COS433/Math 473: Cryptography

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Previously on COS 433…

Perfect Security for Multiple Messages

Definition: A stateless scheme **(Enc,Dec)** has **perfect secrecy for n messages** if, for any two sequences of messages $(m_0^{(i)})_{i\in[d]}$, $(m_1^{(i)})_{i\in[d]} \in M^d$

$$\left(Enc(K,\ m_0^{(i)}\ )\right)_{i\in[d]} \overset{d}{=} \left(Enc(K,\ m_1^{(i)}\ )\right)_{i\in[d]}$$

Notation: $(\ f(i)\ )_{i\in[d]} = (\ f(1),\ f(2),\ \ldots,\ f(n)\ )$

Theorem: No stateless deterministic encryption scheme can have perfect security for multiple messages

Randomized Encryption

Syntax:

- Key space **K** (usually $\{0,1\}^\lambda$)
- Message space **M** (usually $\{0,1\}^n$)
- Ciphertext space **C** (usually $\{0,1\}^m$)
- **Enc: K×M → C** (potentially probabilistic)
- **Dec: K×C → M** (usually deterministic)

Correctness:

- For all $k \in K$, $m \in M$,

$$\Pr[\ \mathrm{Dec}(k, \mathrm{Enc}(k,m)\,) = m\ ] \quad = \quad 1$$

Theorem: No stateless *randomized* encryption scheme can have perfect security for multiple messages

What do we do now?

Tolerate tiny probability of distinguishing

- If $\Pr[c^{(0)} = c^{(1)}] = 2^{-128}$, in reality never going to happen

How small is ok?

- Usually 2^{-80}, 2^{-128}, or maybe 2^{-256}

Next time: formalize weaker notion of secrecy to allow for small probability of detection

Statistical Distance

Given two distributions $\mathbf{D_1}$, $\mathbf{D_2}$ over a set $\mathbf{X}$, define

$$\Delta(D_1,D_2) = ½\Sigma_x \mid Pr[D_1=x] - Pr[D_2=x] \mid$$

Observations:

$$0 \leq \Delta(D_1,D_2) \leq 1$$

$$\Delta(D_1,D_2) = 0 \iff D_1 \overset{d}{=} D_2$$

$$\Delta(D_1,D_2) \leq \Delta(D_1,D_3) + \Delta(D_3,D_2)$$

(**Δ** is a metric)

Another View of Statistical Distance

Theorem: $\Delta(D_1,D_2) \geq \varepsilon$ iff $\exists A$ s.t.

$$| \Pr[A(D_1) = 1] - \Pr[A(D_2) = 1] | \geq \varepsilon$$

Terminology: for any A,

$$| \Pr[A(D_1) = 1] - \Pr[A(D_2) = 1] |$$

is called the “advantage” of A in distinguishing D_1 and D_2

Another View of Statistical Distance

Theorem: $\Delta(D_1,D_2) \geq \varepsilon$ iff $\exists A$ s.t.

$$\left|\ \Pr[A(D_1) = 1] - \Pr[A(D_2) = 1]\ \right| \geq \varepsilon$$

To lower bound **Δ**, just need to show adversary **A** with that advantage

Examples

D_1 = Uniform distribution over $\{0,1\}^n$

- $\Pr[D_1=x] = 2^{-n}$

D_2 = Uniform subject to even parity

- $\Pr[D_2=x] = 2^{-(n-1)}$ if x has even parity, 0 otherwise

$$\begin{aligned}\Delta(D_1,D_2) &= \tfrac{1}{2}\Sigma_{\text{even } x}\, |2^{-n} - 2^{-(n-1)}| \\ &\quad + \tfrac{1}{2}\Sigma_{\text{odd } x}\, |2^{-n} - 0| \\ &= \tfrac{1}{2}\Sigma_{\text{even } x}\, 2^{-n} + \tfrac{1}{2}\Sigma_{\text{odd } x}\, 2^{-n} \\ &= \tfrac{1}{2}\end{aligned}$$

Examples

$D_1 =$ Uniform over $\{1,\ldots,n\}$
$D_2 =$ Uniform over $\{1,\ldots,n+1\}$

$$\Delta(D_1,D_2) = \tfrac{1}{2}\Sigma_{x=1}^{n} |1/n - 1/(n+1)| + \tfrac{1}{2} |0 - 1/(n+1)|$$

$$= \tfrac{1}{2}\Sigma_{x=1}^{n} 1/n(n+1) + \tfrac{1}{2} 1/(n+1)$$

$$= \tfrac{1}{2} 1/(n+1) + \tfrac{1}{2} 1/(n+1) = 1/(n+1)$$

Statistical Security

Definition: A scheme **(Enc,Dec)** has **ε-statistical secrecy for d messages** if ∀ two sequences of messages $(m_0^{(i)})_{i\in[d]}$, $(m_1^{(i)})_{i\in[d]} \in M^d$

$$\Delta\left[\left(Enc(K, m_0^{(i)})\right)_{i\in[d]}, \left(Enc(K, m_1^{(i)})\right)_{i\in[d]} \right] < \varepsilon$$

We will call such a scheme **(d,ε)**-secure

Statistical Security

We will consider a scheme “secure” for **d** messages if it is **(d,ε)**-secure for very small **ε**

- E.g. 2^{-80}, 2^{-128}, etc

For comparison: chance of

- Being struck by lightning twice: 2^{-23}
- Winning the lottery: 2^{-26}
- World-ending asteroid while on this slide: 2^{-46}

Stateless Encryption with Multiple Messages

Ex:

$\mathbf{M} = \mathbf{C} = \mathbb{Z}_{\mathbf{p}}$ ($\mathbf{p}$ a prime of size $\mathbf{2^{-128}}$)
$\mathbf{K} = \mathbb{Z}_{\mathbf{p}}^{*} \times \mathbb{Z}_{\mathbf{p}}$
$\mathbf{Enc((a,b), m) = (am + b) \bmod p}$
$\mathbf{Dec((a,b), c) = (c-b)/a \bmod p}$

Q: Is this statistically secure for two messages?

Example

Ex:

$M = \mathbb{Z}_p$ (p a prime of size 2^{-128})

$C = \mathbb{Z}_p{}^2$

Random in $\mathbb{Z}_p$

$K = \mathbb{Z}_p{}^2$

$\mathrm{Enc}(\ (a,b),\ m) = (r,\ \ (ar+b) + m\ )$

$\mathrm{Dec}(\ (a,b),\ (r,c)\ ) = c - (ar+b)$

Q: Is this statistically secure for two messages?

Proof of Example

Let $\mathbf{D_b}$ be distribution of $\mathbf{(\ Enc(k,m_b^{(i)})\ )_I}$
Let $\mathbf{D_b}'$ be $\mathbf{D_b}$, but conditioned on $\mathbf{r_0 \neq r_1}$

Fix $\mathbf{r_0 \neq r_1,\ m_0, m_1, c_0, c_1}$

$$\Pr_{(a,b)}[ar_0+b+m_0=c_0,\ ar_1+b+m_1=c_1] = 1/p^2$$

So $\mathbf{D_0' \overset{d}{=} D_1'}$ $\mathbf{(\ \Delta(D_0', D_1') = 0\ )}$

Proof of Example

Lemma: $\Delta(D_1,D_2) \leq \Pr[\text{bad}|D_1] + \Pr[\text{bad}|D_2] + \Delta(D_1', D_2')$

Where:

- "**bad**" is some event
- $\Pr[\text{bad}|D_b]$ is probability "**bad**" when sampling from D_b
- D_b' is D_b, but conditioned on **not** "**bad**"

Proof of Lemma

$$\Delta(D_1,D_2) = \Sigma_x | \Pr[D_1=x] - \Pr[D_2=x] |$$

$$= \Sigma_{x:bad} | \Pr[D_1=x\ ] - \Pr[D_2=x\ ] |$$
$$+ \Sigma_{x:good} | \Pr[D_1=x\ ] - \Pr[D_2=x\ ] |$$

$$\leq \Sigma_{x:bad} | \Pr[D_1=x\ ] | + \Sigma_{x:bad} | \Pr[D_2=x\ ] |$$
$$+ \Sigma_{x:good} | \Pr[D_1=x] - \Pr[D_2=x] |$$

$$\leq \Pr[bad|D_1] + \Pr[bad|D_2] + \Delta(D_{1,good},D_{2,good})$$

Proof of Example

Let $\mathbf{D_b}$ be distribution of $\mathbf{(\ Enc(k,m_b^{(i)})\ )_I}$
Let **bad** be when $\mathbf{r_0=r_1}$
Let $\mathbf{D_b'}$ be $\mathbf{D_b}$, but conditioned on **not bad**

