COS433/Math 473: Cryptography

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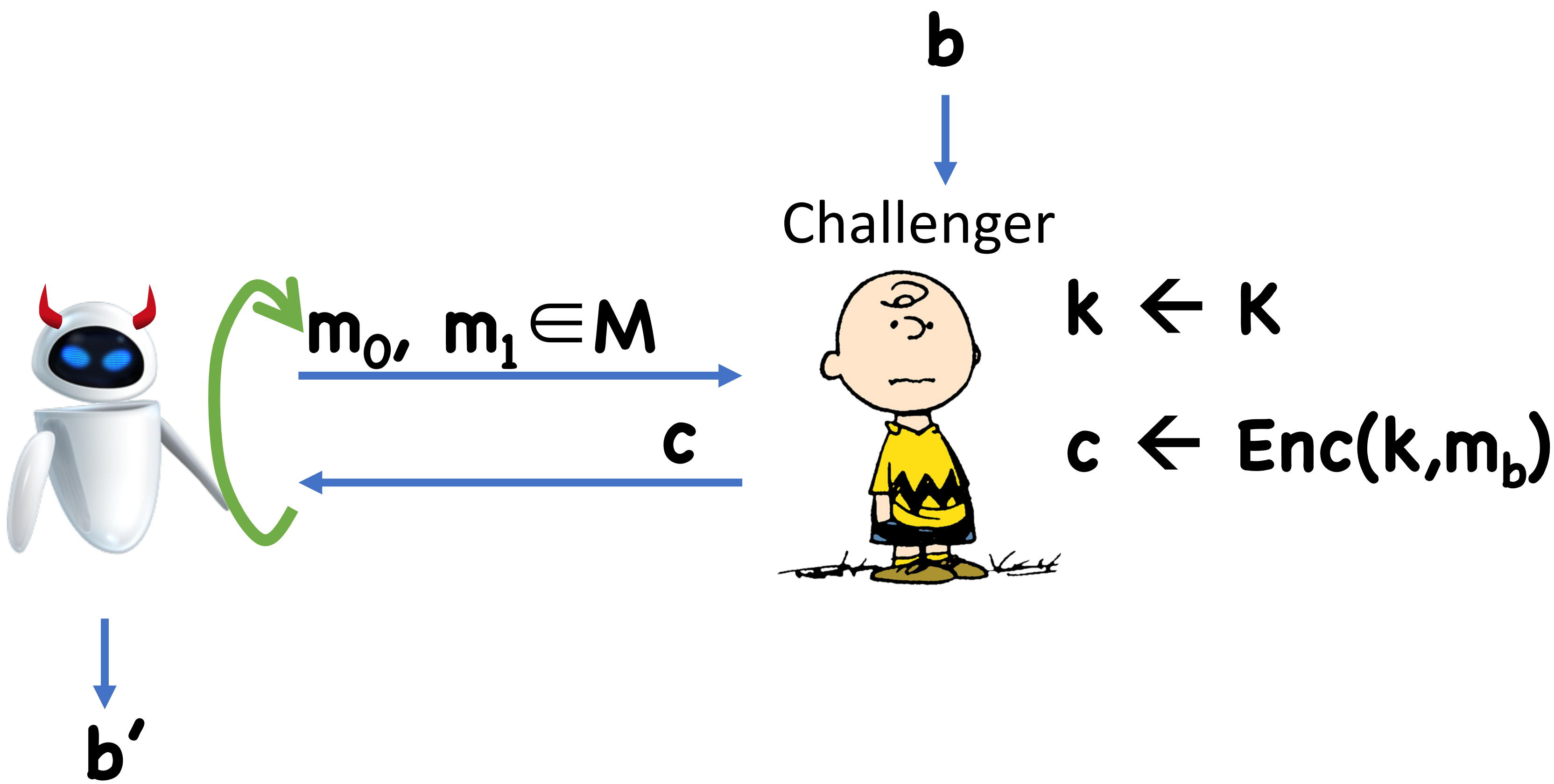
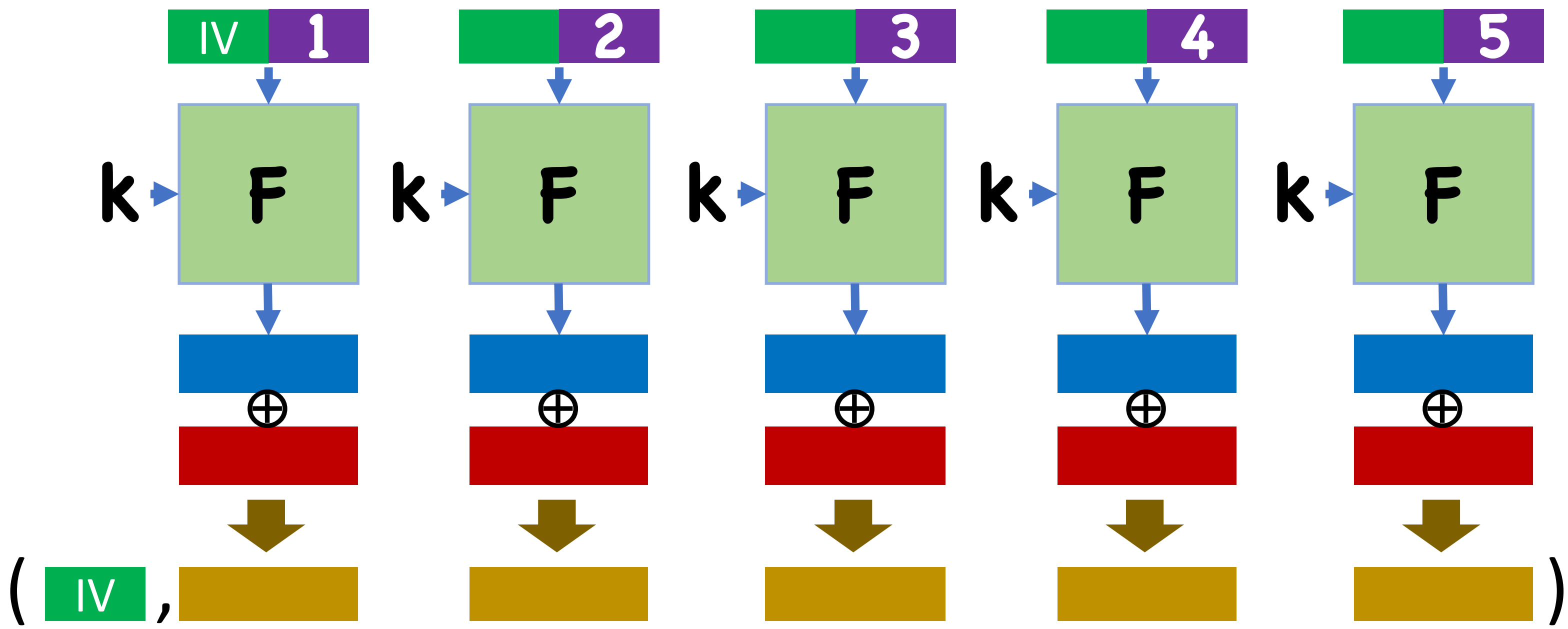
Message Integrity

