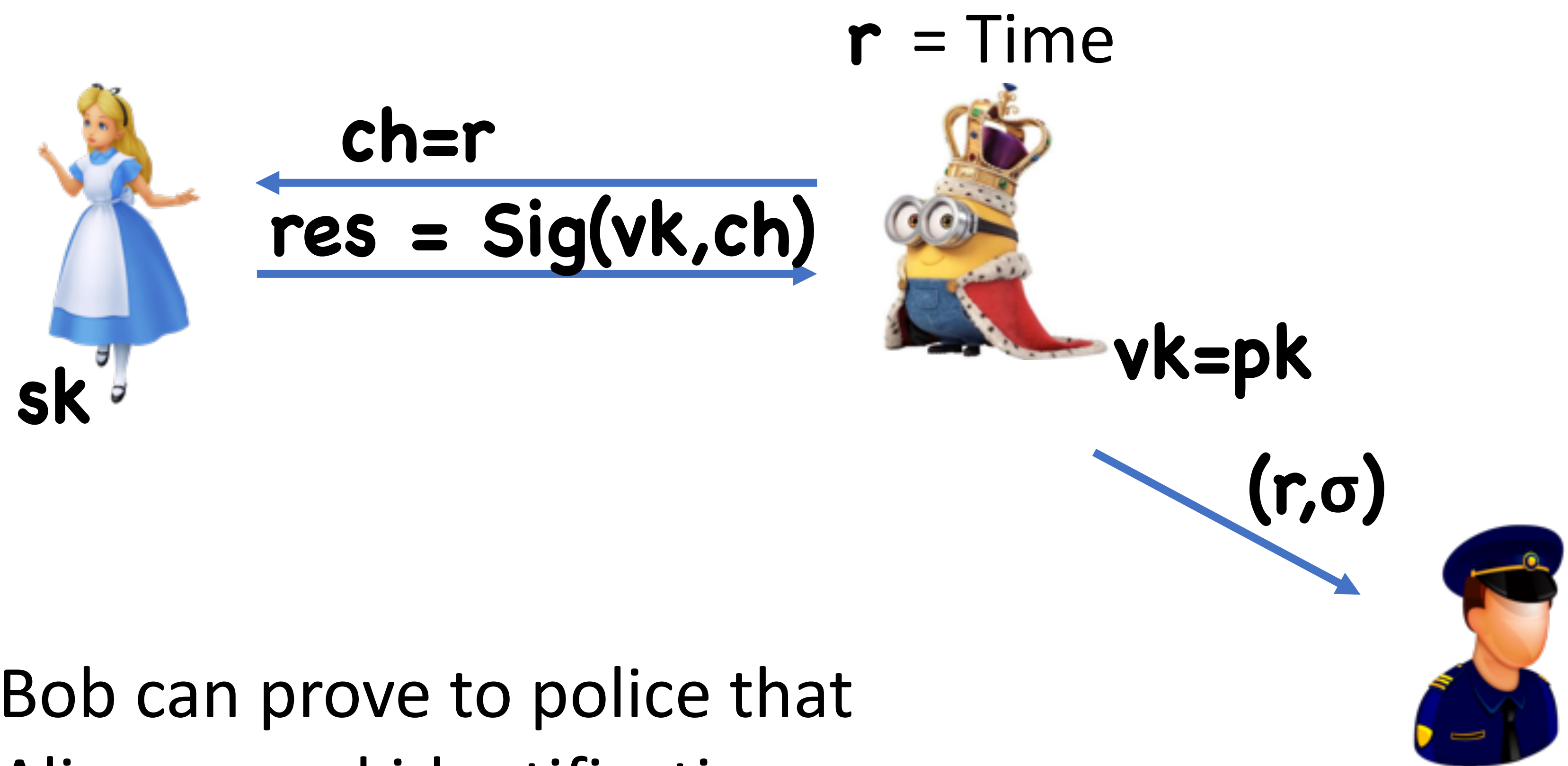
For enc-based C-R, CPA-secure is insufficient

- Instead need CCA-security (lunch-time sufficient)

For MAC/Sig-based C-R, CMA-security is sufficient

Non-Repudiation

Consider signature-based C-R



Bob can prove to police that
Alice passed identification

Zero Knowledge

What if Bob could have come up with a valid transcript, without ever interacting with Alice?