$\mathbf{Pr[bad|D_b] = 1/p}$
$\mathbf{\Delta(D_0', D_1') = 0}$

Therefore, $\mathbf{\Delta(D_0, D_1) \leq 2/p}$

Summary so Far

Stateless encryption for multiple messages ✓

But, key length grows with number of messages ✘

And, key length grows with length of message ✘

Limits of Statistical Security

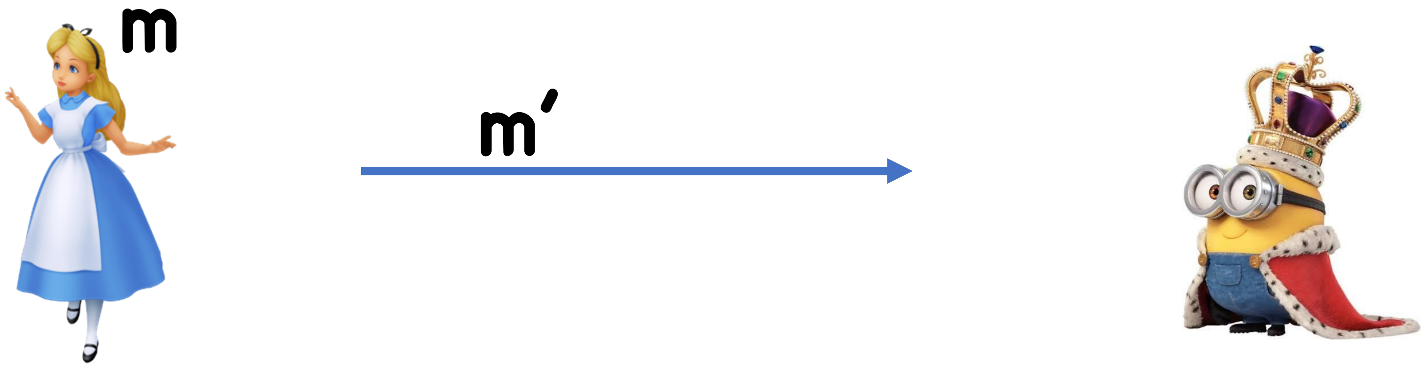
Theorem: Suppose **(Enc,Dec)** has plaintext space $\mathbf{M} = \{0,1\}^n$ and key space $\mathbf{K} = \{0,1\}^t$. Moreover, assume it is **(d,⅓)**-secure. Then:

$$t \geq d\, n$$

In other words, the key must be at least as long as the total length of all messages encrypted

Proof Idea

Use an encryption protocol to build a compression protocol

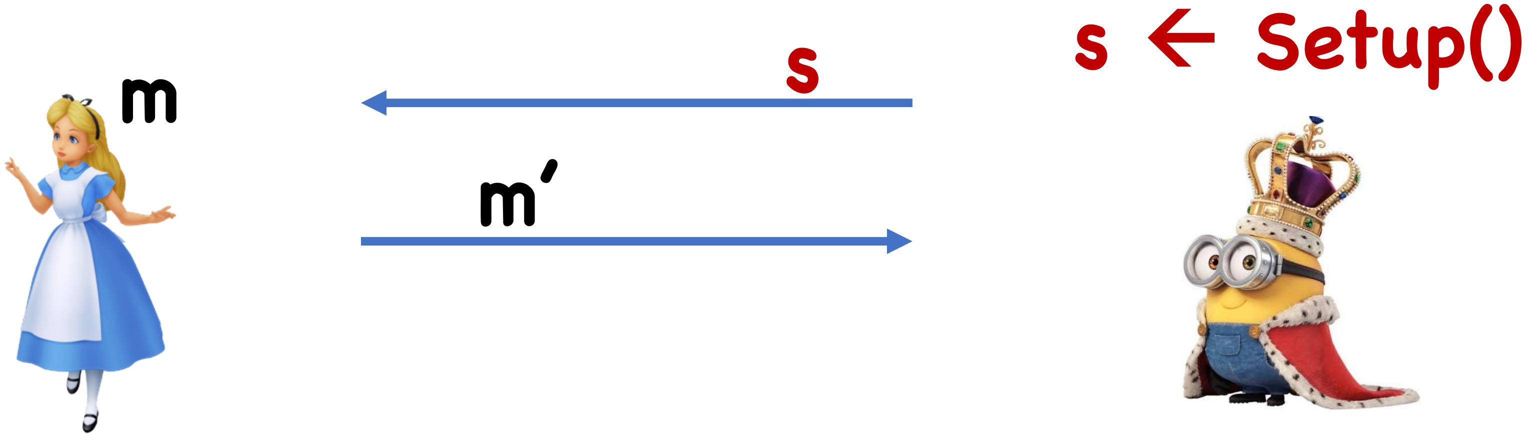


m′ ← Comp(m)

m ← Decomp(m′)

Goal: |m′| < |m|

For Now: Easier Goal



m

$s \leftarrow \mathrm{Setup}()$

s

m′

m′ ← Comp(s,m)

m ← Decomp(s,m′)

Goal: $|m'| < |m|$

The Protocol

Let $\mathbf{m_0}$ be some message in $\mathbf{M}$

Setup():

- Choose random $\mathbf{k_0 \leftarrow K}$
- **Let** $\mathbf{c_1 \leftarrow Enc(k_0,m_0), \ldots, c_d \leftarrow Enc(k_0,m_0)}$
- **Output** $\mathbf{(c_1,\ldots,c_d)}$

In $\mathbf{M^d}$

Comp($(c_1,\ldots,c_d)$, $(m_1,\ldots,m_d)$):

- Find $\mathbf{k,r_1,\ldots,r_d}$ such that $\mathbf{c_i=Enc(k,m_i;\ r_i)}$ $\forall \mathbf{i}$
- If no such values exist, abort
- Output $\mathbf{k}$

The Protocol

Let $\mathbf{m_0}$ be some message in $\mathbf{M}$

In $\mathbf{M^d}$

$\mathbf{Comp(\ (c_1,\ldots,c_d),\ (m_1,\ldots,m_d)\ ):}$

- Find $\mathbf{k,r_1,\ldots,r_d}$ such that $\mathbf{c_i=Enc(k,m_i;\ r_i)}\ \forall\mathbf{i}$
- If no such values exist, abort
- Output $\mathbf{k}$

$\mathbf{Decomp((c_1,\ldots,c_d),\ k\ ):}$

- Compute $\mathbf{m_i = Dec(k,c_i)}$
- Output $\mathbf{(m_1,\ldots,m_d)}$

Analysis of Protocol

If **Comp** succeeds, **Decomp** must succeed by correctness

- Since $\mathbf{c_i=Enc(k,m_i;\ r_i)}$, $\mathbf{Dec(k,c_i)}$ must give $\mathbf{m_i}$

Therefore, must figure out when **Comp** succeeds

Claim: For any sequence of messages $\mathbf{m_1,\ldots,m_d}$, **Comp** succeeds with probability at least $\mathbf{1-\varepsilon}$

(Probability over the randomness used by **Setup()**)

Claim: For any sequence of messages $\mathbf{m_1,\ldots,m_d}$, **Comp** succeeds with probability at least $\mathbf{1-\varepsilon}$

Proof:

- Suppose **Comp** succeeds with probability $\mathbf{1-p}$ for messages $\mathbf{m_1,\ldots,m_d}$
- Let $\mathbf{A(c_1,\ldots,c_d)}$ be the algorithm that runs $\mathbf{Comp((c_1,\ldots,c_d),\ (m_1,\ldots,m_d))}$ and outputs **1** if **Comp** succeeds

- If $\mathbf{c_i = Enc(k_0,m_i)}$, then $\mathbf{Pr[A(c_1,\ldots,c_d)=1] = 1}$
- If $\mathbf{c_i = Enc(k_0,m_0)}$, then $\mathbf{Pr[A(c_1,\ldots,c_d)=1] = 1-p}$

- By $\mathbf{(d,\varepsilon)}$statistical security of **Enc**, **p** must be $\mathbf{\leq\varepsilon}$

Claim: For any sequence of messages $\mathbf{m_1,\ldots,m_d}$, **Comp** succeeds with probability at least $\mathbf{1-\varepsilon}$

Claim: For **a random** sequence of messages $\mathbf{m_1,\ldots,m_d}$, **Comp** succeeds with prob at least $\mathbf{1-\varepsilon}$

(Probability over the randomness used by **Setup()** and the random choices of $\mathbf{m_1,\ldots,m_d}$)

Next step: Removing Setup

We know:

$$\Pr\left[\text{Comp succeeds: } \begin{matrix}(c_1,\ldots,c_d)\leftarrow\text{Setup}(), \\ m_i\leftarrow M\end{matrix}\right] \geq 1-\varepsilon$$