Recall: CPA Security



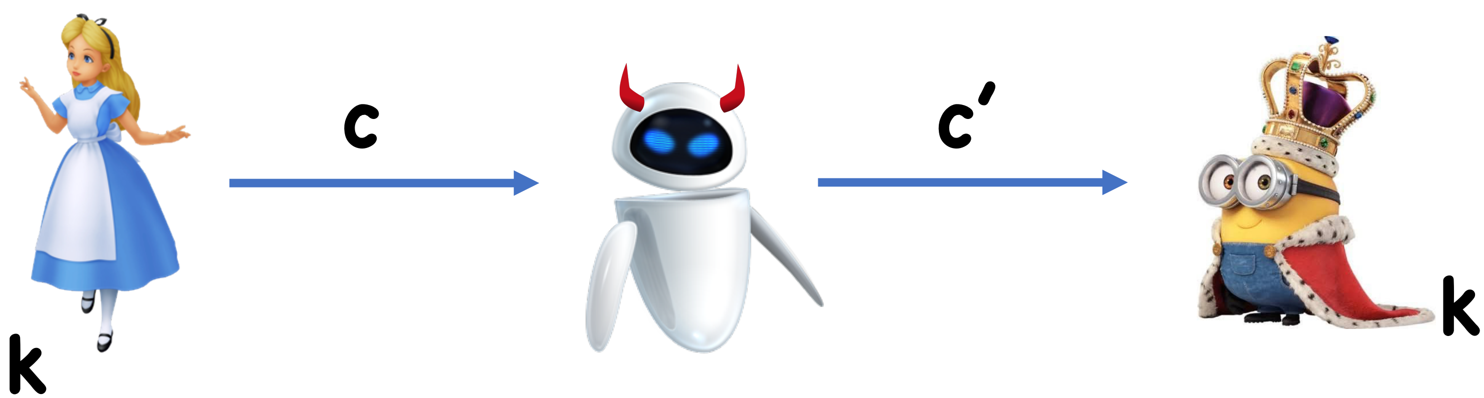
LoR-Exp$_b$( **)**

Recall: Counter Mode (CTR)



Limitations of CPA security

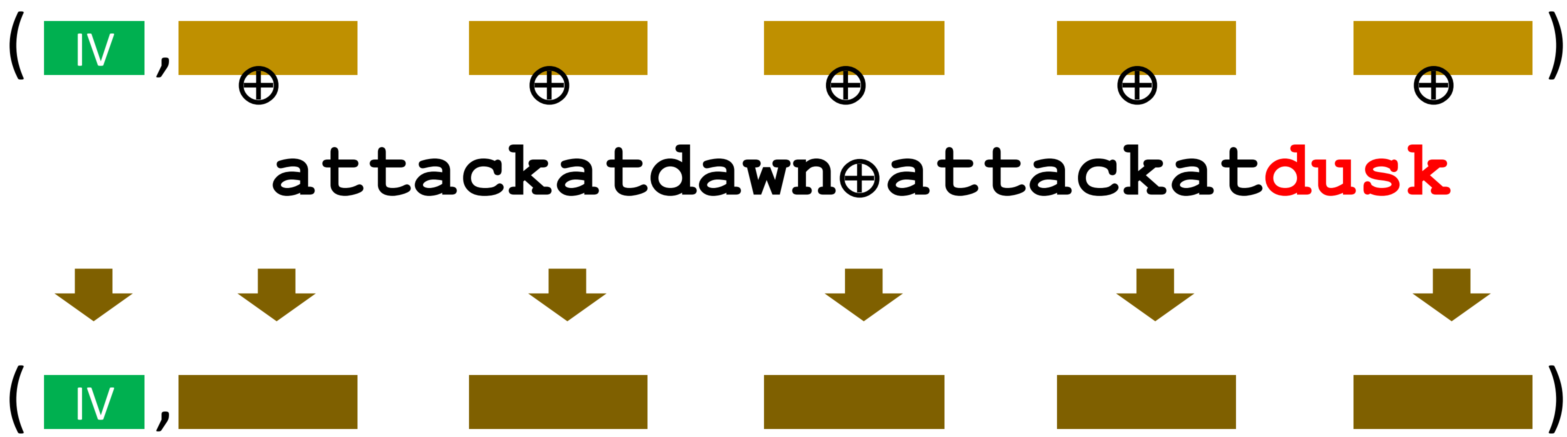
attackatdawn



attackat**dusk**

How?