- Then Bob cannot prove to police that Alice authenticated

Seems impossible:

- If (public) **vk** is sufficient to come up with valid transcript, why can't an adversary do the same?

Zero Knowledge

Adversary CAN come up with valid transcripts, but Bob doesn't accept transcripts

- Instead, accepts i*nteractions*

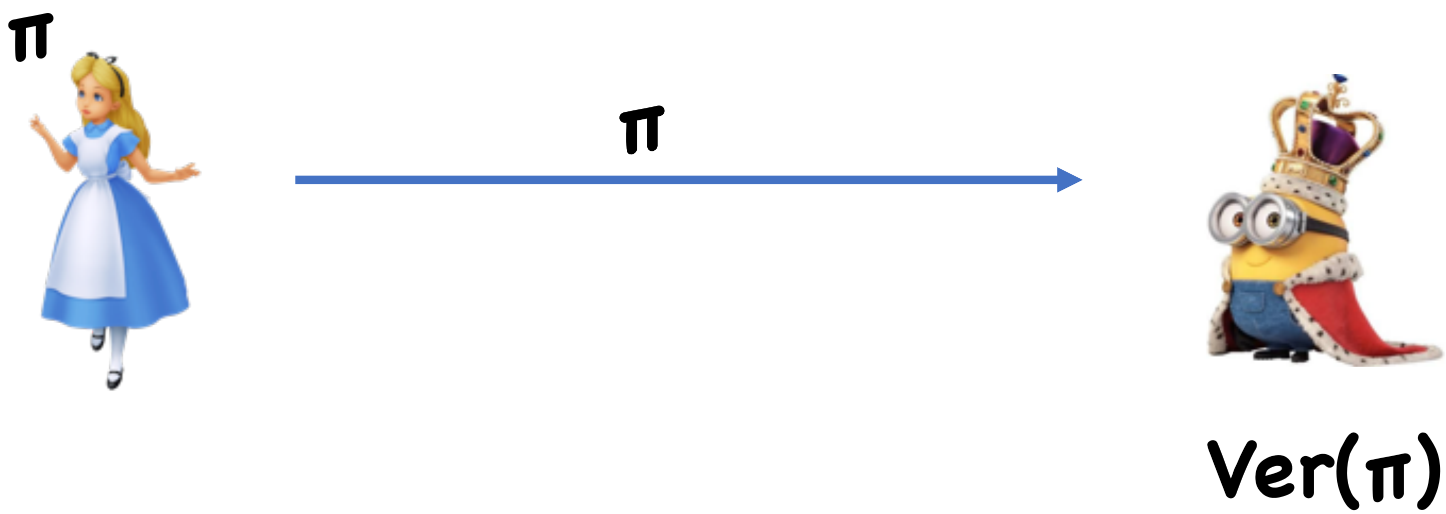
Ex: public key Enc-based C-R

- Valid transcript: **(c,r)** where **c** encrypts **r**
- Anyone can come up with a valid transcript
- However, only Alice can generate the transcript for a given **c**

Takeaway: order of messages matters

Zero Knowledge Proofs

Mathematical Proof



Mathematical Proof

Statement **x**

Witness **w**

w →

Ver(x,w)

Interactive Proof

Statement **x**

Witness **w**

Properties of Interactive Proofs

Let **(P,V)** be a pair of probabilistic interactive algorithms for the proof system

Completeness: If **w** is a valid witness for **x**, then **V** should always accept

Soundness: If **x** is false, then no cheating prover can cause **V** to accept

- Perfect: accept with probability **0**
- Statistical: accept with negligible probability
- Computational: cheating prover is comp. bounded

Zero Knowledge

Intuition: verifier doesn't learn anything by engaging in the protocol (other than the truthfulness of **x**)

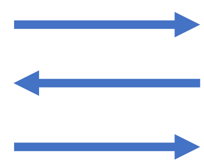
How to characterize what adversary "knows"?

