COS433/Math 473: Cryptography

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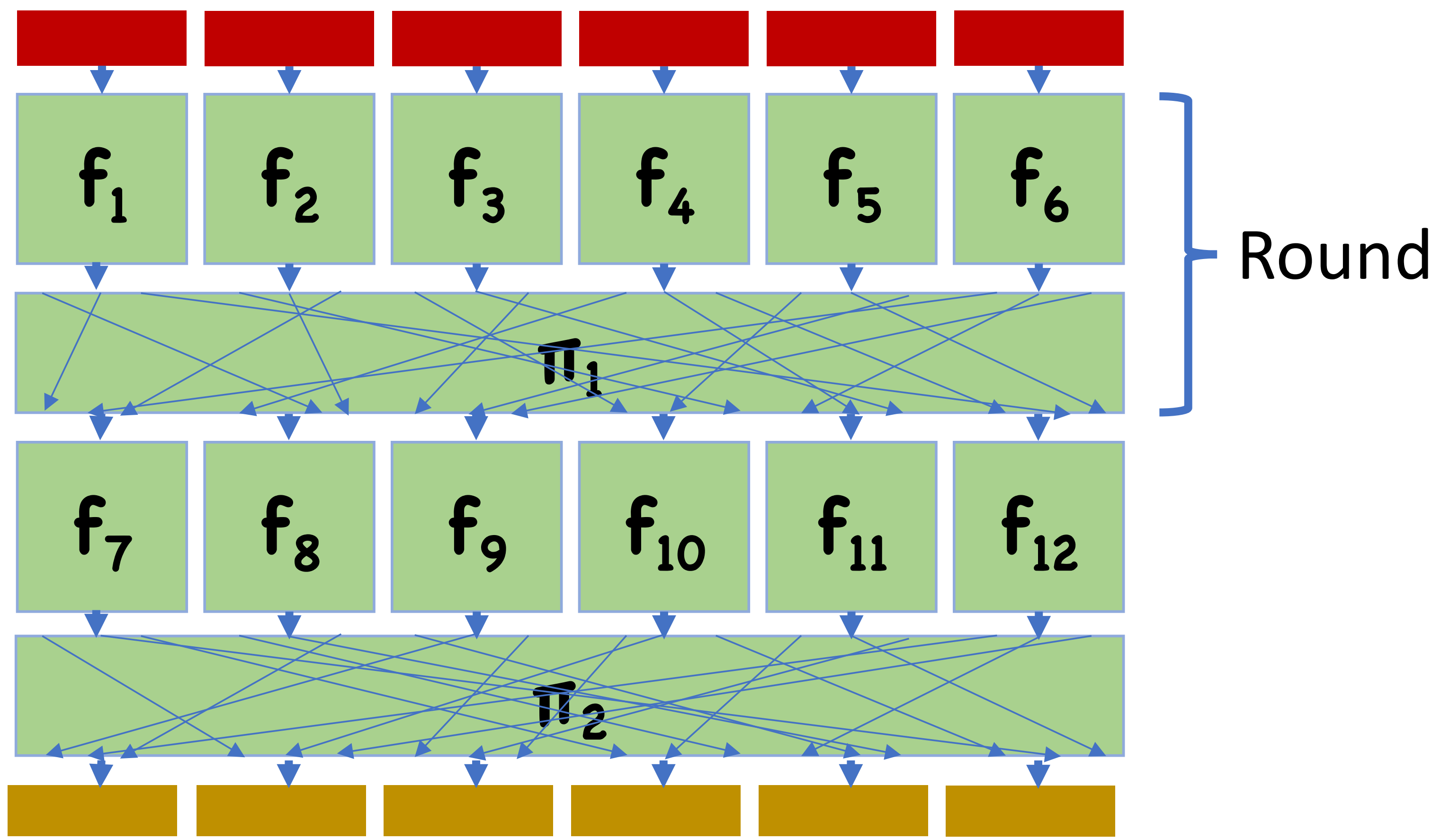
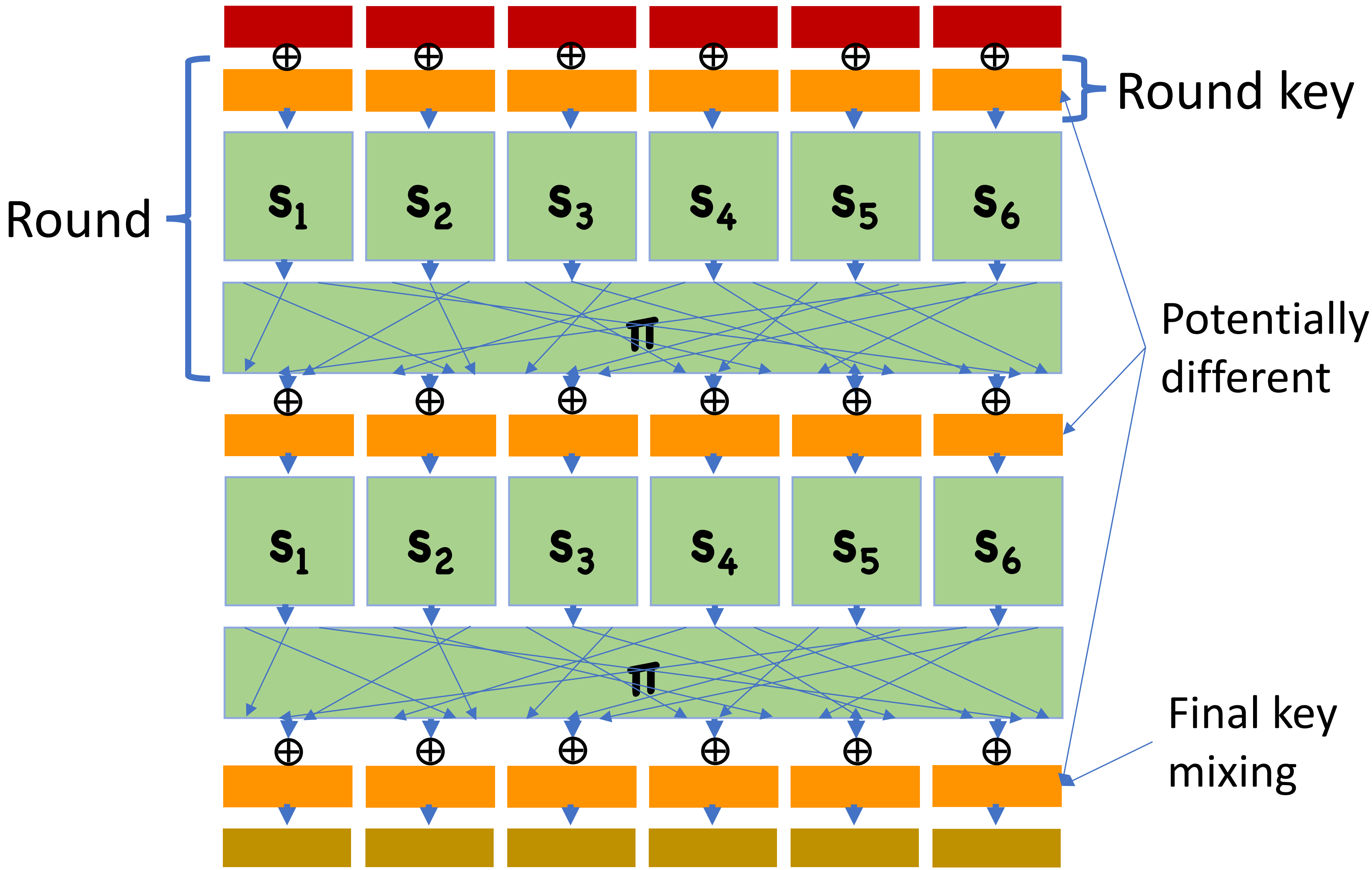
Previously on COS 433…

Confusion/Diffusion Paradigm



Substitution Permutation Networks



Designing SPNs

Avalanche Affect:

- Need S-boxes and mixing permutations to cause every input bit to ”affect” every output bit

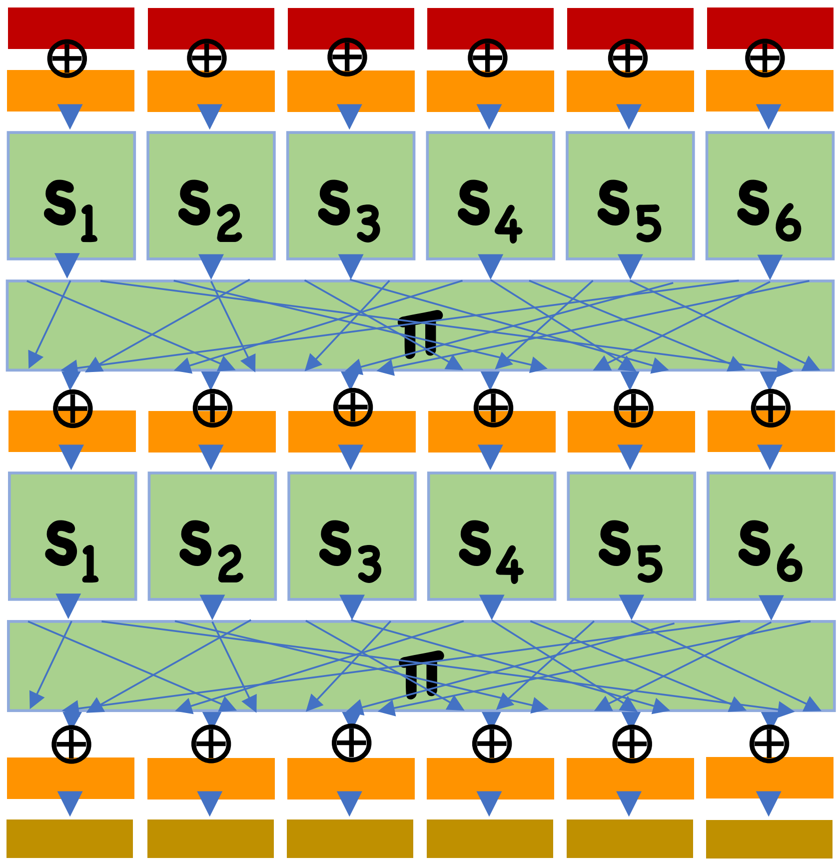
One way to guarantee this:

- Changing any bit of S-box input causes at least 2 bits of output to change
- Mixing permutations send outputs of S-boxes into at least 2 different S-boxes for next round
- Sufficiently many rounds are used
- At least how many rounds should be used?

Linearity?