Limitations of CPA Security



Malleability

Some encryption schemes are *malleable*

- Can modify ciphertext to cause predictable changes to plaintext

Examples: basically everything we’ve seen so far

- Stream ciphers
- CTR
- CBC
- ECB
- …

Message Integrity

We cannot stop adversary from changing the message in route to Bob

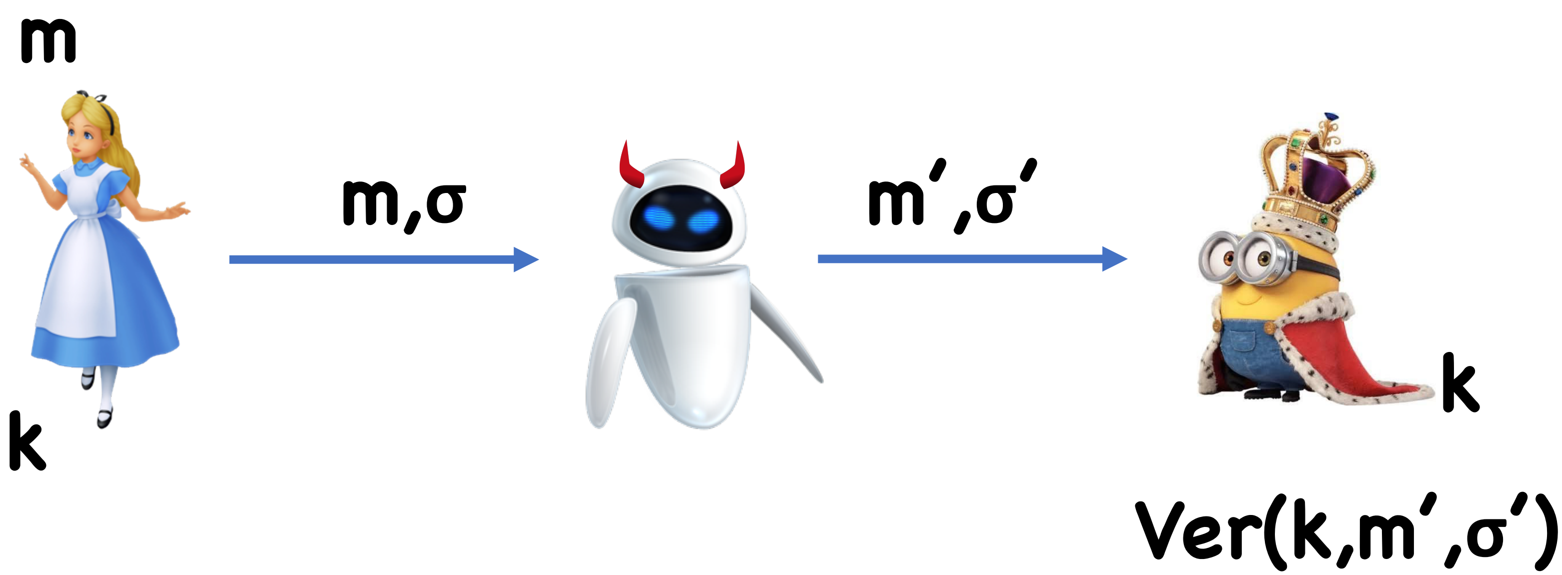
However, we can hope to have Bob perform some check on the message he receives to ensure it was sent by Alice

- If check fails, Bob rejects the message

For now, we won't care about message secrecy

- We will add it back in later

Message Authentication



Goal: If Eve changed **m**, Bob should reject

Message Authentication Codes

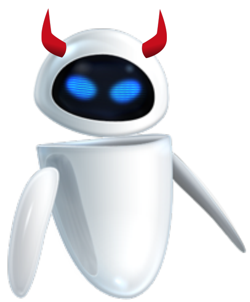
Syntax:

- Key space **K**
- Message space **M**
- Tag space **T**
- **MAC(k,m) → σ**
- **Ver(k,m,σ) → 0/1**

Correctness:

- $\forall$**m,k, Ver(k,m, MAC(k,m)) = 1**

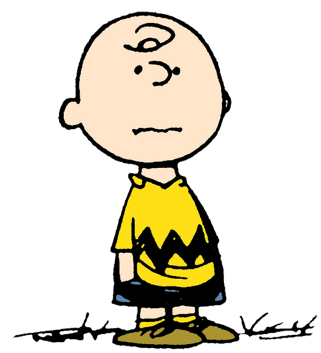
1-time Security For MACs



$m \in M \rightarrow$

$\leftarrow \sigma$

$(m^*, \sigma^*) \rightarrow$

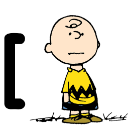


$k \leftarrow K$

$\sigma \leftarrow MAC(k,m)$

Output 1 iff:

- $m^* \neq m$
- $Ver(k, m^*, \sigma^*) = 1$

1CMA-Adv() = Pr[ outputs 1]

Definition: (MAC,Ver) is **ε–**1-time secure under a chosen message attack (**1CMA-secure)** if, for all ,

1CMA–Adv() ≤ ε

Question

Is perfect **0–**security possible?