Therefore, there must exist *some* $(c_1{}^*,\ldots,c_d{}^*)$ such that

$$\Pr[\text{Comp succeeds: } m_i\leftarrow M] \geq 1-\varepsilon$$

Define: $M' = \{(m_1,\ldots,m_d)\text{: Comp}$ succeeds$\}$

- Note that $|M'| \geq (1-\varepsilon)\ |M|^d$

The Protocol

$m \in M'$

k →

Find $k, r_1, \ldots, r_d$ such that
$c_i^* = Enc(k, m_{i;}\ r_i)\ \forall i$

For each i,
Let $m_i \leftarrow Dec(k, c_i^*)$
Output $(m_1, \ldots, m_d)$

By previous analysis,

- Alice always successfully compresses
- Bob always successfully decompresses

Final Touches

Can compress messages in $\mathbf{M'}$ into keys in $\mathbf{K}$

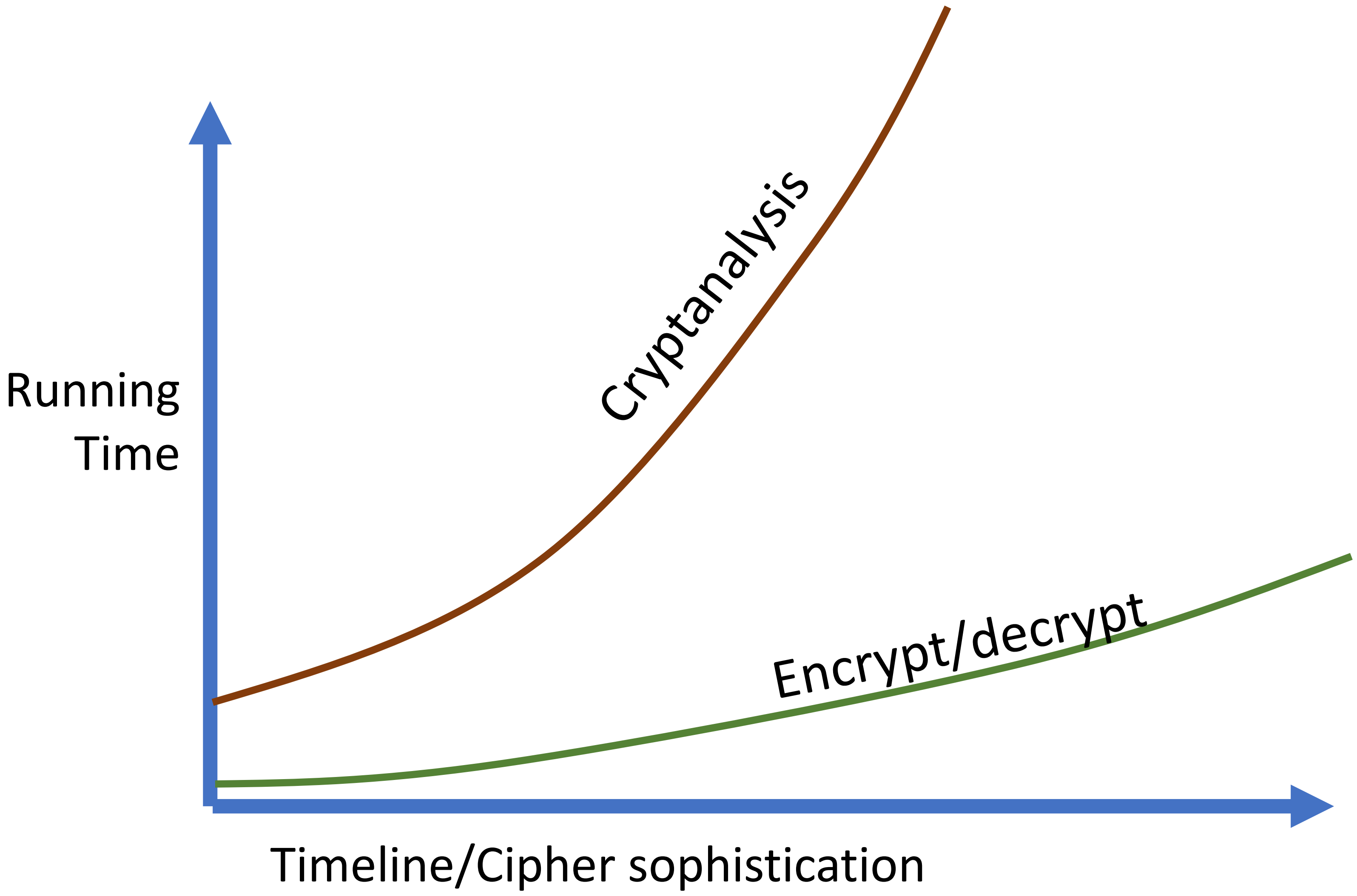
Therefore, it must be that $|M'| \leq |K|$

$$
\begin{aligned}
\textbf{Meaning } t &= \log |K| \\
&\geq \log |M'| \\
&\geq \log [\, (1-\varepsilon)\, |M|^d \,] \\
&= d \log |M| + \log [1-\varepsilon] \\
&\geq dn - 2\varepsilon \\
&\geq dn \text{ (as long as } \varepsilon<½\text{)}
\end{aligned}
$$

Takeaway

If you don’t want to physically exchange keys frequently, you cannot obtain statistical security

So, now what?


Running Time
Cryptanalysis
Encrypt/decrypt
Timeline/Cipher sophistication

Computational Security

We are ok if adversary takes a really long time

Usually measure in machine operations

- Though depends on architecture, so rough approx
- 2^{80}, 2^{128}, or maybe 2^{256} are probably ok

For comparison:

- Lifetime of universe in nanoseconds: 2^{58}
- Number of atoms in known universe: 2^{265}

Brute Force Attacks

Simply try every key until find right one

Relevant as long as key length is smaller than total length of messages encrypted

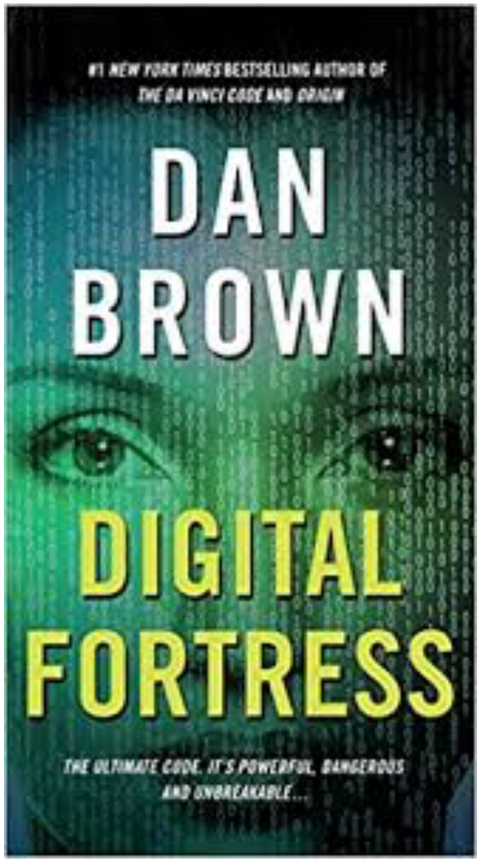
If keys have length $\boldsymbol{\lambda}$, $\mathbf{2}^{\lambda}$ is upper bound on attack

Crypto and P vs NP

What if P = NP?

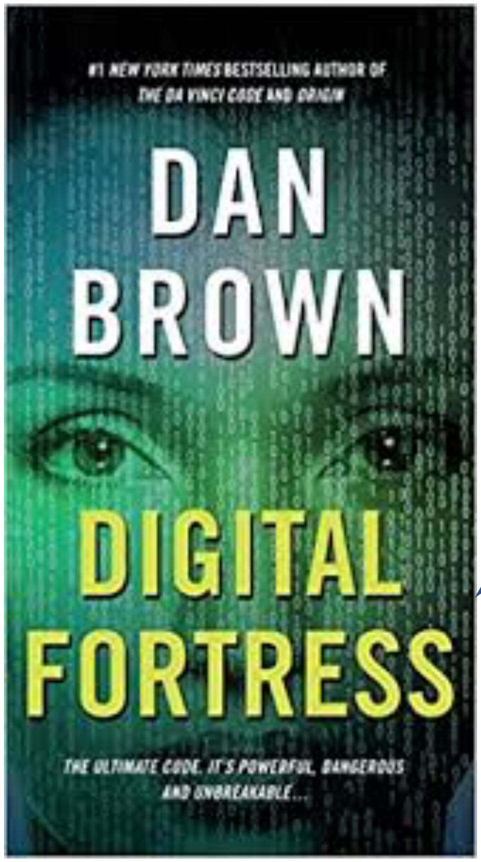
From this point forward, almost all crypto we will see depends on computational assumptions

Holiwudd Criptoe!


#1 NEW YORK TIMES BESTSELLING AUTHOR OF
THE DA VINCI CODE AND ORIGIN
DAN BROWN
DIGITAL FORTRESS
THE ULTIMATE CODE. IT'S POWERFUL, DANGEROUS
AND UNBREAKABLE...

[TRANSLTR]'s three million processors would all work in parallel ... trying every new permutation as they went

Holiwudd Criptoe!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THE ULTIMATE CODE. IT'S POWERFUL, DANGEROUS AND UNBREAKABLE...

“What’s the longest you’ve ever seen TRANSLTR take to break a code?”