- Only outputs a bit
- May "know" witness, but hidden inside the programs state

Zero Knowledge

First Attempt:

∃ "simulator"  s.t. for every true statement **x**, valid witness **w**,

**(x)** $\approx_c$ **P(x,w)**  **V(x)**

Zero Knowledge

First Attempt:

Assumes Bob obeys protocol

- "Honest Verifier"

But what if Bob deviates from specified prover algorithm to try and learn more about the witness?

Zero Knowledge

For every malicious verifier **V***, ∃ "simulator" 
s.t. for every true statement **x**, valid witness **w**,

**(x)** $\approx_c$ **P(x,w)** ⇄ **V*(x)**

QR Protocol

Statements: $\mathbf{x}$ is a Q.R. mod $\mathbf{N}$

Witness: $\mathbf{w}$ s.t. $\mathbf{w^2 \bmod N = x}$

Protocol:

w

$u \leftarrow Z_N^*$

$y \leftarrow u^2 \bmod N$

y (→)

b (←)

$z = w^b u \bmod N$ (→)

$b \leftarrow \{0,1\}$

$z^2 == x^b y \bmod N?$

QR Protocol

Zero Knowledge:

What does Bob see?

- A random QR **y**,
- A random bit **b**,
- A random root of $x^b y$

Idea: simulator chooses **b**, then **y**,

- Can choose **y** s.t. it always knows a square root of $x^b y$

QR Protocol

Honest Verifier Zero Knowledge:

 **(x):**

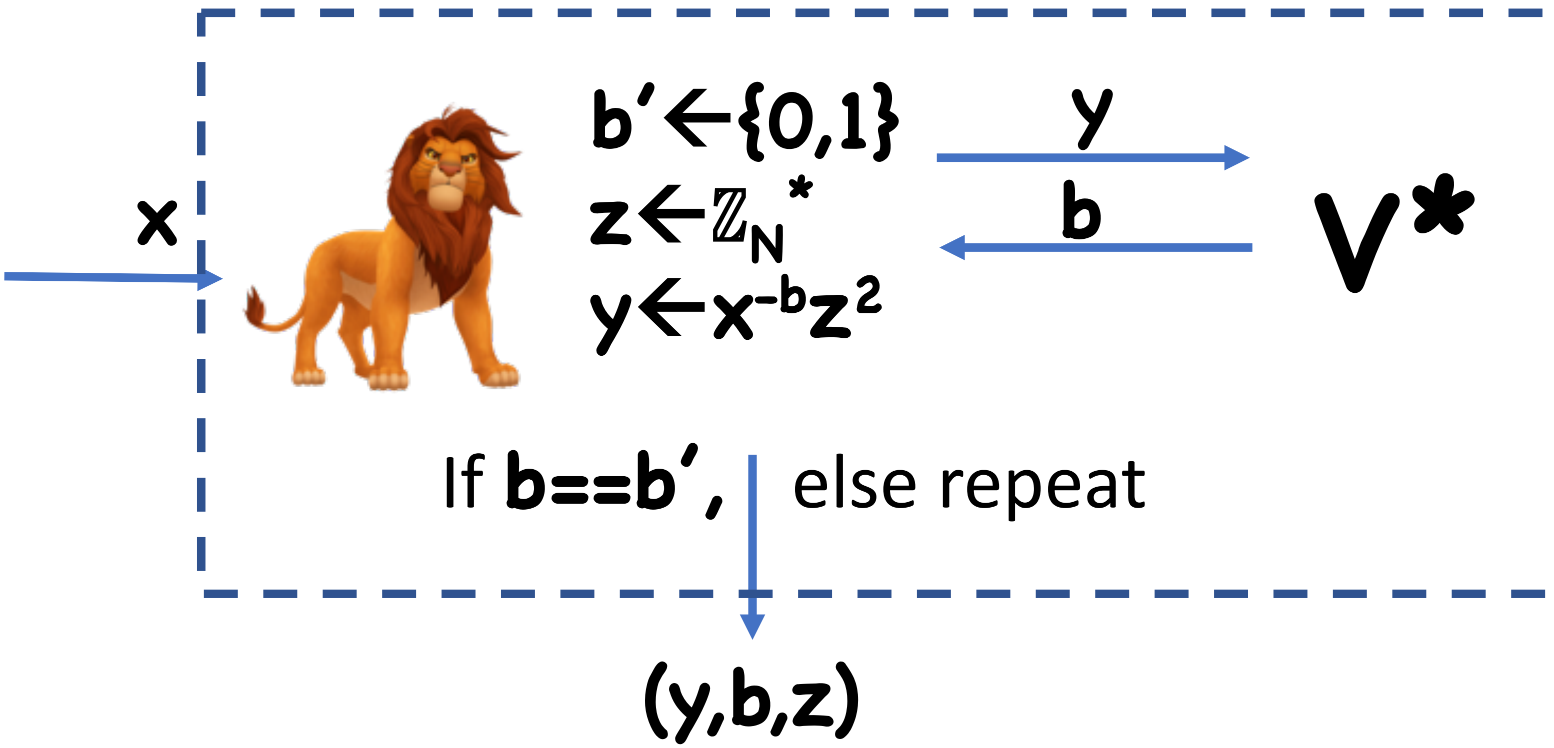
- Choose a random bit **b**
- Choose a random string **z**
- Let $y = x^{-b}z^2$
- Output **(y,b,z)**