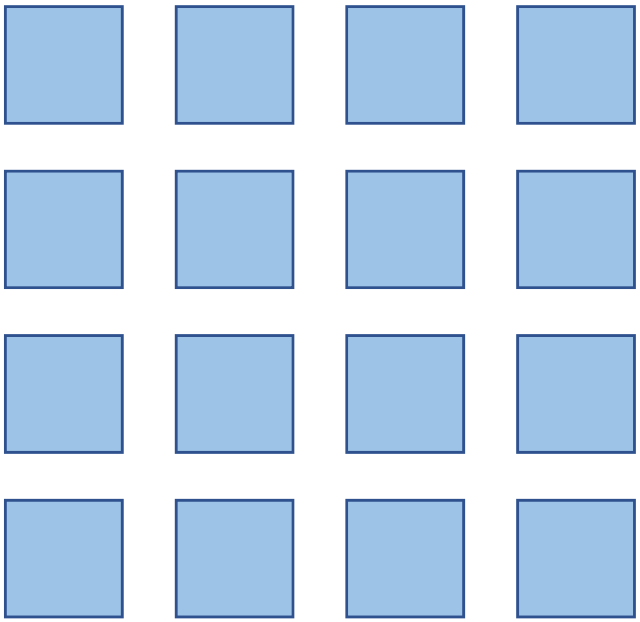
Can S-Boxes be linear?

- That is, $\mathbf{S(x_0)} \oplus \mathbf{S(x_1)} = \mathbf{S(x_0 \oplus x_1)}$?



AES

State = **4×4** grid of bytes



AES

One fixed S-box, applied to each byte

- Step 1: multiplicative inverse over finite field $\mathbb{F}_8$
- Step 2: fixed affine transformation
- Implemented as a simple lookup table

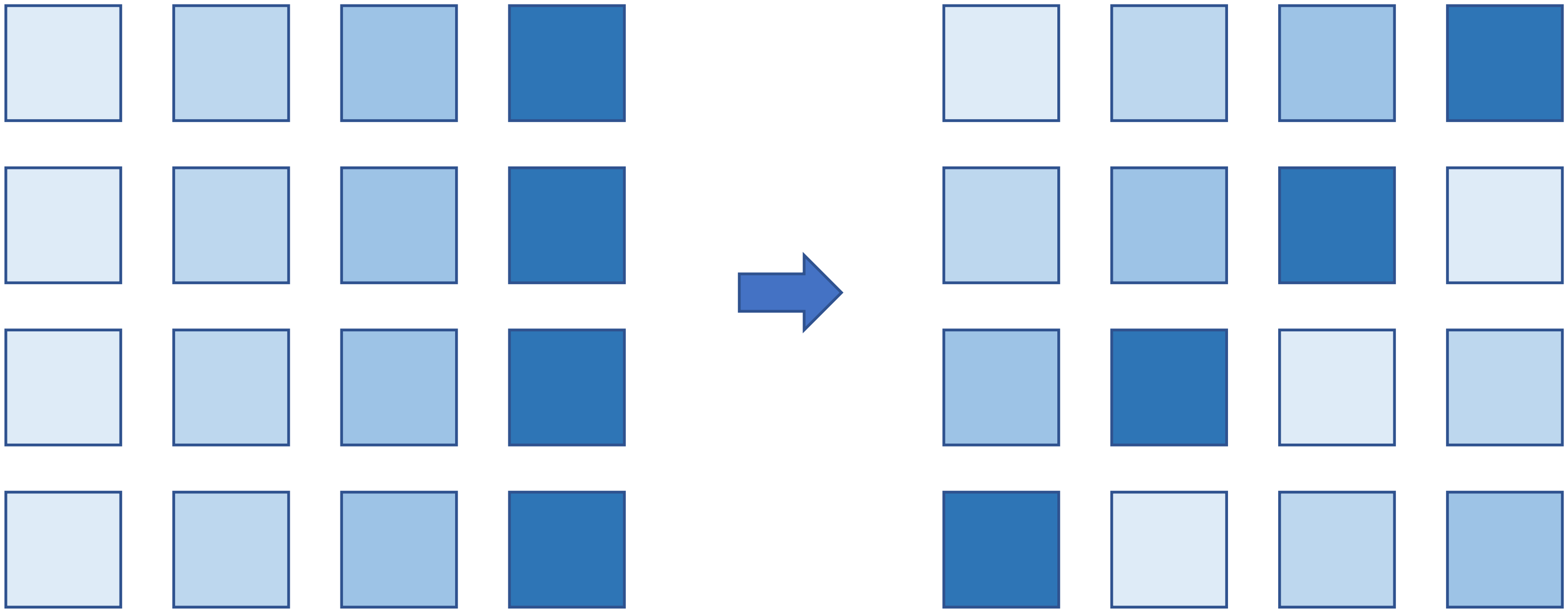
AES

Diffusion (not exactly a P-box):

- Step 1: shift rows
- Step 2: mix columns

AES

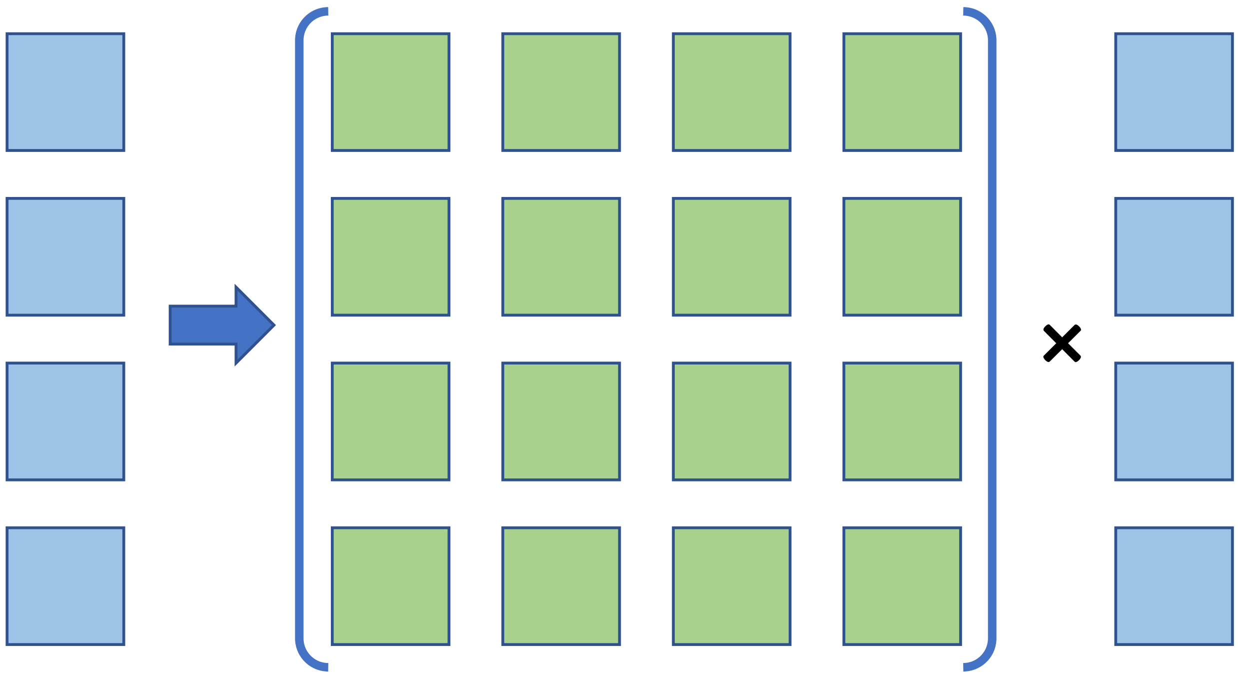
Shift Rows:



AES

Mix Columns

- Each byte interpreted as element of $\mathbb{F}_8$
- Each column is then a length-4 vector
- Apply fixed linear transformation to each column



AES

Number of rounds depends on key size

- 128-bit keys: 10 rounds
- 192-bit keys: 12 rounds
- 256-bit keys: 14 rounds

Key schedule:

- Won’t describe here, but involves more shifting, S-boxes, etc
- Can think of key schedule as a weak PRG

Today

Feistel Networks

Attacks on block ciphers and PRFs

Feistel Networks

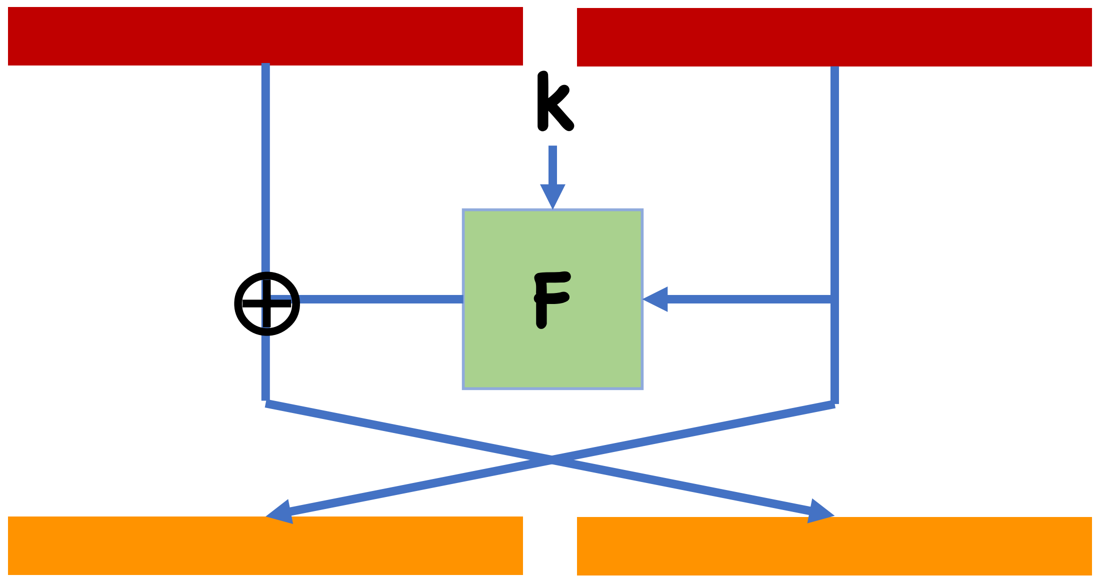
Feistel Networks

Designing permutations with good security properties is hard

What if instead we could built a good permutation from a function with good security properties...