“About an hour, but it had a ridiculously long key—ten thousand bits”

Defining Security

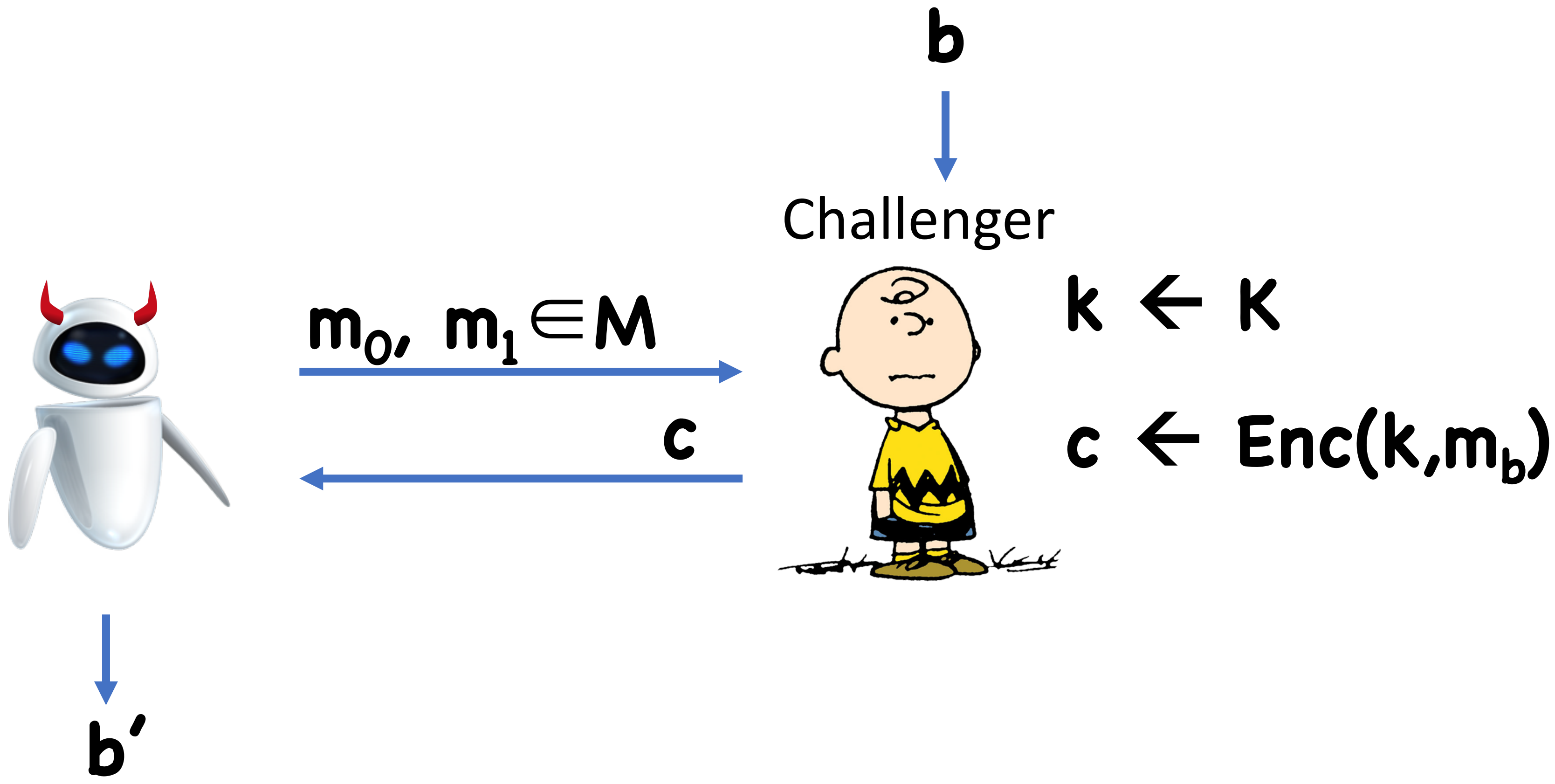
Consider an attacker as a probabilistic efficient algorithm

Attacker gets to choose the messages

All attacker has to do is distinguish them

Security Experiment/Game

(One-time setting)



b

Challenger

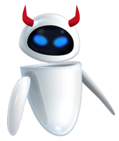
$m_0, m_1 \in M$

c

$k \leftarrow K$

$c \leftarrow \mathrm{Enc}(k,m_b)$

b′

$\mathrm{IND\text{-}Exp}_b($$)$

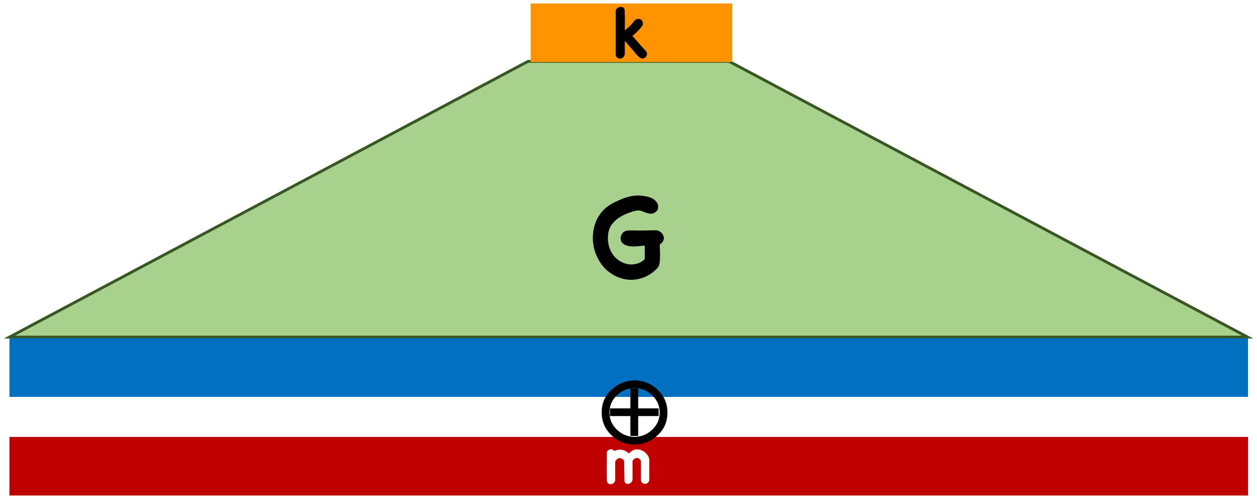
Security Definition (One-time setting)

Definition: (Enc, Dec) has **(t,ε)–ciphertext indistinguishability** if, for all 🤖 running in time at most **t**

$$\Big| \Pr[1 \leftarrow \text{IND-Exp}_0(\text{🤖})\,] \\ \quad - \Pr[1 \leftarrow \text{IND-Exp}_1(\text{🤖})\,] \Big| \leq \varepsilon$$

Construction with |k| << |m|

Idea: use OTP, but have key generated by some expanding function **G**



What Do We Want Out of **G**?

Definition: $G:\{0,1\}^\lambda \rightarrow \{0,1\}^n$ is a **(t,ε)-secure pseudorandom generator** (PRG) if:

- $n > \lambda$
- **G** is deterministic
- For all running in time at most **t**,

$\Big|\, \Pr[$ $(G(s))=1 : s \leftarrow \{0,1\}^\lambda]$

$-\ \Pr[$ $(x)=1 : x \leftarrow \{0,1\}^n] \,\Big| \leq \varepsilon$

Secure PRG → Ciphertext Indistinguishability

$K = \{0,1\}^{\lambda}$
$M = \{0,1\}^{n}$
$C = \{0,1\}^{n}$

$\mathrm{Enc}(k,m) = \mathrm{PRG}(k) \oplus m$
$\mathrm{Dec}(k,c) = \mathrm{PRG}(k) \oplus c$

Security?

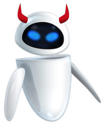
Intuitively, security is obvious:

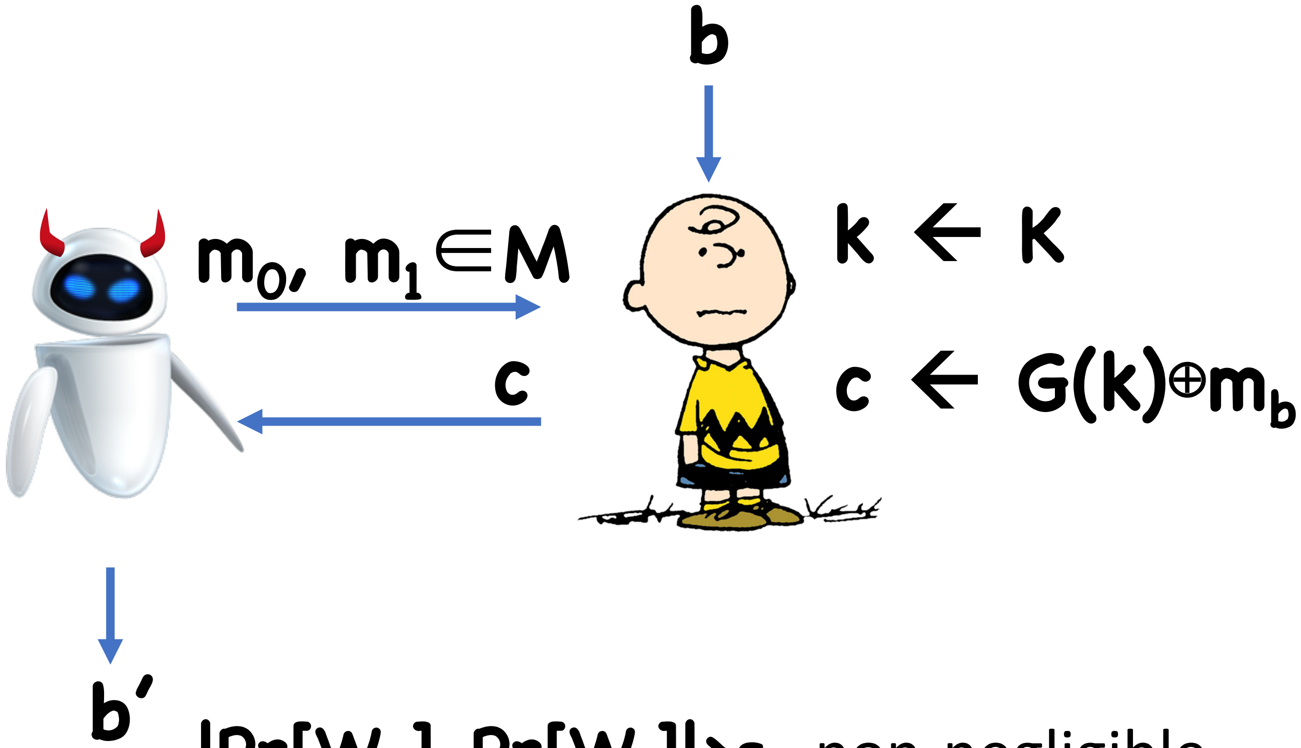
- **PRG(k)** ”looks” random, so should completely hide **m**
- However, formalizing this argument is non-trivial.