A Simple 1-time MAC

Suppose **H** is a family of pairwise independent functions from **M** to **T**

For any $m_0 \neq m_1 \in M$, $\sigma_0, \sigma_1 \in T$

$$\Pr_{h \leftarrow H}[\ h(m_0)=\sigma_0 \wedge h(m_1)=\sigma_1] = 1/|T|^2$$

$$K = H$$

$$MAC(h, m) = h(m)$$

$$Ver(h,m,\sigma) = (h(m) == \sigma)$$

Theorem: (MAC,Ver) is $(1/|T|)$-1-time secure

Intuition: after seeing one message/tag pair, adversary learns nothing about tag on any other message

So to have security, just need $|T_\lambda|$ to be large

Ex: $T_\lambda = \{0,1\}^{128}$

Constructing Pairwise Independent Functions

$\mathbf{T} = \mathbb{F}$ (finite field of size $\approx 2^\lambda$)

- Example: $\mathbb{Z}_p$ for some prime p

Easy case: let $\mathbf{M}=\mathbb{F}$

- $\mathbf{H} = \{h(x) = a\,x + b:\ a,b \in \mathbb{F}\}$

Slightly harder case: Embed $\mathbf{M} \subseteq \mathbb{F}^n$

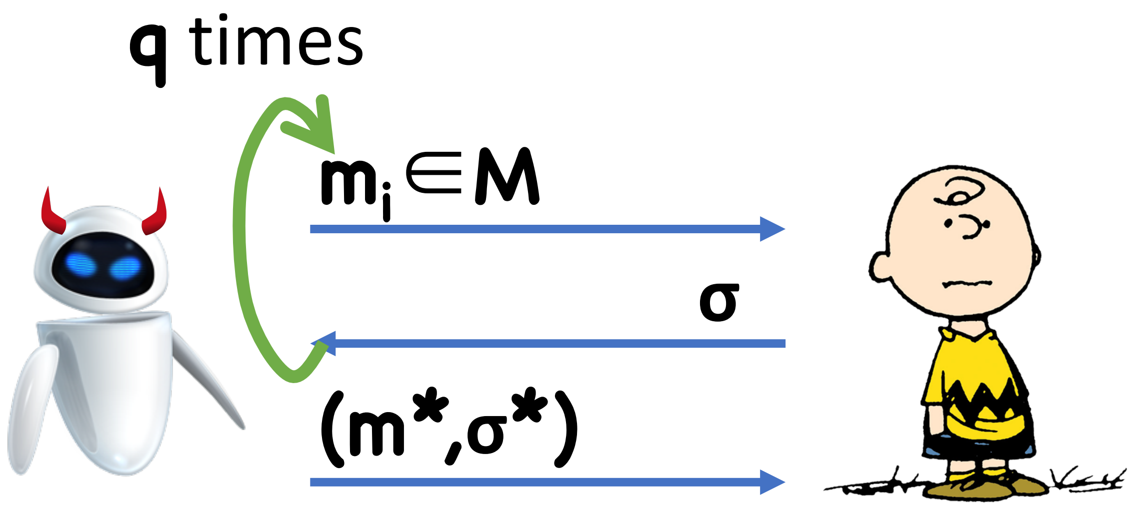
- $\mathbf{H} = \{h(x) = \langle a,x \rangle + b:\ a \in \mathbb{F}^n,\ b \in \mathbb{F}\}$

Multiple Use MACs?

Just like with OTP, if use 1-time twice, no security

Why?

q-Time MACs

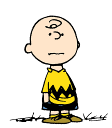


$k \leftarrow K$

$\sigma \leftarrow MAC(k,m)$

Output 1 iff:

- $m^* \notin \{m_1,\ldots,m_q\}$
- $Ver(k,m^*,\sigma^*) = 1$

qCMA-Adv( ,) = Pr[ outputs 1]

Definition: **(MAC,Ver)** is **(q,ε)-**secure under a chosen message attack (**CMA-secure)** if, for all  making at most **q** queries,

CMA-Adv( **) ≤ ε**

Constructing **q**-time MACs

Ideas?

Limitations?