Feistel Network

Convert functions into permutations



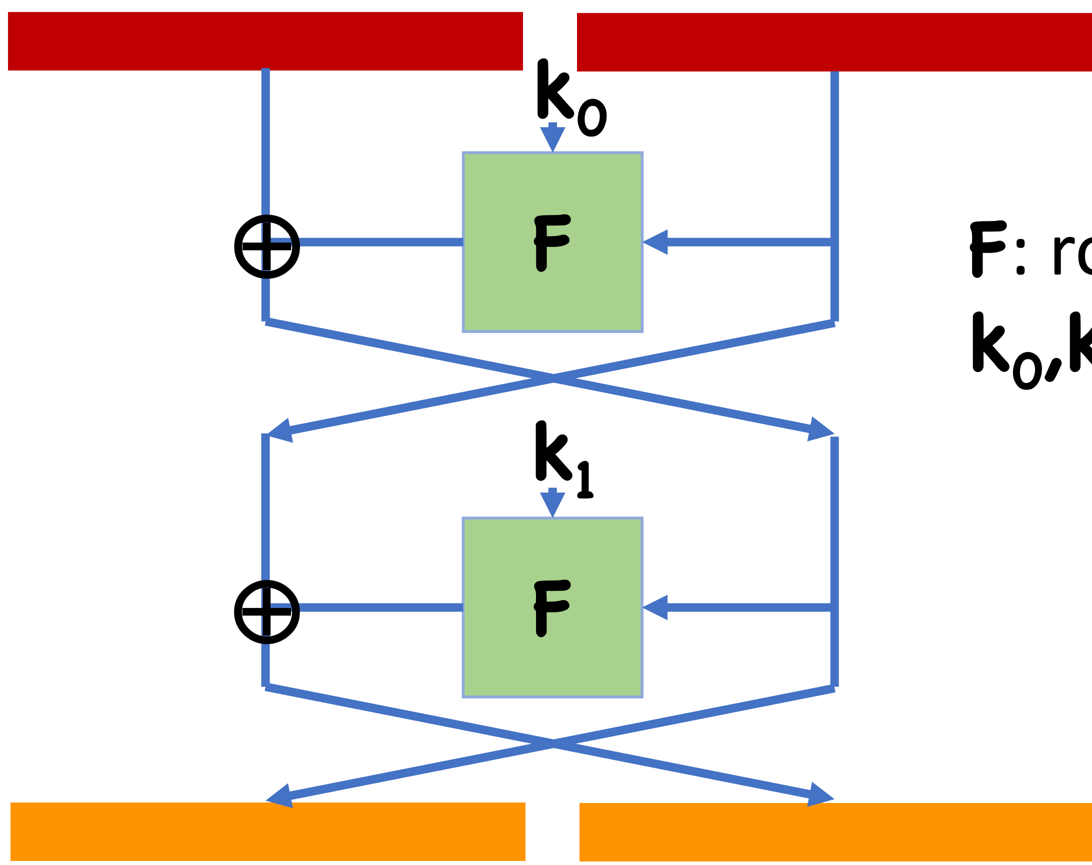
F public

k secret

Can this possibly give a secure PRP?

Feistel Network

Convert functions into permutations



F: round function

k_0,k_1: round keys

Feistel Network

Depending on specifics of round function, different number of rounds may be necessary

- Number of rounds must always be at least 3
- (Need at least 4 for a strong PRP)
- Maybe need even more for weaker round functions

Luby-Rackoff

3- or 4-round Feistel where round function is a PRF

Theorem: If F is a secure PRF, then 3 rounds of Feistel (with independent round keys) give secure PRP. 4 rounds give a strong PRP

- Proof non-trivial, won't be covered in this class

Limitations of Feistel Networks

Turns out Feistel requires block size to be large
- If number of queries ~$2^{\text{block size}/2}$, can attack

Format preserving encryption:
- Encrypted data has same form as original
- E.g. encrypted SSN is an SSN
- Useful for encrypting legacy databases

Sometimes, want a very small block size

Constructing Round Functions

Ideally, “random looking” functions

Similar ideas to constructing PRPs
- Confusion/diffusion
- SPNs, S-boxes, etc

Key advantage is that we no longer need the functions to be permutations
- S-boxes can be non-permutations

DES

Block size: 64 bits
Key size: 56 bits
Rounds: 16

DES

Key Schedule:

- Round keys are just 48-bit subsets of master key

Round function:

- Essentially an SPN network

DES S-Boxes

8 different S-boxes, each

- 6-bit input, 4-bit output
- Table lookup: 2 bits specify row, 4 specify column

- Each row contains every possible 4-bit output
- Changing one bit of input changes at least 2 bits of output

DES History

Designed in the 1970's

- At IBM, with the help of the NSA
- At the time, many in academia were suspicious of NSA's involvement
  - Mysterious S-boxes
  - Short key length
- Turns out, S-box probably designed well
  - Resistant to "differential cryptanalysis"
  - Known to IBM and NSA in 1970's, but kept secret
- Essentially only weakness is the short key length
  - Maybe secure in the 1970's, definitely not today

DES Security Today

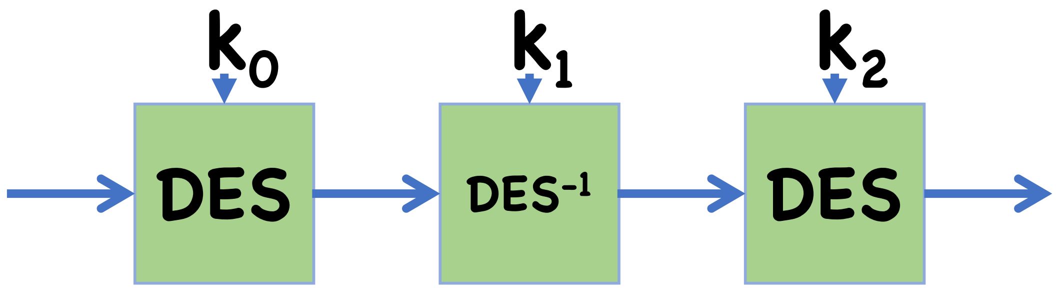
Seems like a good cipher, except for its key length and block size

What's wrong with a small block size?

- Remember for e.g. CTR mode, IV is one block
- If two identical IV's seen, attack possible
- After seeing q ciphertext, probability of repeat IV is roughly $q^2/2^{\text{block length}}$
- Attack after seeing ≈ billion messages

3DES: Increasing Key Length

3DES key = Apply DES three times with different keys



Why three times?

- Next time: “meet in the middle attack” renders 2DES no more secure than 3DES

Why inverted second permutation?

Attacks on block ciphers

Brute Force Attacks

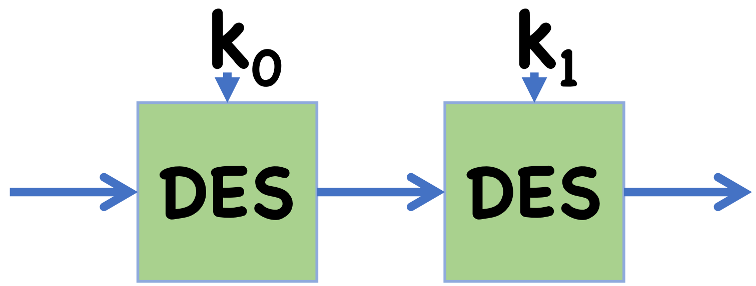
Suppose attacker is given a few input/output pairs

Likely only one key could be consistent with this input/output

Brute force search: try every key in the key space, and check for consistency

Attack time: $2^{\text{key length}}$

Insecurity of 2DES



DES key length: 56 bits

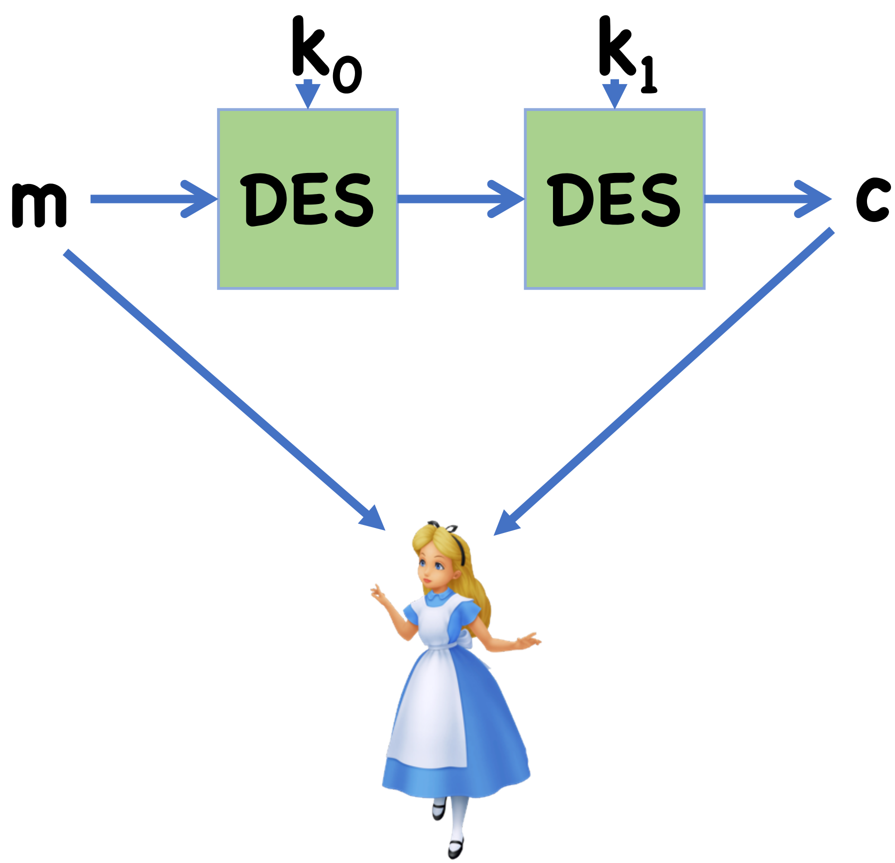
2DES key length: 112 bits

Brute force attack running time: 2^{112}

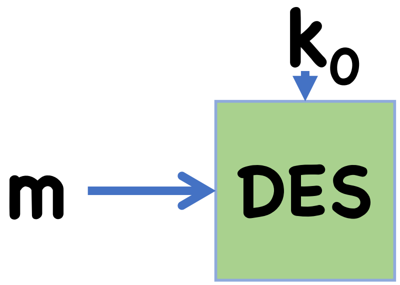
Meet In The Middle Attacks

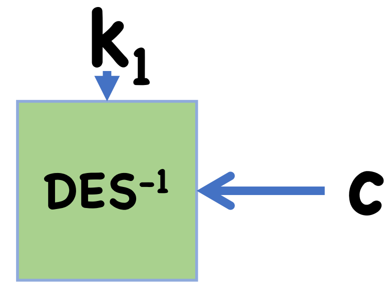
For 2DES, can actually find key in 2^{56} time