Solution: reductions

- Assume toward contradiction an adversary for the encryption scheme, derive an adversary for the PRG

Security

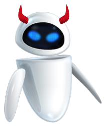
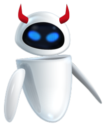
Assume towards contradiction that there is a  such that

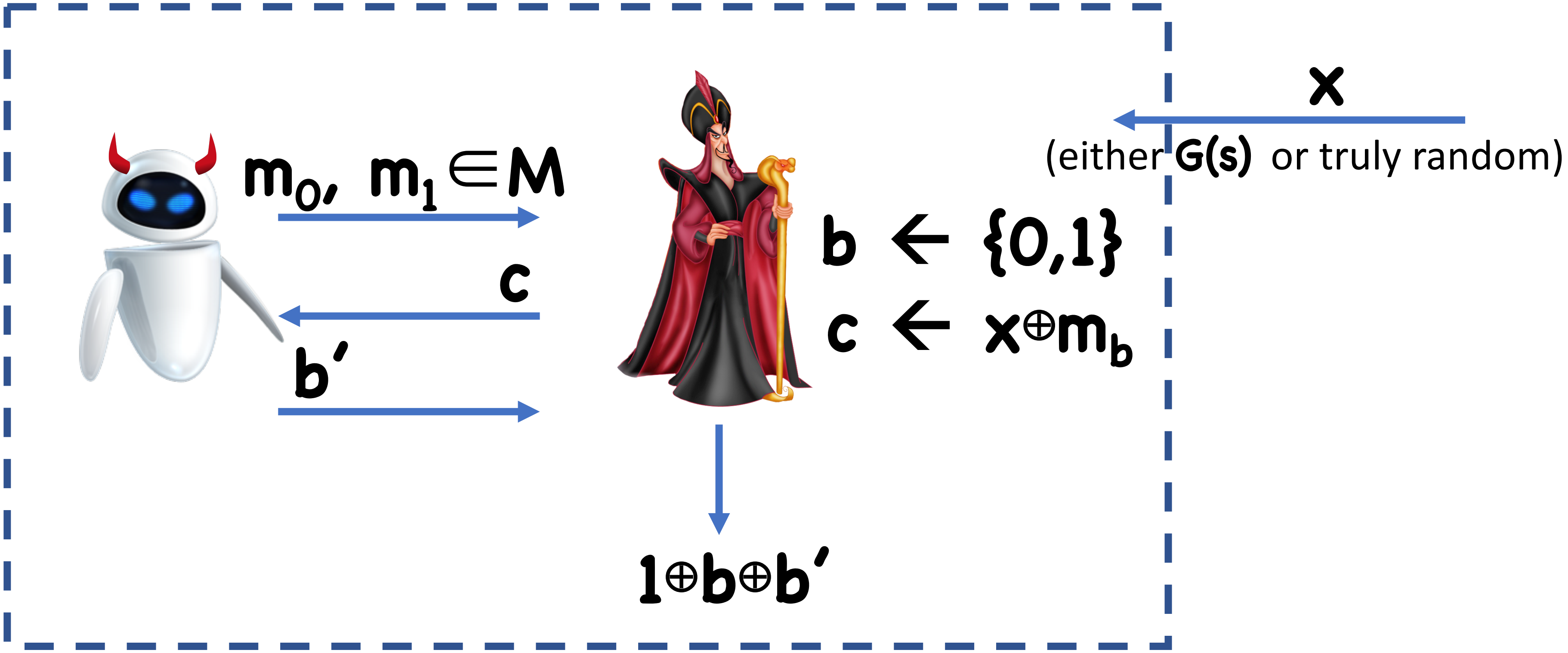


$|\Pr[W_0] - \Pr[W_1]| \geq \varepsilon$, non-negligible

W_b: $b' = 1$ in IND-Exp_b

Security

Use  to build . will run  as a subroutine, and pretend to be



$m_0, m_1 \in M$

c

b'

$b \leftarrow \{0,1\}$

$c \leftarrow x \oplus m_b$

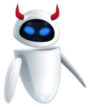
x

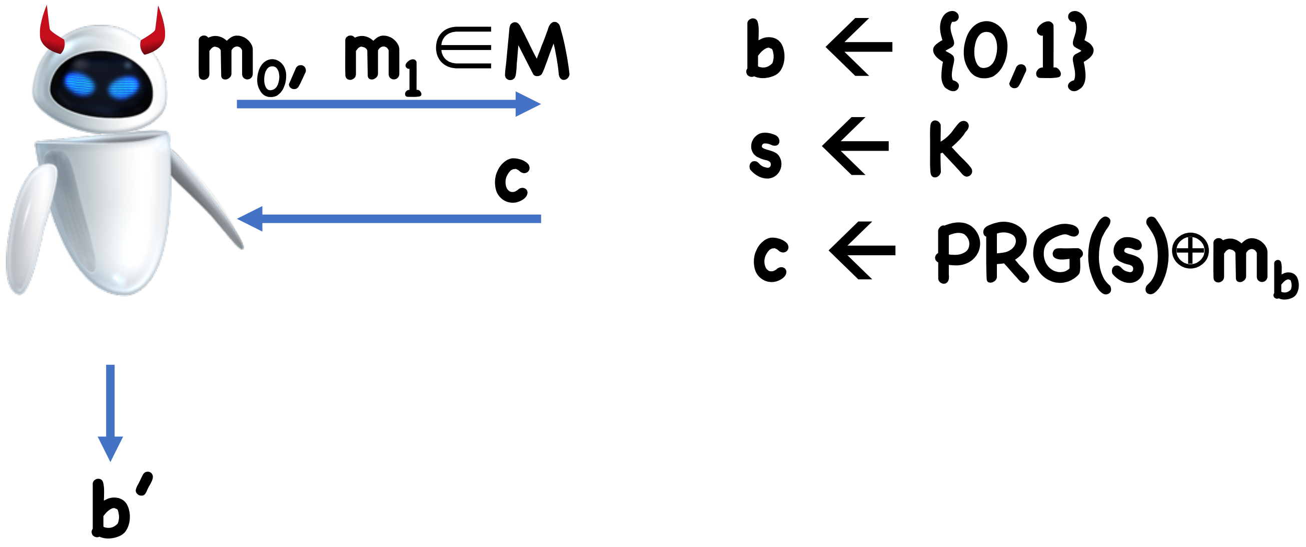
(either $G(s)$ or truly random)

$1 \oplus b \oplus b'$

Security

Case 1: **x = PRG(s)** for a random seed **s**

-  "sees" **IND-Exp**$_b$ for a random bit **b**



$m_0, m_1 \in M$

c

b'

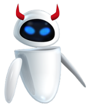
$b \leftarrow \{0,1\}$

$s \leftarrow K$

$c \leftarrow PRG(s) \oplus m_b$

Security

Case 1: $\mathbf{x} = \mathbf{PRG(s)}$ for a random seed **s**

-  “sees” $\mathbf{IND\text{-}Exp_b}$ for a random bit **b**
- $\Pr[1 \oplus b \oplus b' = 1] = \Pr[b = b']$

$$= \tfrac{1}{2} \Pr[b'=1 \mid b=1\ ]$$

$$+ \tfrac{1}{2}\ (1 - \Pr[b'=1 \mid b=0])$$

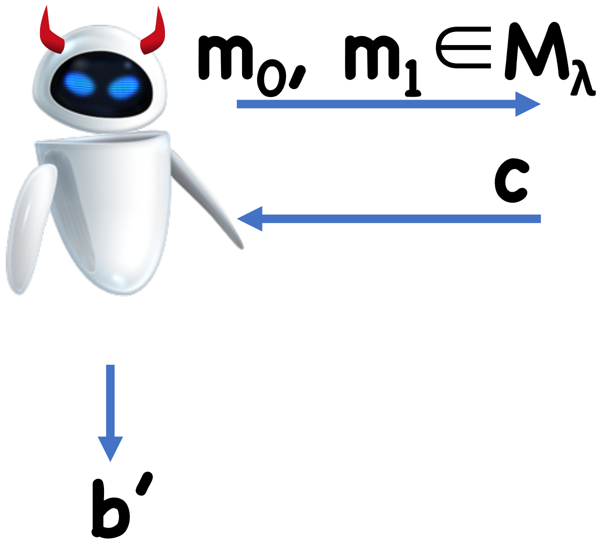
$$= \tfrac{1}{2}(1 + \Pr[W_0] - \Pr[W_1])$$

$$= \tfrac{1}{2}(\ 1 \pm \varepsilon\ )$$

Security

Case 2: **x** is truly random

- “sees” OTP encryption



$m_0, m_1 \in M_\lambda$

c

b'