- If **x** is a QR, then **y** is a random QR, no matter what **b** is
- **z** is a square root of $x^b y$

(y,b,z) is distributed identically to **(P,V)(x)**

QR Protocol

(Malicious Verifier) Zero Knowledge:



QR Protocol

(Malicious Verifier) Zero Knowledge:

Proof:

- If **x** is a QR, then **y** is a random QR, independent of **b'**
- Conditioned on **b'=b**, then **(y,b,z)** is identical to random transcript seen by **V***
- **b'=b** with probability **1/2**

Repetition and Zero Knowledge

(sequential) repetition also preserves ZK

Unfortunately, parallel repetition might not:

-  makes guesses $b_1', b_2', \ldots$
- Generates valid transcript only if all guesses were correct
- Probability of correct guess: 2^{-t}

Maybe other simulators will work?

- Known to be impossible in general, but nothing known for QR

Zero Knowledge Proofs

Known:

- Proofs for any NP statement assuming statistically-binding commitments
- Non-interactive ZK proofs for any NP statement using trapdoor permutations

Proofs of Knowledge

Sometimes, not enough to prove that statement is true, also want to prove "knowledge" of witness

Ex:

- Identification protocols: prove knowledge of key
- Discrete log: always exists, but want to prove knowledge of exponent.