- Also $\approx 2^{56}$ space



Meet In The Middle Attacks





k_0	d = DES(k_0,m)
0	52
1	93
2	03
3	96
4	20
5	49
…	…

k_1	d = DES^{-1}(k_1,m)
0	69
1	10
2	86
3	49
4	99
5	08
…	…

Meet In The Middle Attacks

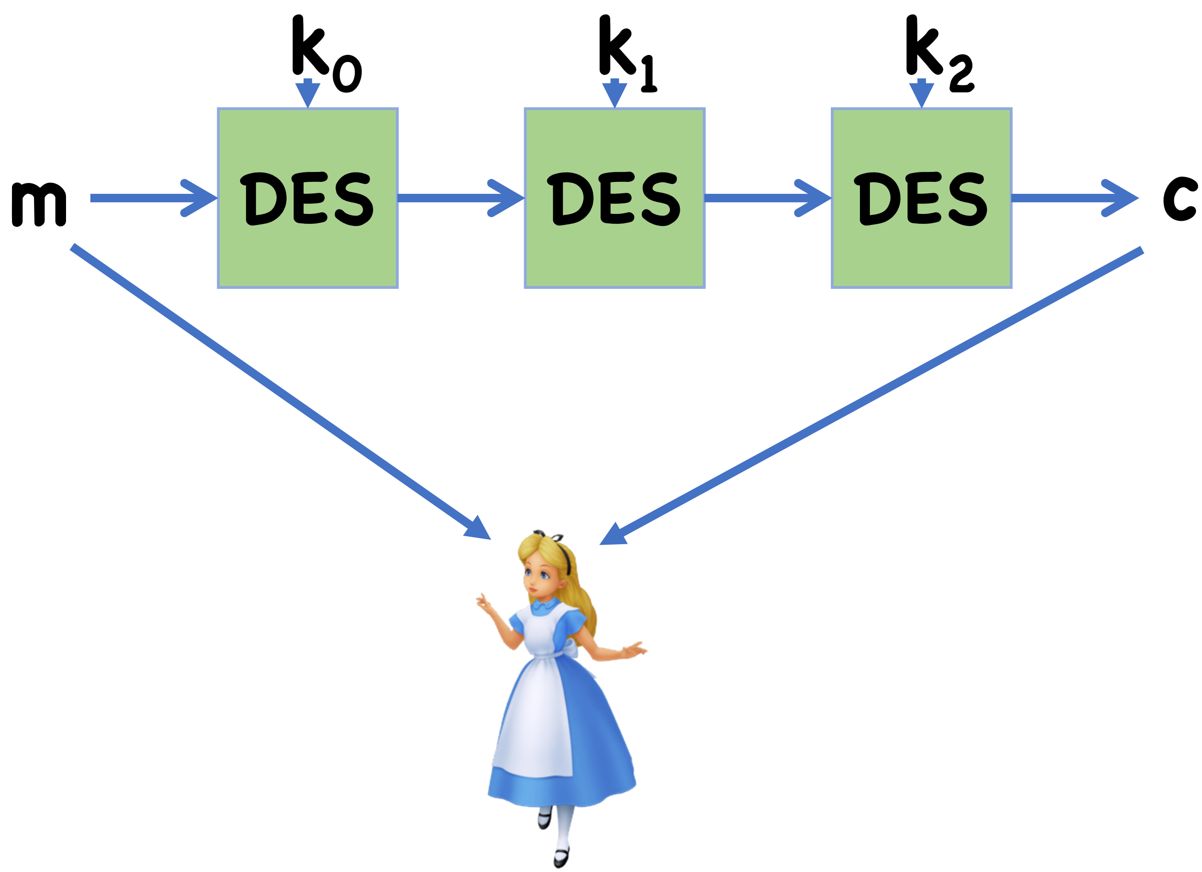
Complexity of meet in the middle attack:

- Computing two tables: time, space $2 \times 2^{\text{key length}}$
- Slight optimization: don't need to actually store second table

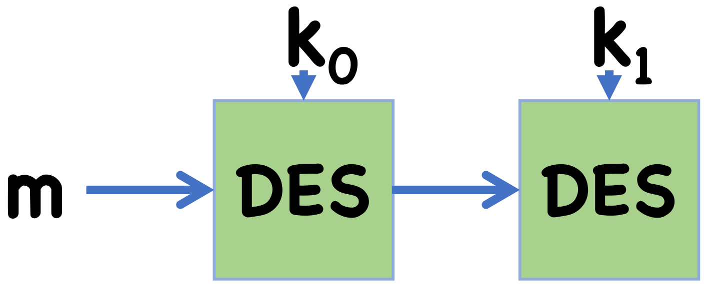
On 2DES, roughly same time complexity as brute force on DES

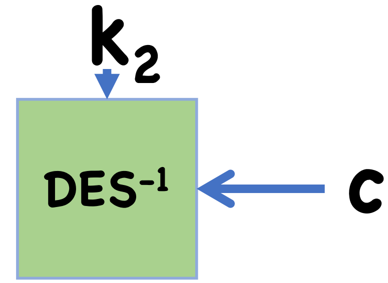
MITM Attacks on 3DES

MITM attacks also apply to 3DES…



MITM for 3DES





k_0	k_1	$d = DES(k_0,m)$
0	0	52
0	1	93
…	…	03
5	6	96
5	7	20
5	8	49
…		…

k_2	$d = DES^{-1}(k_2,m)$
0	69
1	10
2	86
3	49
4	99
5	08
…	…

MITM for 3DES

No matter where "middle" is, need to have two keys on one side

- Must go over 2^{112} different keys

Space?

While 3DES has 168 bit keys, effective security is 112 bits

Generalizing MITM

In general, given r rounds of a block cipher with t-bit keys,

- Attack time: $2^{t\lceil r/2 \rceil}$
- Attack space: $2^{t\lfloor r/2 \rfloor}$

Brute Force vs. Generic Attacks

MITM attacks on iterated block ciphers are *generic*

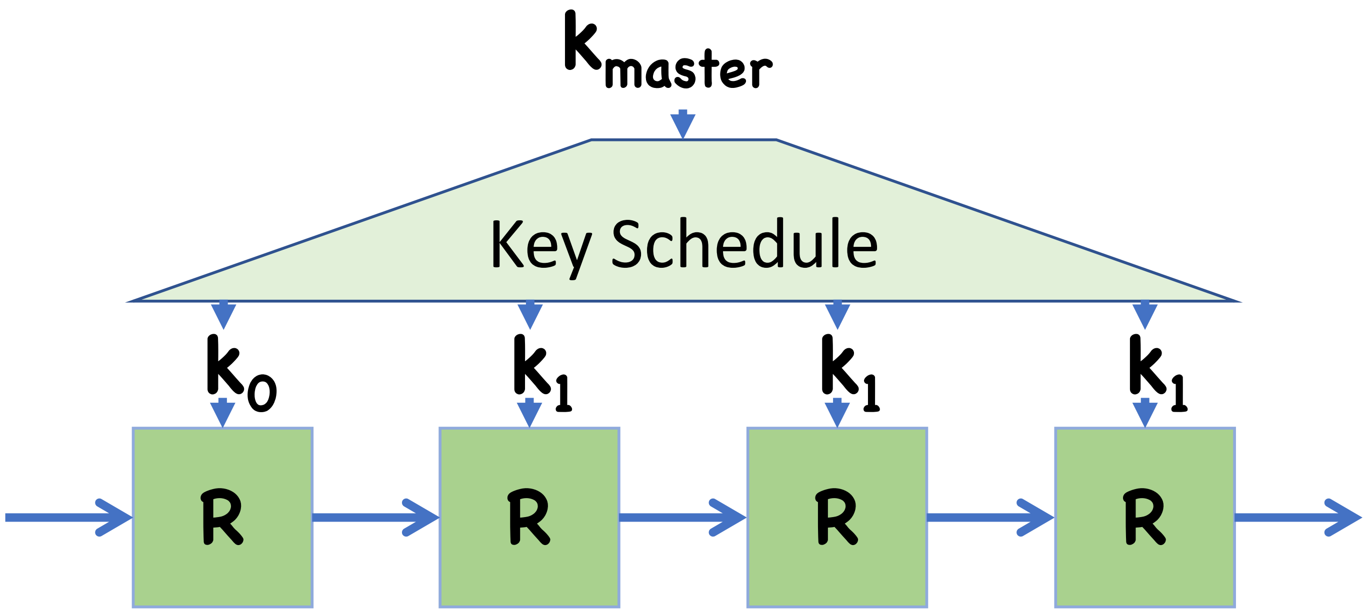
- Attack exists independent of implementation details of block cipher

However, still beats a *brute force*

- Doesn’t simply try every key

MITM Attacks

MITM attacks can also be applied to plain single block ciphers



Can yield reasonable attacks if the key schedule produces highly independent round keys

Time-Space Tradeoffs

MITM attack requires significant space

In contrast, brute force requires essentially no space, but runs slower

Known as a time-space tradeoff

Another Time-Space Trade-off Example

Given $y=F(k,x)$, find x

- Allowed many queries to $F(k,x)$ oracle
  (That is, standard block cipher oracle)
- Assume $|k| >> |x|$

Option 1:

- Brute force search over entire domain looking for x
- Time: 2^l
- Space: 1

Another Time-Space Trade-off Example

Given **y=F(k,x)**, find **x**

- Allowed many queries to **F(k,x)** oracle
  (That is, standard block cipher oracle)
- Assume **|k| >> |x|**

Option 2: Preprocessing

- Before seeing **y**, compute giant table of **(x,F(k,x))** pairs, sorted by **F(k,x)**
- Preprocessing Time: 2^l
- Space: 2^l
- Online time: ?