$b \leftarrow \{0,1\}$

$x \leftarrow \{0,1\}^n$

$c \leftarrow x \oplus m_b$

Security

Case 2: x is truly random

- 😈 “sees” OTP encryption
- Therefore $\Pr[b'=1 \mid b=0] = \Pr[b'=1 \mid b=1]$
- $\Pr[1 \oplus b \oplus b'=1] = \Pr[b=b']$

$$= \tfrac{1}{2} \Pr[b'=1 \mid b=1\ ]$$

$$+ \tfrac{1}{2} (1 - \Pr[b'=1 \mid b=0])$$

$$= \tfrac{1}{2}$$

Security

Putting it together:

- $\Pr[$ $(G(s))=1 : s \leftarrow \{0,1\}^{\lambda}] = \tfrac{1}{2}(\, 1 \pm \varepsilon(\lambda)\,)$
- $\Pr[$ $(x)=1 : x \leftarrow \{0,1\}^{n}] = \tfrac{1}{2}$
- Absolute Difference: **½ε,** $\Rightarrow$ Contradiction!

Security

Thm: If **G** is a **(t+t',ε/2)**-secure PRG, then **(Enc,Dec)** is has **(t,ε)**-ciphertext indistinguishability, where **t'** is the time to:

- Flip a random bit **b**
- XOR two **n**-bit strings

Security

Thm: If **G** is a **$(t+\text{poly},\varepsilon/2)$**-secure PRG, then **(Enc,Dec)** is has **(t,ε)**-ciphertext indistinguishability

An Alternate Proof: Hybrids

Idea: define sequence of “hybrid” experiments “between” **IND-Exp**$_0$ and **IND-Exp**$_1$

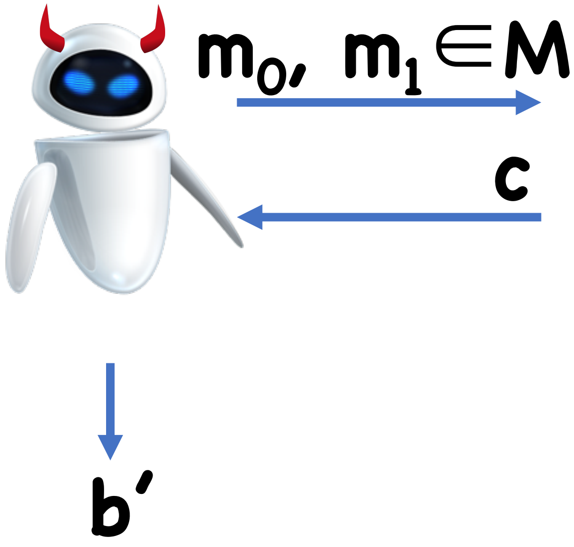
In each hybrid, make small change from previous hybrid

Hopefully, each small change is undetectable

Using triangle inequality, overall change from **IND-Exp**$_0$ and **IND-Exp**$_1$ is undetectable

An Alternate Proof: Hybrids

Hybrid 0: IND-Exp$_0$



$m_0, m_1 \in M$

c

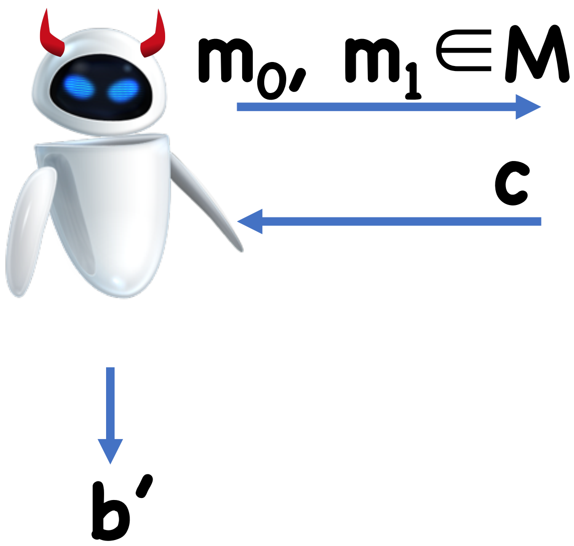
b'

$k \leftarrow K$

$c \leftarrow G(k) \oplus m_0$

An Alternate Proof: Hybrids

Hybrid 1:



$m_0, m_1 \in M$

c

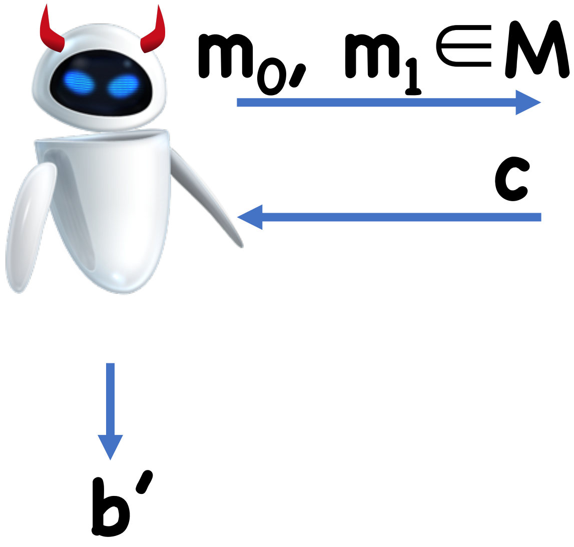
b'

$x \leftarrow \{0,1\}^n$

$c \leftarrow x \oplus m_0$

An Alternate Proof: Hybrids

Hybrid 2:



$m_0, m_1 \in M$

c

b'

$x \leftarrow \{0,1\}^n$

$c \leftarrow x \oplus m_1$

An Alternate Proof: Hybrids

Hybrid 3: IND-Exp$_1$

$m_0, m_1 \in M$

c

b'

$k \leftarrow K$

$c \leftarrow G(k) \oplus m_1$

An Alternate Proof: Hybrids

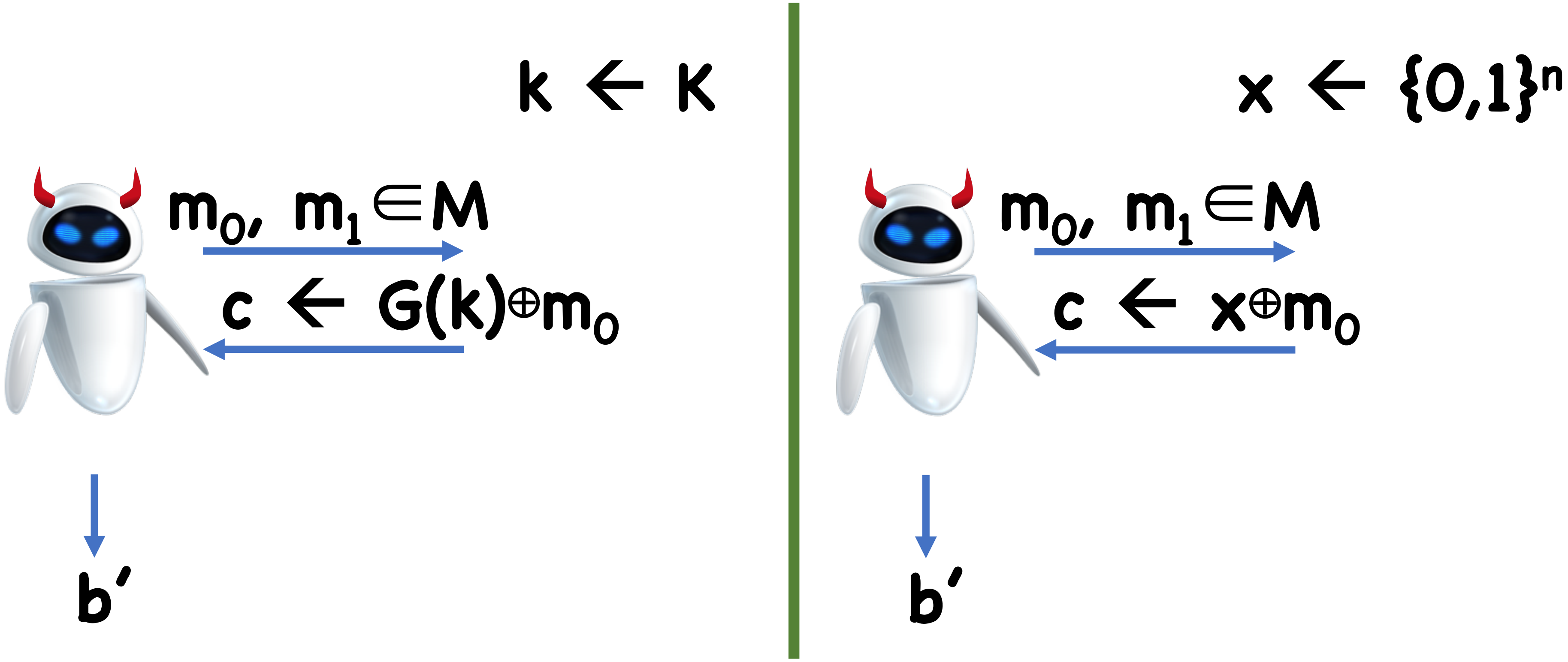
$$
\begin{aligned}
&|\ \Pr[b'=1 : \text{IND-Exp}_0] - \Pr[b'=1 : \text{IND-Exp}_1]\ | \\
&\qquad = |\ \Pr[b'=1 : \text{Hyb } 0] - \Pr[b'=1 : \text{Hyb } 3]\ | \\
&\qquad \leq |\ \Pr[b'=1 : \text{Hyb } 0] - \Pr[b'=1 : \text{Hyb } 1]\ | \\
&\qquad\quad + |\ \Pr[b'=1 : \text{Hyb } 1] - \Pr[b'=1 : \text{Hyb } 2]\ | \\
&\qquad\quad + |\ \Pr[b'=1 : \text{Hyb } 2] - \Pr[b'=1 : \text{Hyb } 3]\ |
\end{aligned}
$$