Proofs of Knowledge

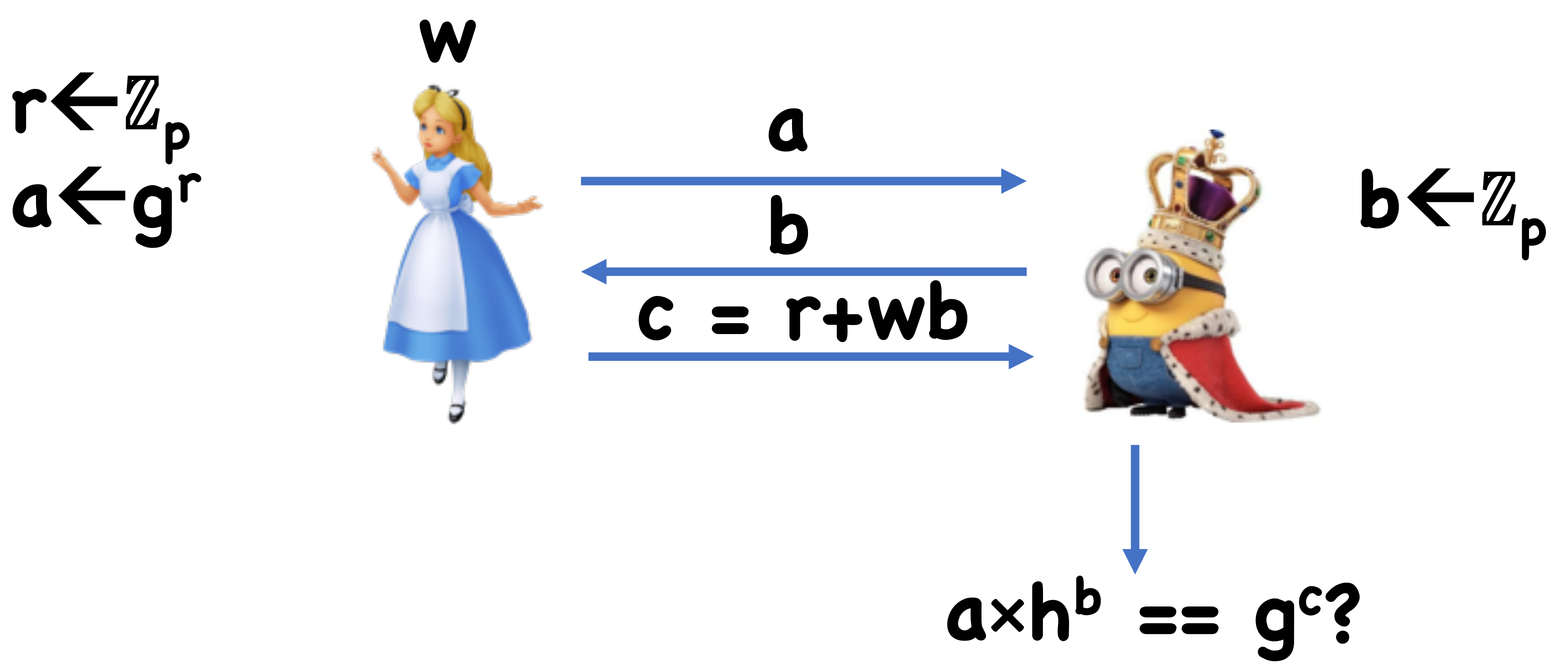
We won't formally define, but here's the intuition:

Given any (potentially malicious) PPT prover $\mathbf{P^*}$ that causes $\mathbf{V}$ to accept, it is possible to "extract" from $\mathbf{P^*}$ a witness $\mathbf{w}$

Schnorr PoK for DLog

Statement: **(g,h)**
Witness: **w** s.t. **$h=g^w$**

Protocol:



Schnorr PoK for DLog

Completeness:

- $g^c = g^{r+wb} = a \times h^b$

Honest Verifier ZK:

- Transcript = (a,b,c) where $a=g^c/h^b$ and (b,c) random in $\mathbb{Z}_p$
- Can easily simulate. How?

Schnorr PoK for DLog

Proof of Knowledge?

Idea: once Alice commits to $\mathbf{a=g^r}$, show must be able to compute $\mathbf{c = r+bw}$ for any $\mathbf{b}$ of Bob's choosing

- Intuition: only way to do this is to know $\mathbf{w}$
- Run Alice on two challenges, obtain:

$$\mathbf{c_0 = r_0 + b_0\ w},\ \mathbf{c_1 = r_1 + b_1\ w}$$

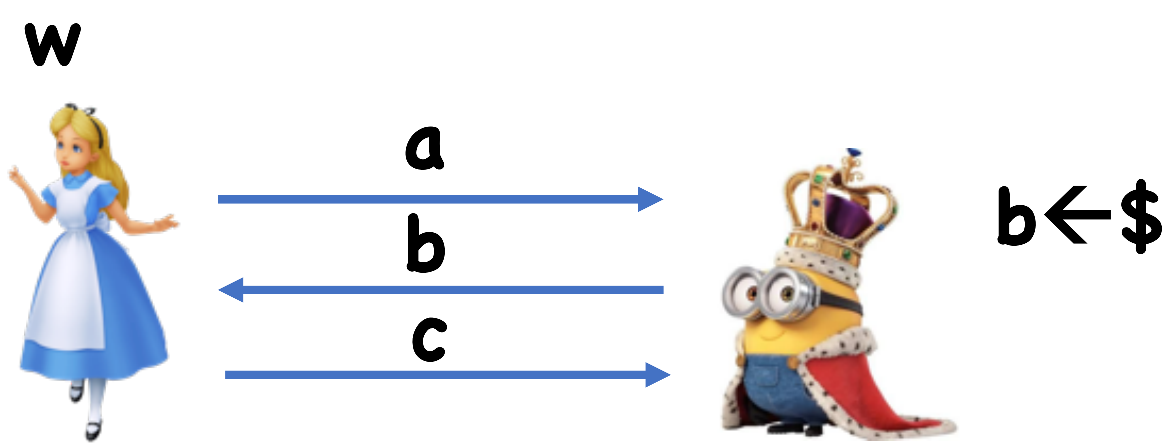
(Can solve linear equations to find $\mathbf{w}$)

Deniability

Zero Knowledge proofs provide deniability:

- Alice proves statement $\mathbf{x}$ is true to Bob
- Bob goes to Charlie, and tries to prove $\mathbf{x}$ by providing transcript
- Charlie not convinced, as Bob could have generated transcript himself
- Alice can later deny that she knows proof of $\mathbf{x}$

∑ Protocols



(fancy name for 3-round "public coin" protocols)

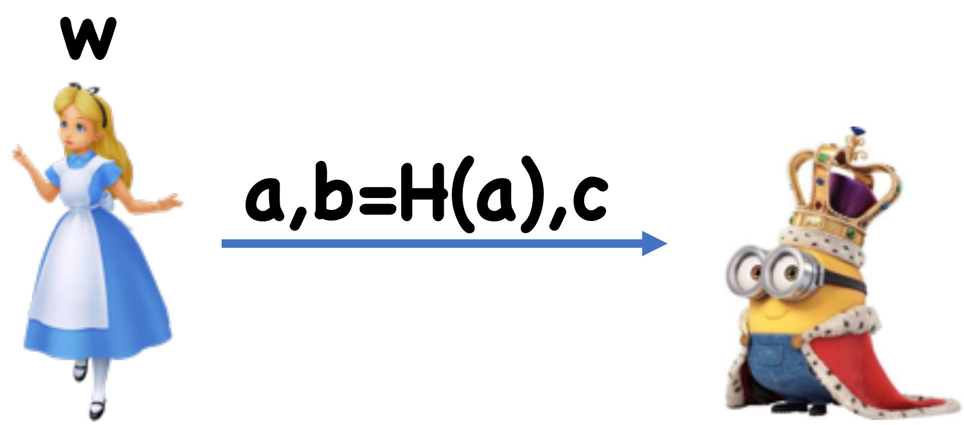
Fiat-Shamir Transform

Idea: set **b = H(a)**

- Since **H** is a random oracle, **a** is a random output

Notice: now prover can compute **b** for themselves!

- No need to actually perform interaction



Theorem: If (P,V) was a secure ZKPoK for honest verifiers, and if H is a random oracle, then compiled protocol is a ZKPoK

Proof idea: second message is exactly what you'd expect in original protocol

Complication: adversary can query H to learn second message, and throw it out if she doesn't like it

Signatures from ∑ Protocols

Idea: what if set **b = H(m,a)**

- Challenge **b** is message specific
- Intuition: proves that someone who knows **sk** engaged in protocol depending on **m**
- Can use resulting transcript as signature on **m**

Schnorr PoK → Schnorr Signatures

Applications of ZK (PoK)

Identification protocols: prove that you know the secret without revealing the secret

Signatures: prove that you know the secret in a “message dependent” way

Protocol Design:

- E.g. CCA secure PKE
  - To avoid mauling attacks, provide ZK proof that ciphertext is well formed
  - Problem: ZK proof might be malleable
  - With a bit more work, can be made CCA secure
- Example: multiparty computation
  - Prove that everyone behaved correctly

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