Option 3: Hellman's Attack

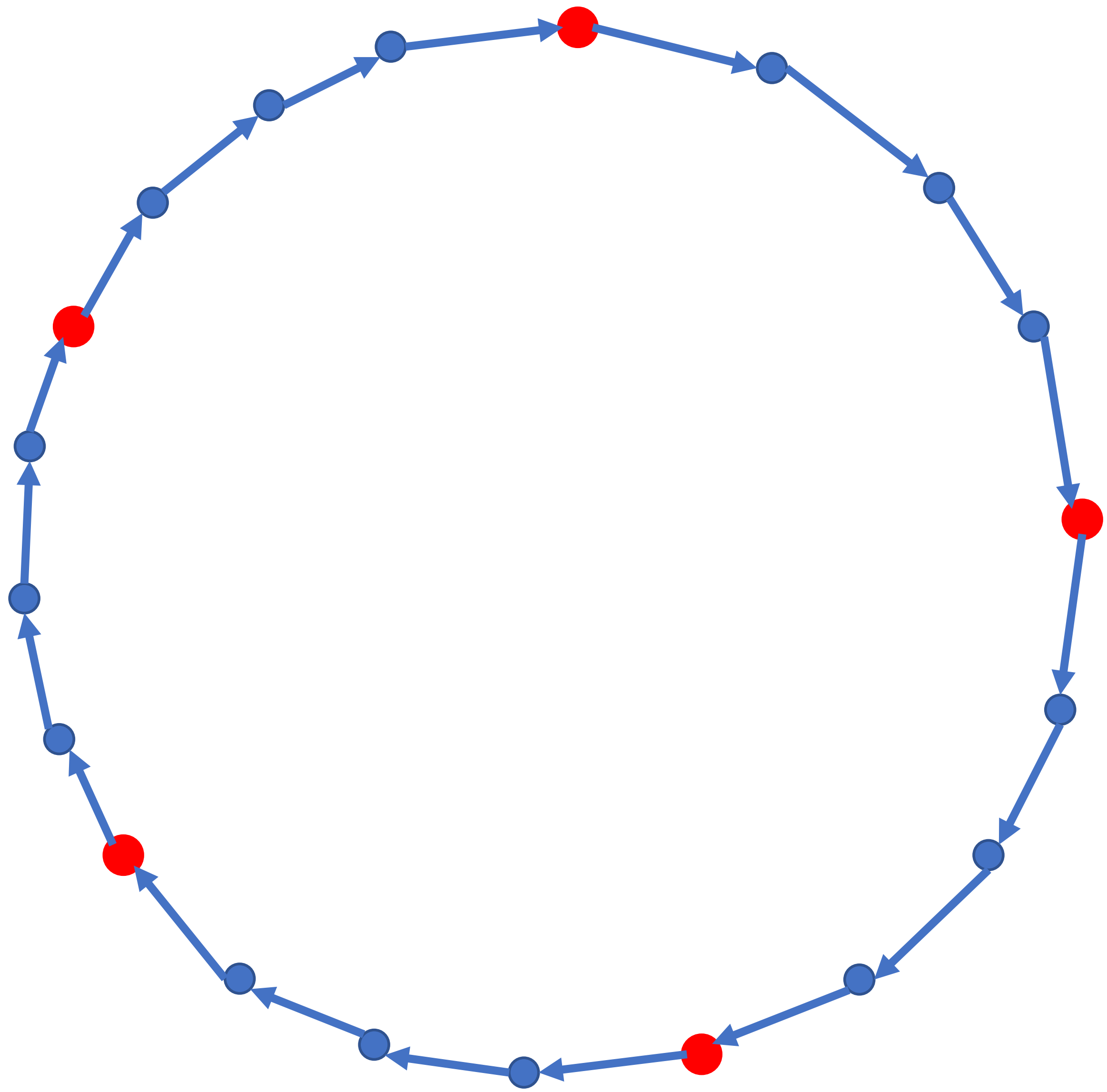
For simplicity, assume **F(k,•)** forms a cycle covering entire domain

- **{0, F(k,0), F(k, F(k,0)), F(k, F(k, F(k,0))),…} = X**

In preprocessing stage:

- Attacker iterates over entire cycle, saving every t^{th} term in a table $(x_1,\ldots,x_{N/t})$ where $N=2^l$

Option 3: Hellman's Attack



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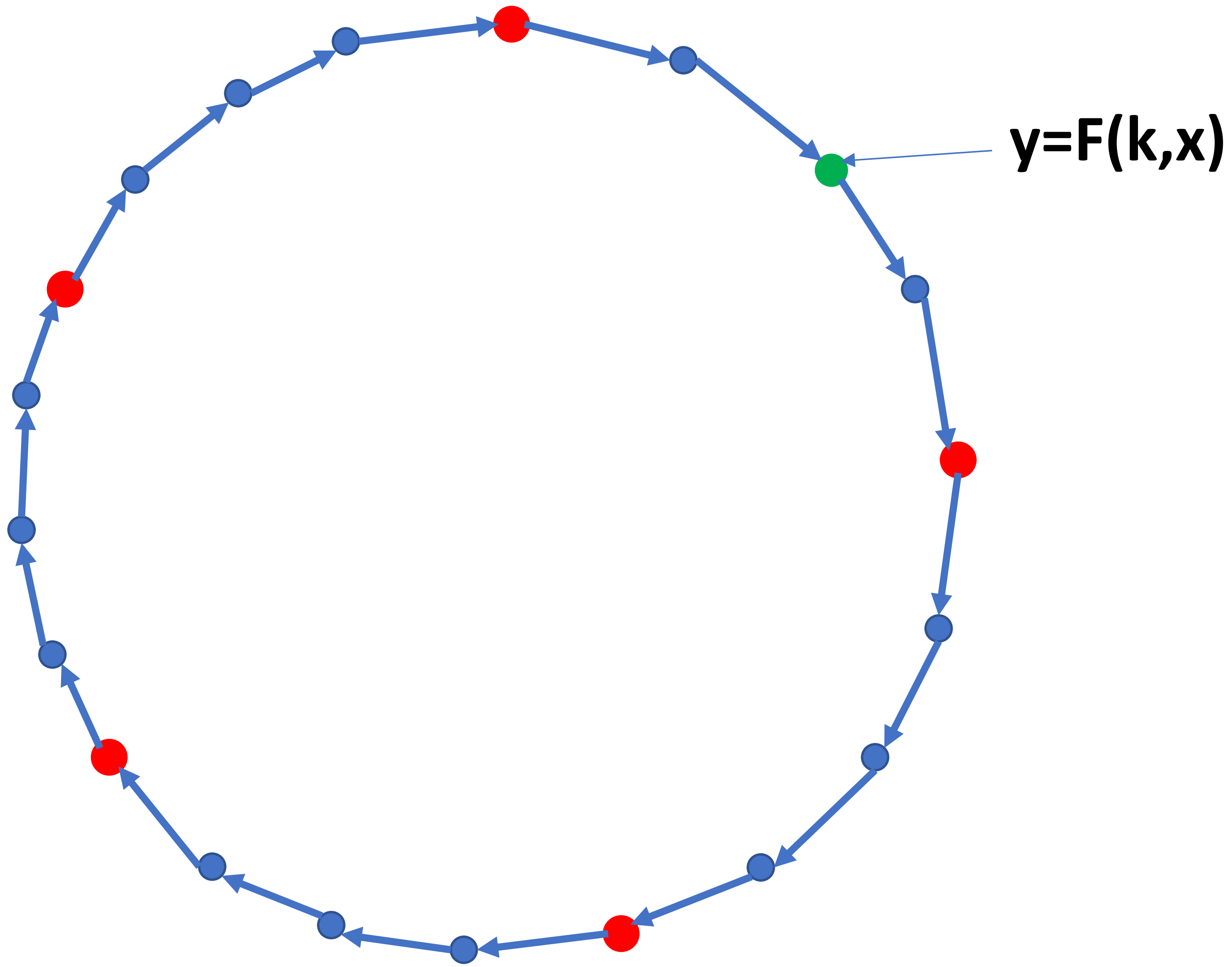

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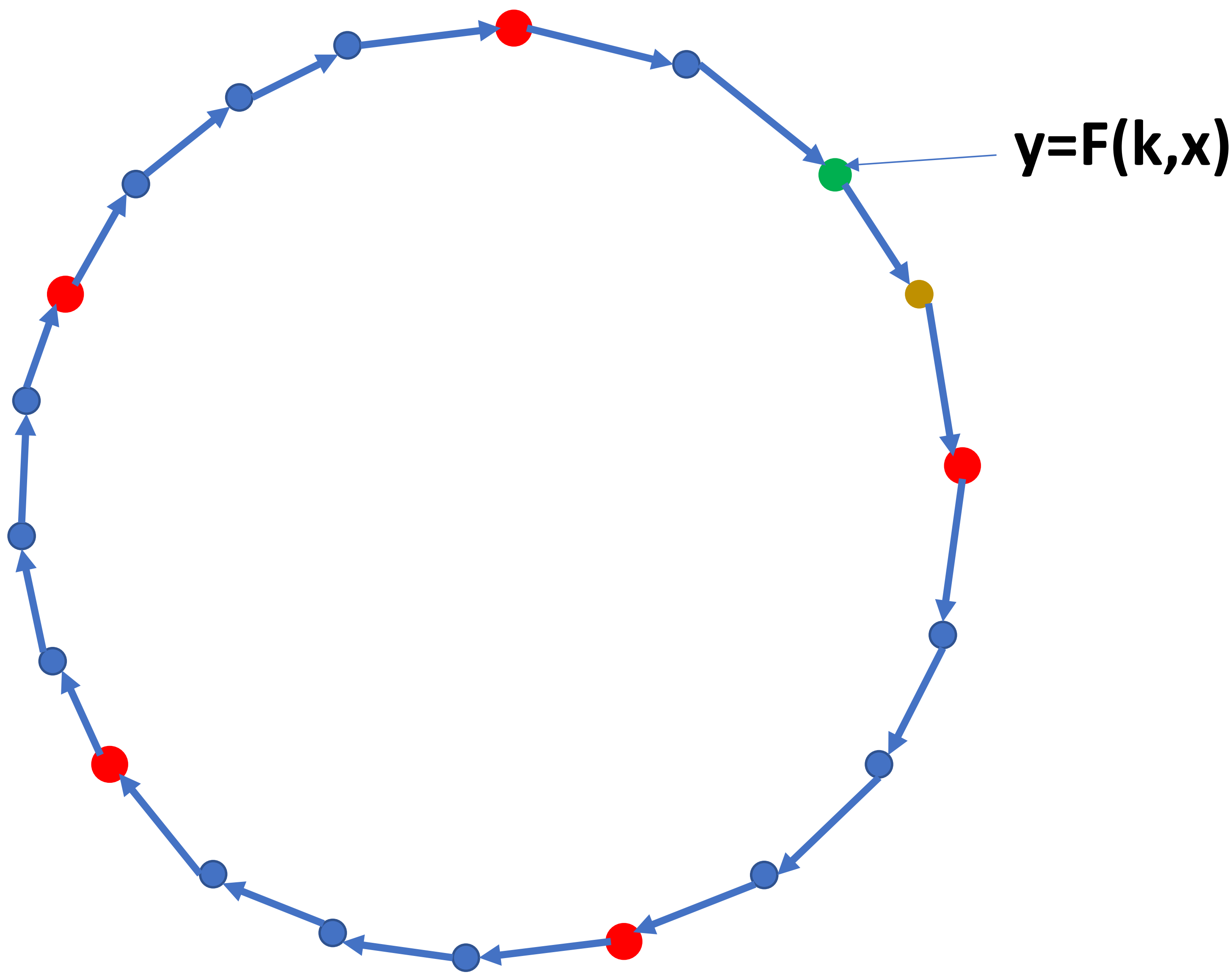