If $|\Pr[b'=1:\text{IND-Exp}_0]-\Pr[b'=1:\text{IND-Exp}_1]| \geq \varepsilon$,
Then for some $i=0,1,2$,

$$|\Pr[b'=1:\text{Hyb } i]-\Pr[b'=1:\text{Hyb } i+1]| \geq \varepsilon/3$$

An Alternate Proof: Hybrids

Suppose distinguishes **Hybrid 0** from **Hybrid 1** with advantage **$\varepsilon/3$**



$k \leftarrow K$

$m_0, m_1 \in M$

$c \leftarrow G(k) \oplus m_0$

b'

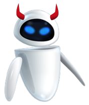
$x \leftarrow \{0,1\}^n$

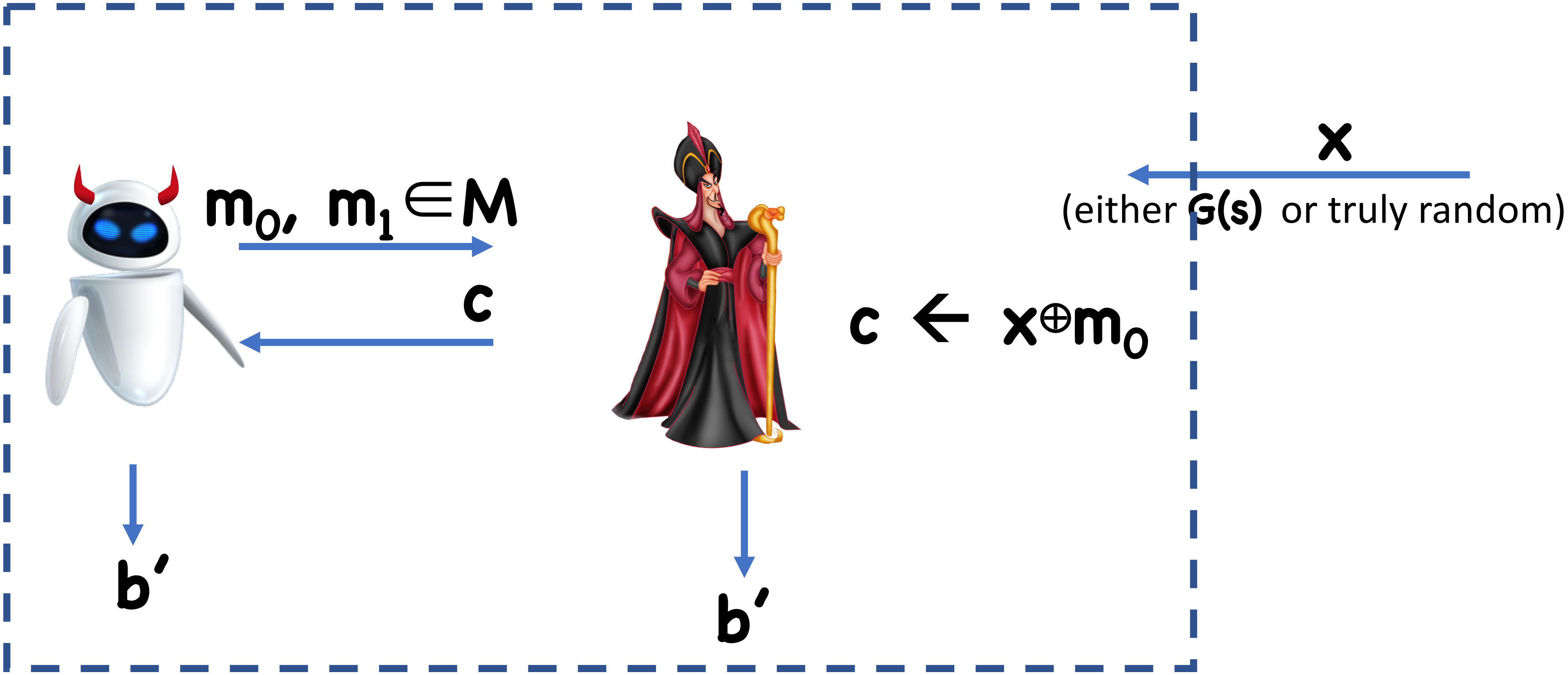
$m_0, m_1 \in M$

$c \leftarrow x \oplus m_0$

b'

An Alternate Proof: Hybrids

Suppose  distinguishes **Hybrid 0** from **Hybrid 1** with advantage **ε/3** ⇒ Construct



$m_0, m_1 \in M$

c

b'

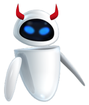
$c \leftarrow x \oplus m_0$

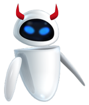
b'

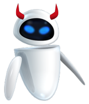
x

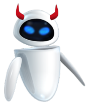
(either **G(s)** or truly random)

An Alternate Proof: Hybrids

Suppose  distinguishes **Hybrid 0** from **Hybrid 1** with advantage **ε/3** ⇒ Construct

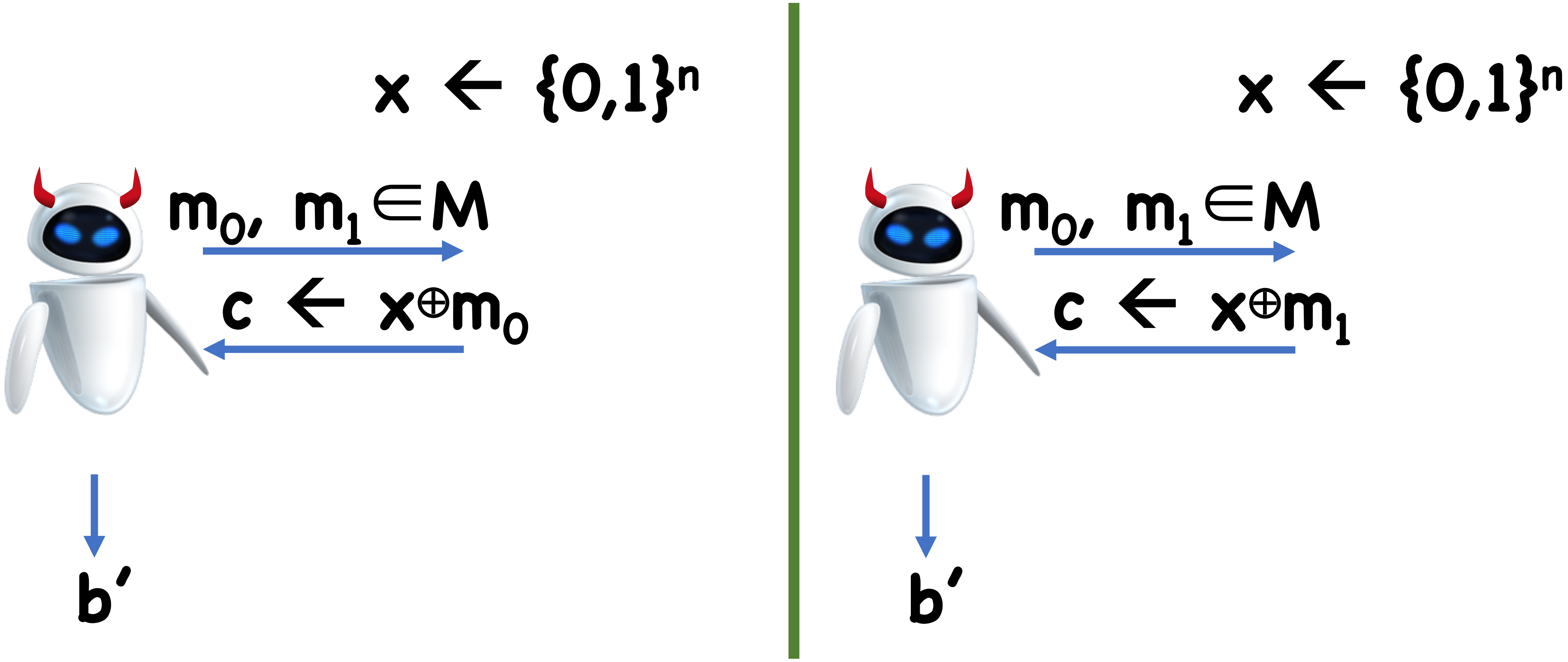
If is given **G(s)** for a random **s**,  sees **Hybrid 0**

If is given x for a random **x**,  sees **Hybrid 1**

Therefore, advantage of is equal to advantage of  which is at least **ε/3** ⇒ Contradiction!