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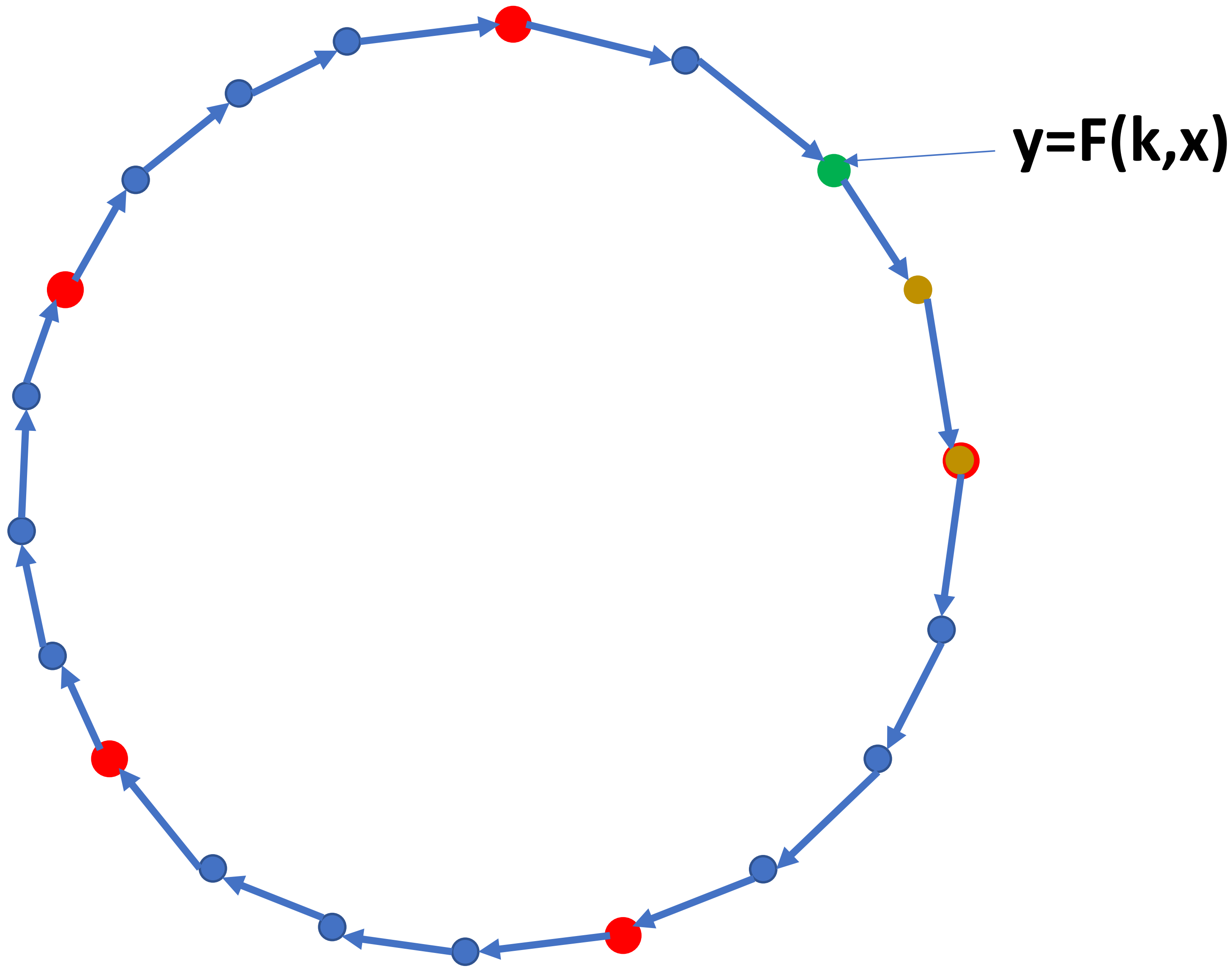

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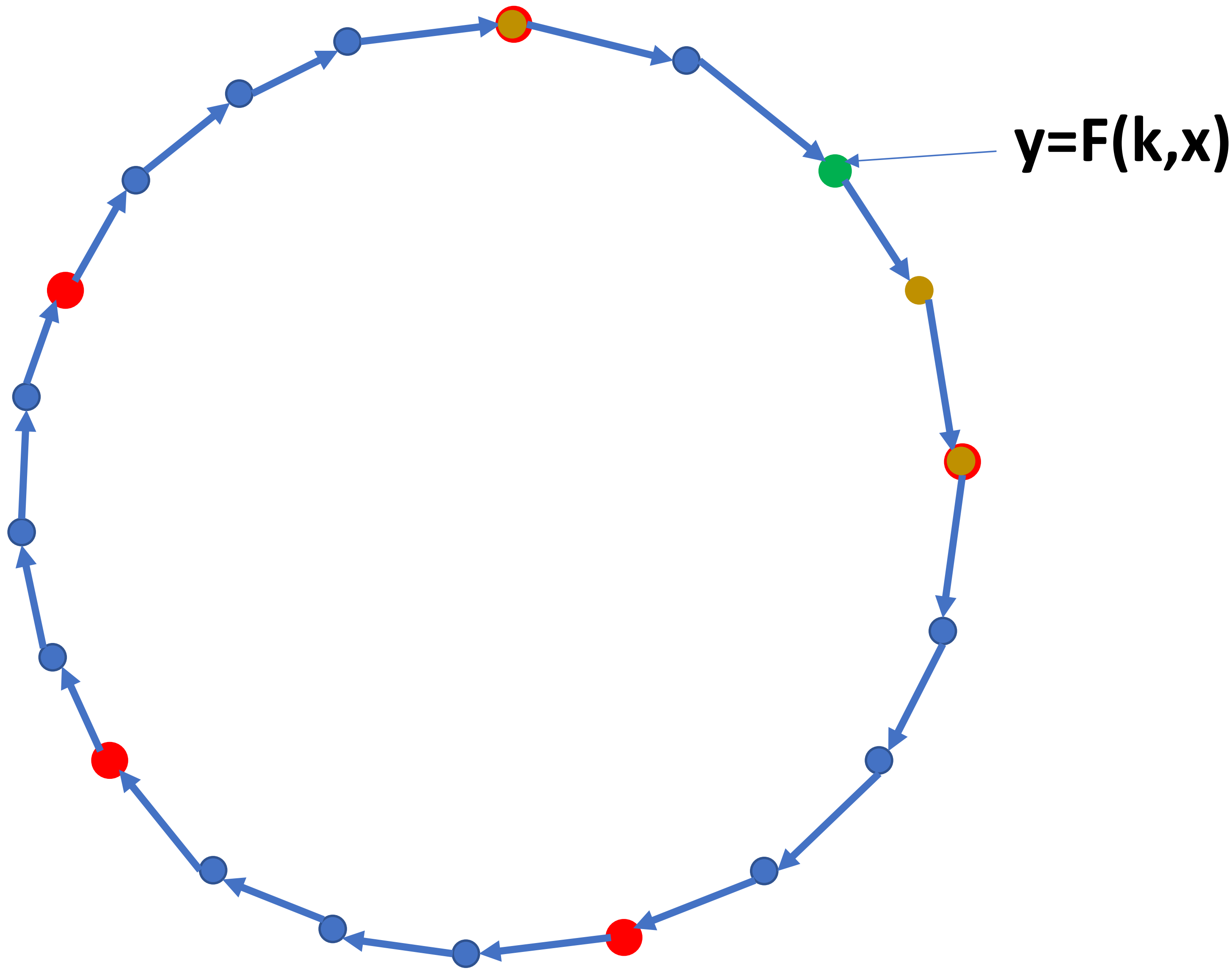

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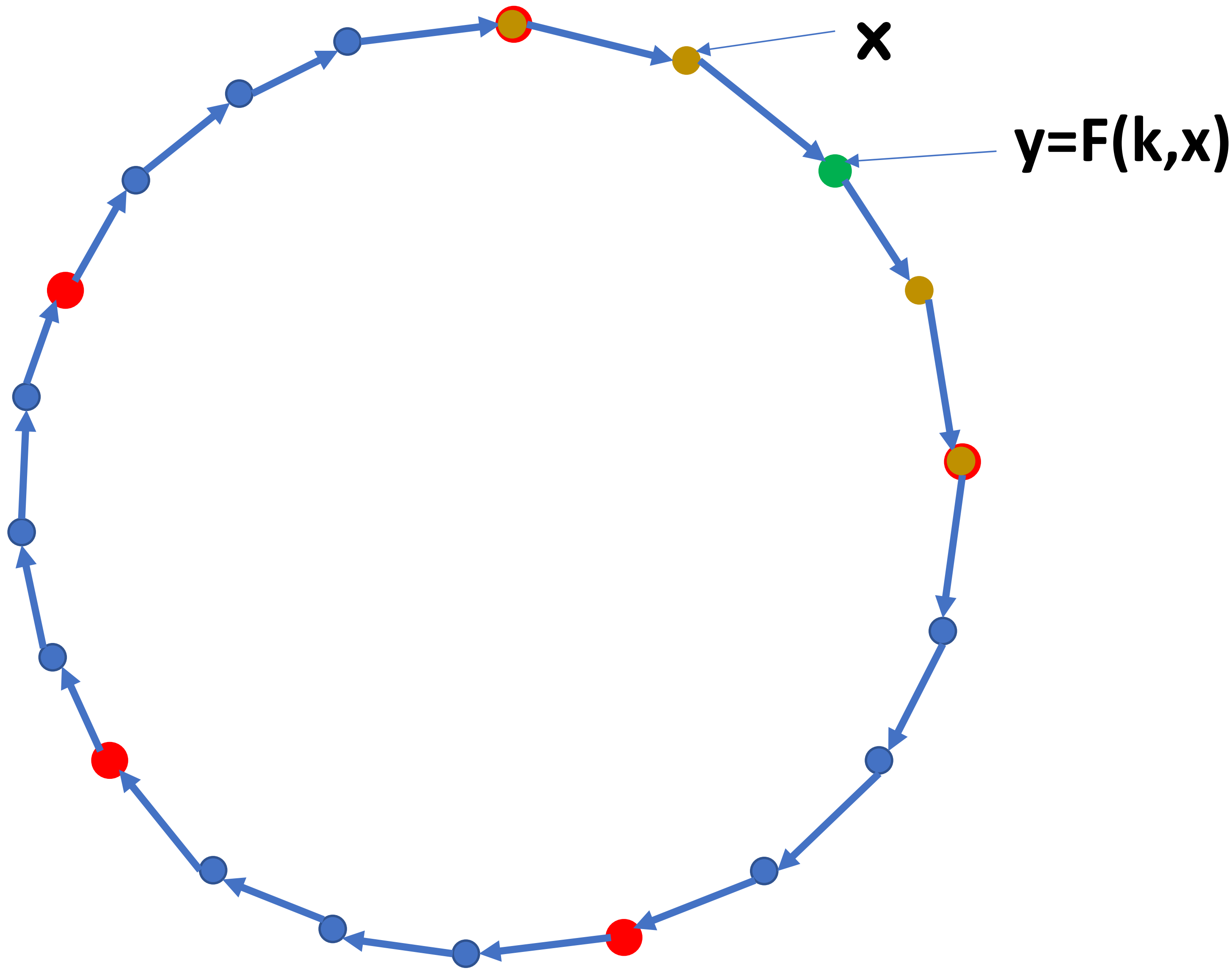