An Alternate Proof: Hybrids

Suppose distinguishes **Hybrid 1** from **Hybrid 2** with advantage **$\varepsilon/3$**



$x \leftarrow \{0,1\}^n$

$m_0, m_1 \in M$

$c \leftarrow x \oplus m_0$

b'

$x \leftarrow \{0,1\}^n$

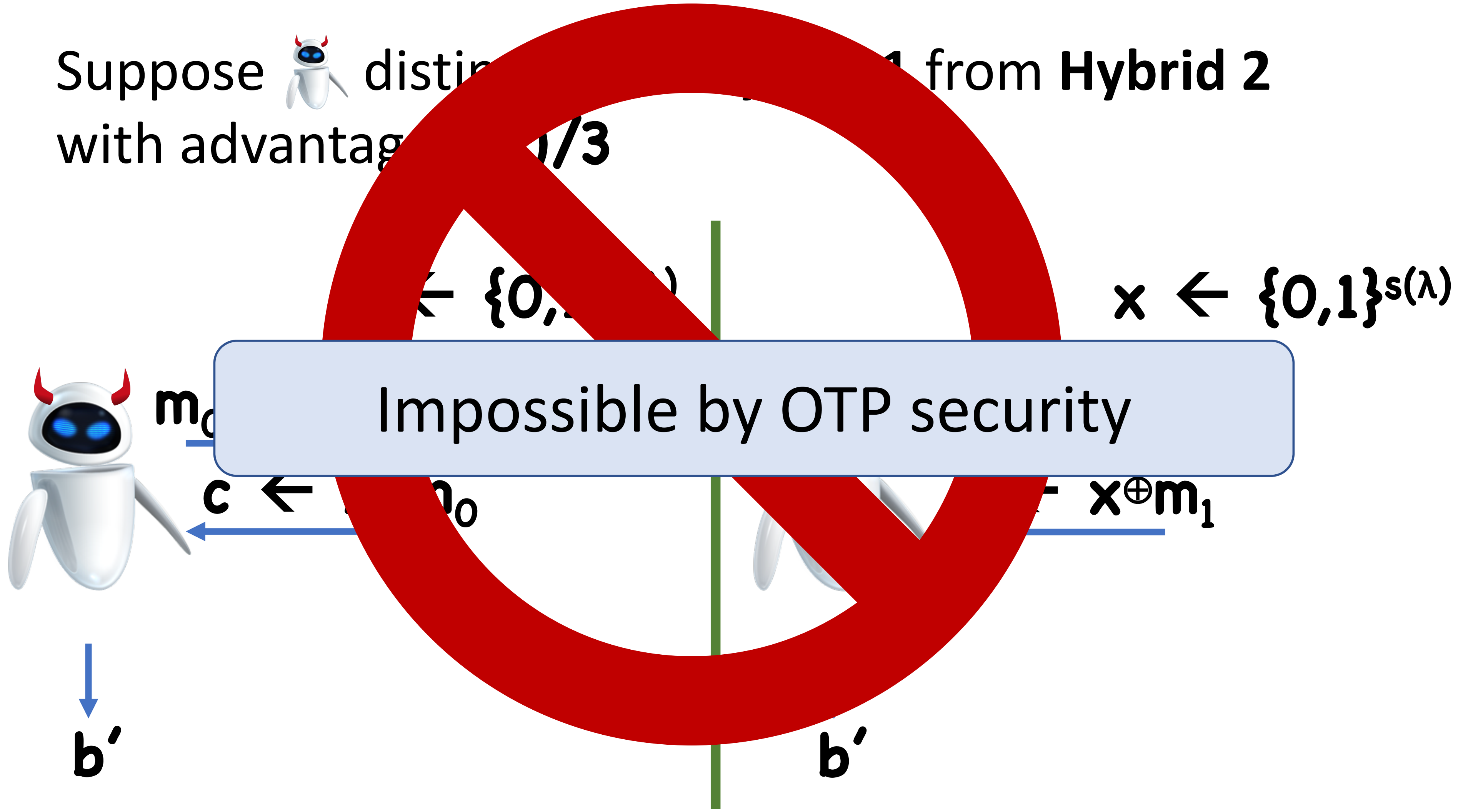
$m_0, m_1 \in M$

$c \leftarrow x \oplus m_1$

b'

An Alternate Proof: Hybrids

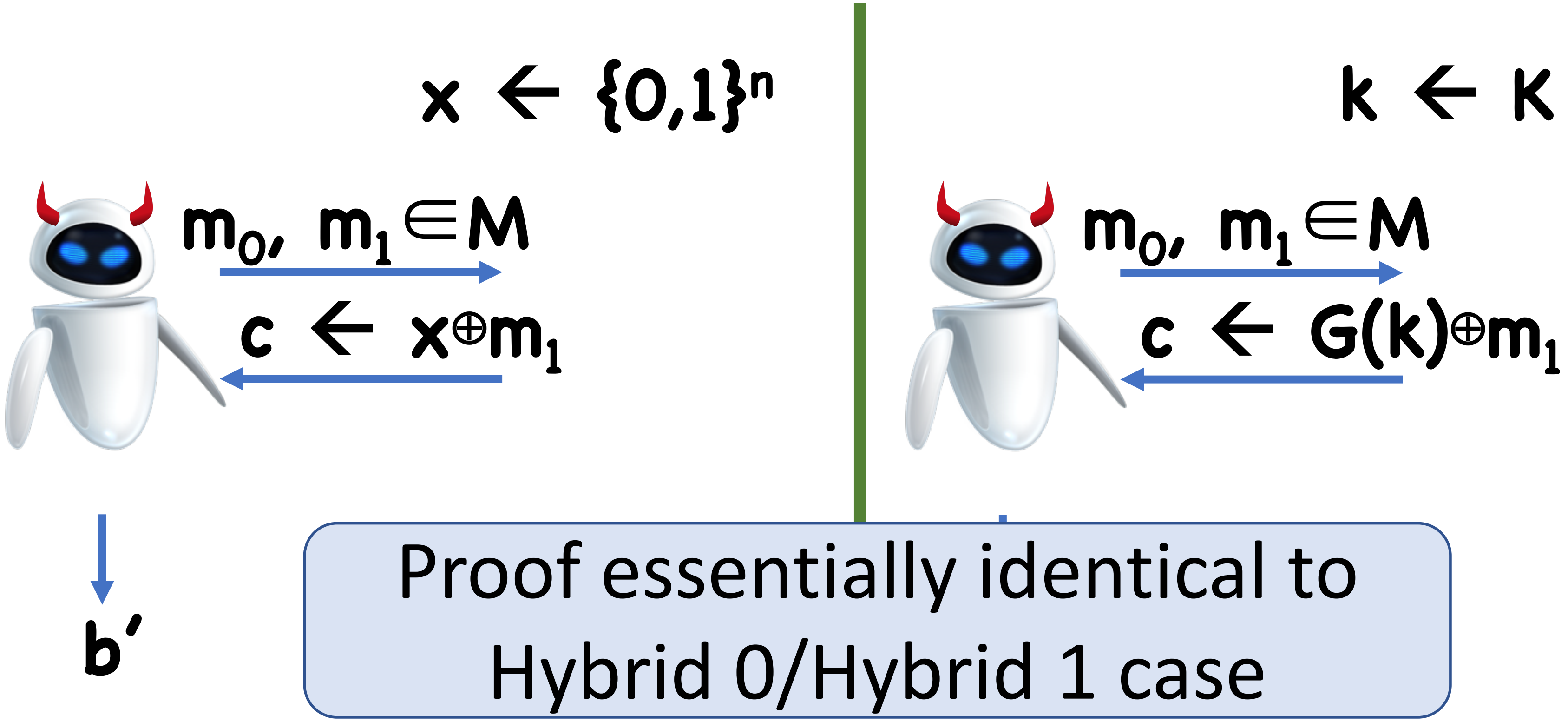
Suppose disti 1 from **Hybrid 2** with advantag **/3**



Impossible by OTP security

An Alternate Proof: Hybrids

Suppose 🤖 distinguishes **Hybrid 2** from **Hybrid 3** with advantage **$\varepsilon/3$**



$x \leftarrow \{0,1\}^n$

$m_0, m_1 \in M$

$c \leftarrow x \oplus m_1$

b'

$k \leftarrow K$

$m_0, m_1 \in M$

$c \leftarrow G(k) \oplus m_1$

Proof essentially identical to Hybrid 0/Hybrid 1 case

Reminders

PR1 Part 1 Due Tuesday, Feb 20th