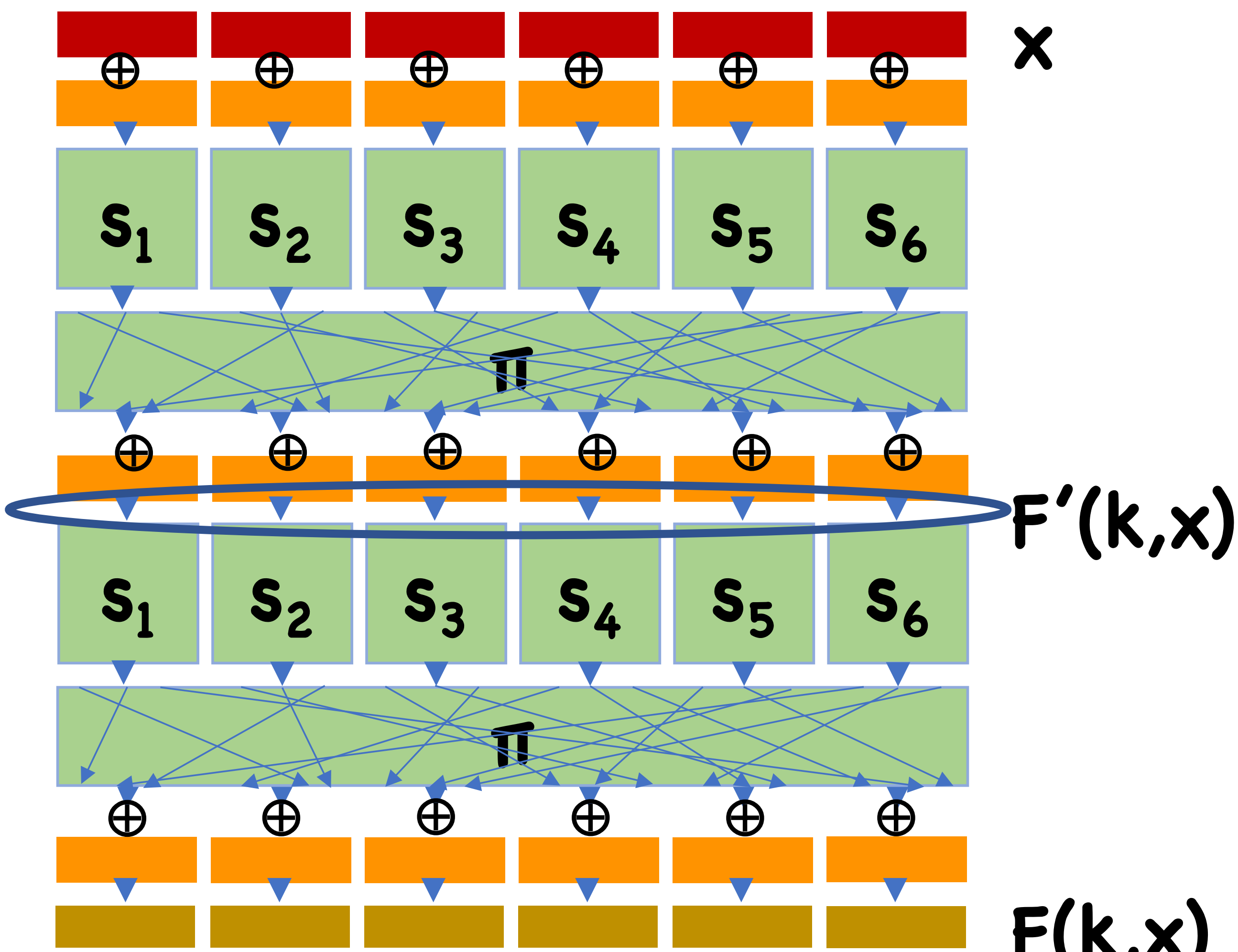
Option 3: Hellman's Attack

Preprocessing Time: $N=2^l$
Space: N/t
Online Time: t

Time-space tradeoff: **space × online time ≈ N**

For non-cycles, attack is a bit harder, but nonetheless possible

Differential Cryptanalysis



Differential Cryptanalysis

Suppose there were Δ_x, Δ_z such that, for random key k and random x_1, x_2 where $x_1 \oplus x_2 = \Delta_x$, $F'(k,x_1) \oplus F'(k,x_2) = \Delta_z$ with probability $p \gg 2^{-l}$

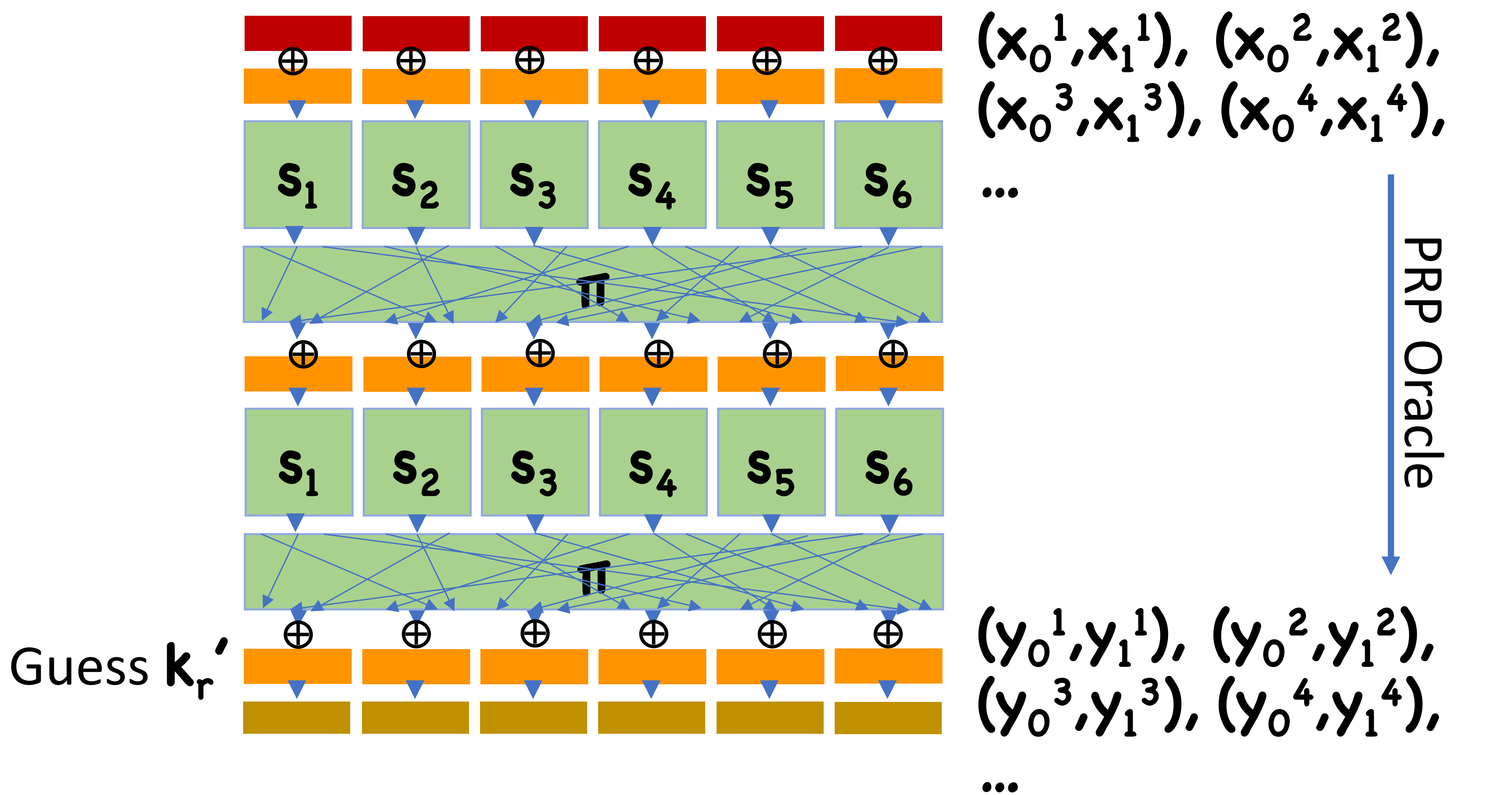
- Call (Δ_x, Δ_z) a differential
- p is probability of differential
- 2^{-l} is probability for random permutation

Differential Cryptanalysis

Attack:

- Choose many random pairs (x_1,x_2) s.t. $x_1 \oplus x_2 = \Delta_x$
- Make queries on each x_1, x_2, obtaining y_1,y_2
- For each round key guess k_r',
    - Use differentials to determine if guess was correct

Differential Cryptanalysis

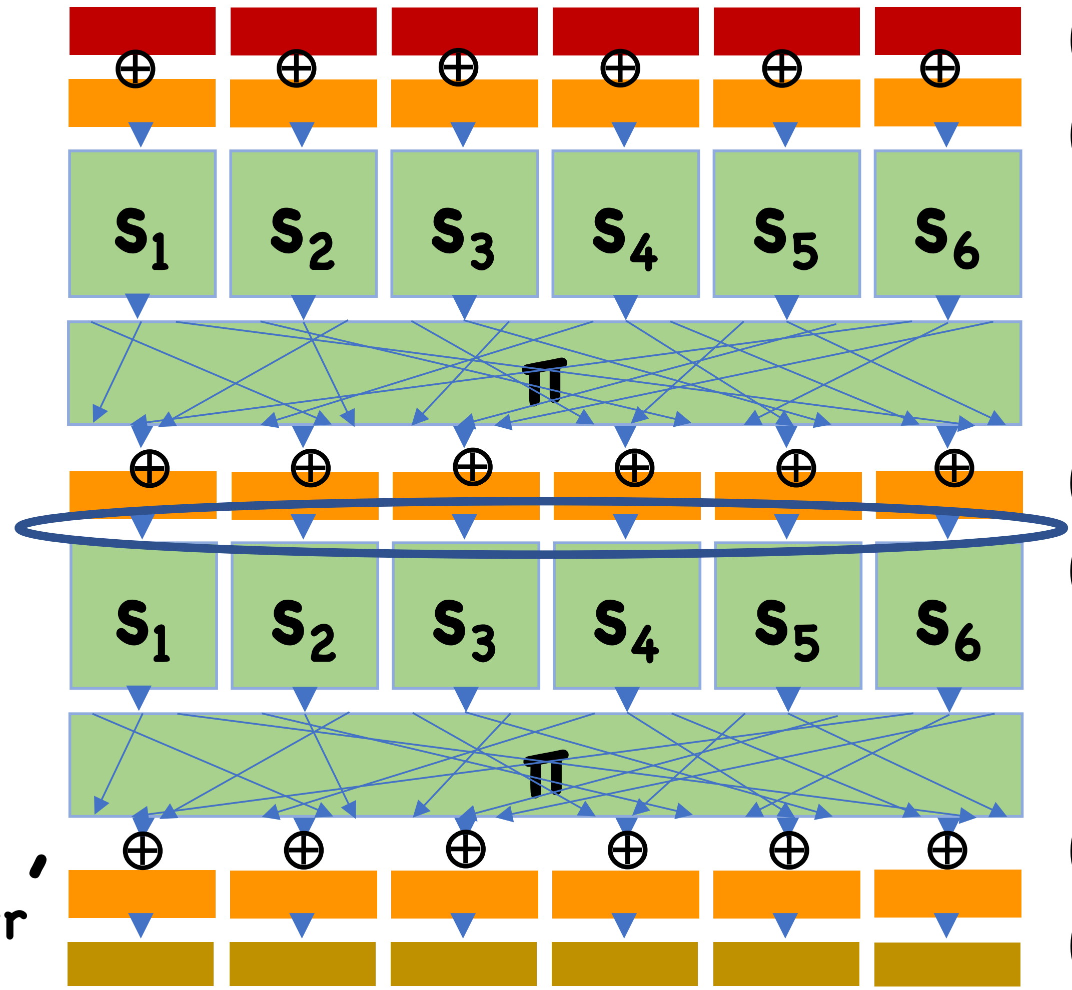


Differential Cryptanalysis

Attack:

- Choose many random pairs (x_1,x_2) s.t. $x_1 \oplus x_2 = \Delta_x$
- Make queries on each x_1, x_2, obtaining y_1,y_2
- For each round key guess k_r',
    - Undo last round assuming k_r', obtaining (z_1',z_2')
    - Look for $z_1' \oplus z_2' = \Delta_z$
    - If right guess, expect $\approx p$ fraction
    - If wrong guess, expect $\approx 2^{-l}$ fraction

Differential Cryptanalysis



(x_0^1,x_1^1), (x_0^2,x_1^2), (x_0^3,x_1^3), (x_0^4,x_1^4), …

$(z'_0{}^1,z'_1{}^1)$, $(z'_0{}^2,z'_1{}^2)$, $(z'_0{}^3,y'_1{}^3)$, $(z'_0{}^4,z'_1{}^4)$, …

k_r'

(y_0^1,y_1^1), (y_0^2,y_1^2), (y_0^3,y_1^3), (y_0^4,y_1^4), …

Guess k_r'

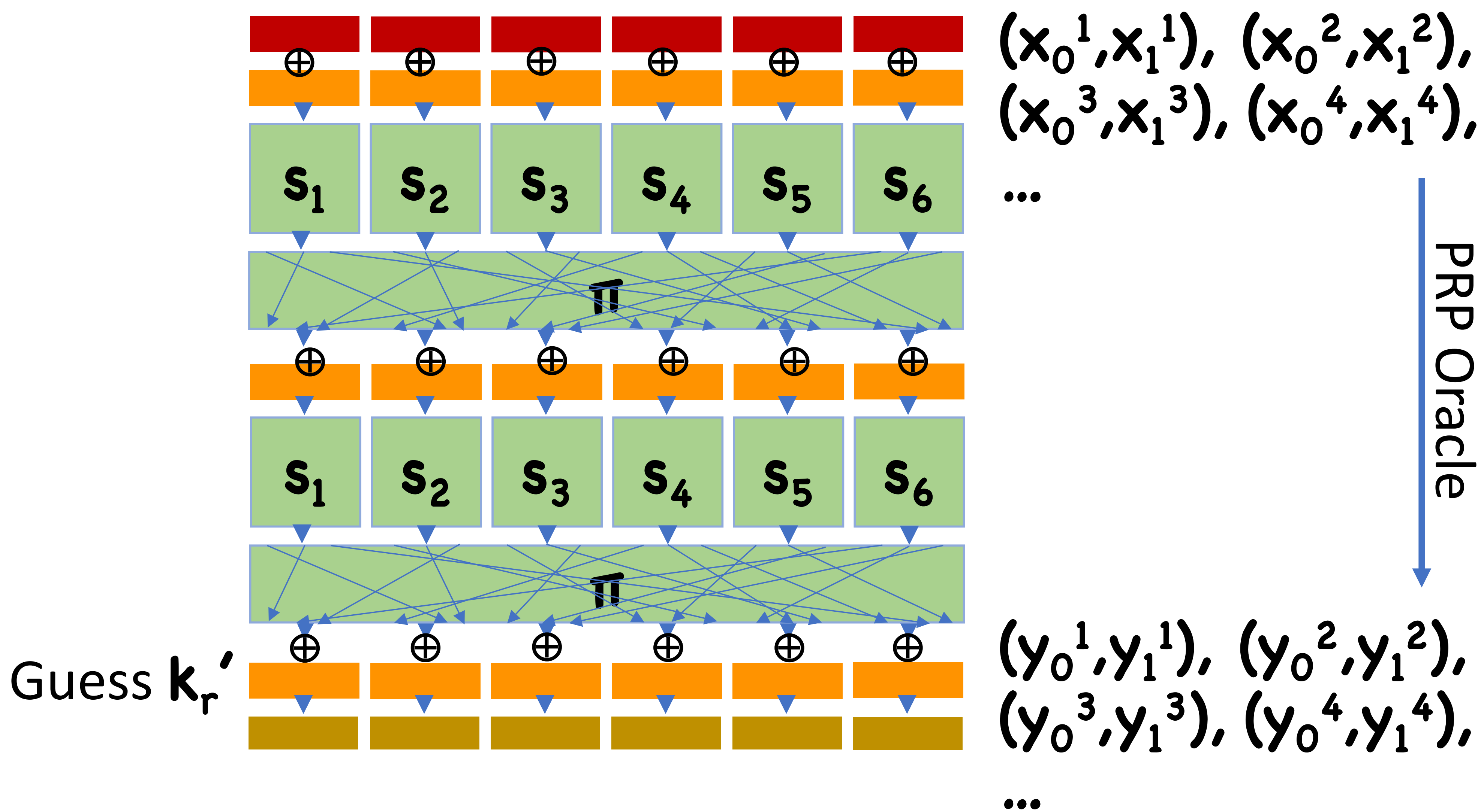
Differential Cryptanalysis

So far, inefficient since we have to iterate over all 2^l possible round keys

Instead, we can learn k_r byte by byte

- Guess 8 bits of k_r at a time
- Iterate through all 2^8 possible values for those 8 bits
  - Compute 8 bits of z_1', z_2', look for (portion of) differential
- Which bits to choose?

Differential Cryptanalysis



Differential Cryptanalysis

Extending to further levels:

- One $\mathbf{k_r}$ is known, can uncompute last layer
- Now perform same attack on round-reduced cipher
- Repeat until all round keys have been found

Finding Differentials

So far, assumed differential given

How do we find it?
- Can’t simply brute force all possible differentials

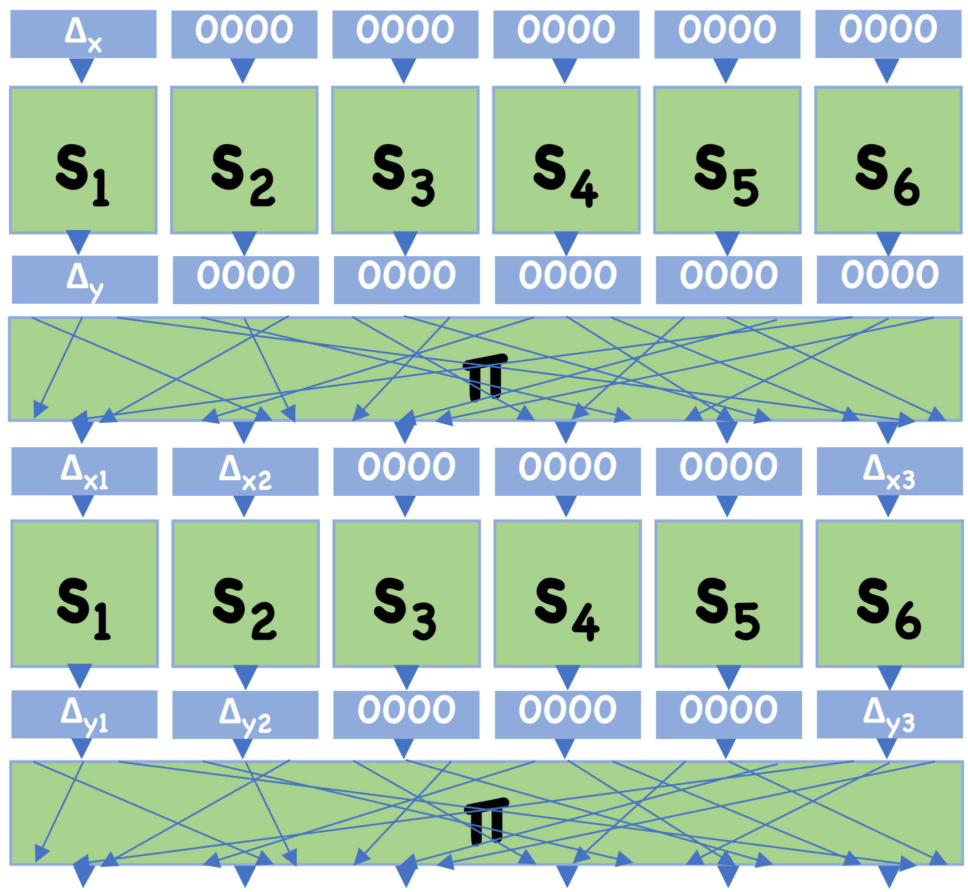
Finding Differentials

Solution: look for differentials in S-boxes

- Only 2^8 possible differences, so we can actually look for all possible differentials

- Then trace differentials through the evaluation
  - Key mixing does not affect differentials
  - Diffusion steps just shuffle differential bits

Differential Cryptanalysis



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Differential Cryptanalysis in Practice

Used to attack real ciphers

- FEAL-8, proposed as alternative to DES in 1987
  - requires just 1000 chosen input/output pairs, 2 minutes computation time in 1990's
- Also theoretical attacks on DES
  - Requires 2^{47} chosen input/output pairs
  - Very difficult to obtain in real world applications
  - Therefore, DES is still considered relatively secure
  - Small changes to S-boxes in DES lead to much better differential attacks

Linear Cryptanalysis

High level idea: look for linear relationships that hold with too-high a probability

- E.g. $x_1 \oplus x_5 \oplus x_{17} \oplus y_3 \oplus y_6 \oplus y_{12} \oplus y_{21} = 0$

Can show that if happen with too-high probability, can completely recover key

Important feature: only requires *known* plaintext as opposed to *chosen* plaintext

- Much easier to carry out in practice
- Ex: DES can be broken with 2^{43} input/output pairs

Block Cipher Design

S-boxes are designed to minimize differential and linear cryptanalysis

- Cannot completely remove differentials/linear relations, but can minimize their probability

Increasing number of rounds helps

- Likelihood of differential decreases each round

Related Key Attacks

Properly designed crypto will always use random, independent keys for every application

However, sometimes people don't follow the rules

Related key attack: have messages encrypted under similar keys

(Recall RC4 used for encryption, **RC4(IV,k)**)

For AES 256, can attack in 2^{110} space/time

Holiwudd Criptoe!

Device is top of the line.
AES cipher locks, brute force decryption is the only way.... It's effective, but slow. Very slow.