Impossibility of Large **q**

Theorem: Any **(q,ε)–CMA-secure** MAC must have **q ≤ log |K|**

Proof

Idea:

- By making $q \gg \log |K|$ queries, you *should* be able to uniquely determine key
- One key is determined, can forge any message

Problem:

- What if certain bits of the key are ignored
- Intuition: ignoring bits of key shouldn't help

Proof

Define $\mathbf{r_q}$ as follows:

- Challenger chooses random key $\mathbf{k}$
- Adversary repeatedly choose random (distinct) messages $\mathbf{m_i}$ in $\mathbf{M}$
- Query the CMA challenger on each $\mathbf{m_i}$, obtaining $\boldsymbol{\sigma_i}$
- Let $\mathbf{K'_q}$ be set of keys $\mathbf{k'}$ such that $\mathbf{MAC(k',m_i)=\sigma_i}$ for $\mathbf{i=1,\ldots,q}$
- Let $\mathbf{r_q}$ be the expected size of $\mathbf{K'_q}$

Claim: If $(\mathbf{MAC},\mathbf{Ver})$ is statistically CMA-secure, then $r_q \leq r_{q-1}/2$

If not, then with probability at least ¼,

$$|K'_q| > |K'_{q-1}|/4$$

Attack:

- Make $q-1$ queries on random messages m_i
- Choose key k from K'_{q-1}
- Choose random m_q, compute $\sigma_q = MAC(k,m_q)$
- Output (m_q, σ_q)

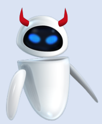
Probability of forgery?

Claim: If $(\mathbf{MAC},\mathbf{Ver})$ is statistically CMA-secure, then $r_q \leq r_{q-1}/2$

Finishing the impossibility proof:

- r_q is always at least 1 (since there is a consistent key)
- $r_0 = |K|$
- $1 \leq r_q \leq r_0/2^q \leq |K|/2^q$
- Setting $q > \log |K|$ gives a contradiction

Computational Security

Definition: (MAC,Ver) is **(tq,ε)-**secure under a chosen message attack (**CMA-secure)** if, for all  running in time at most **t** and making at most **q** queries,

CMA-Adv(**) ≤ ε**

Constructing MACs

Use a PRF

$F: K \times M \rightarrow T$

$\mathrm{MAC}(k,m) = F(k,m)$
$\mathrm{Ver}(k,m,\sigma) = (F(k,m) == \sigma)$

Theorem: If $\mathbf{F}$ is $\mathbf{(t,q,\varepsilon)}$-secure then $\mathbf{(MAC,Ver)}$ is $\mathbf{(t\text{-}t',q,\varepsilon+1/|T|)}$-CMA secure

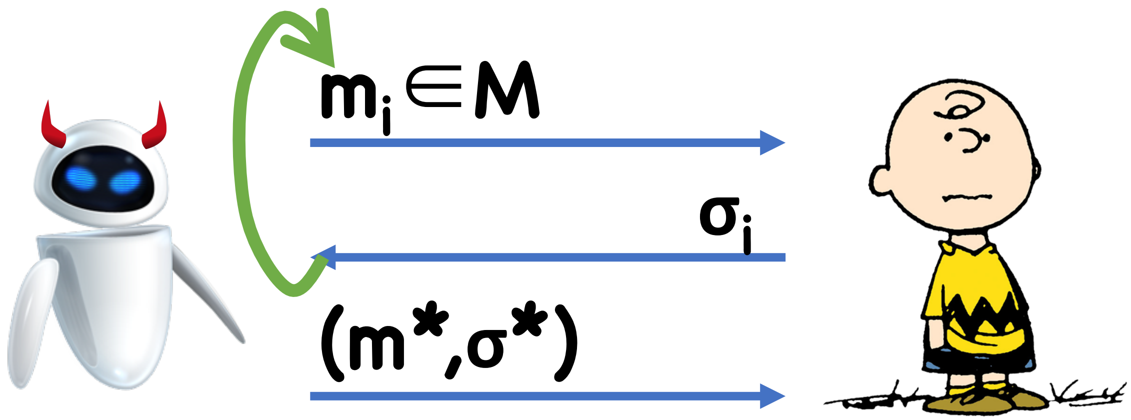
Security Proof

Assume toward contradiction PPT 

Hybrids!

Security Proof

Hybrid 0



$k \leftarrow K$

$\sigma \leftarrow F(k,m_i)$

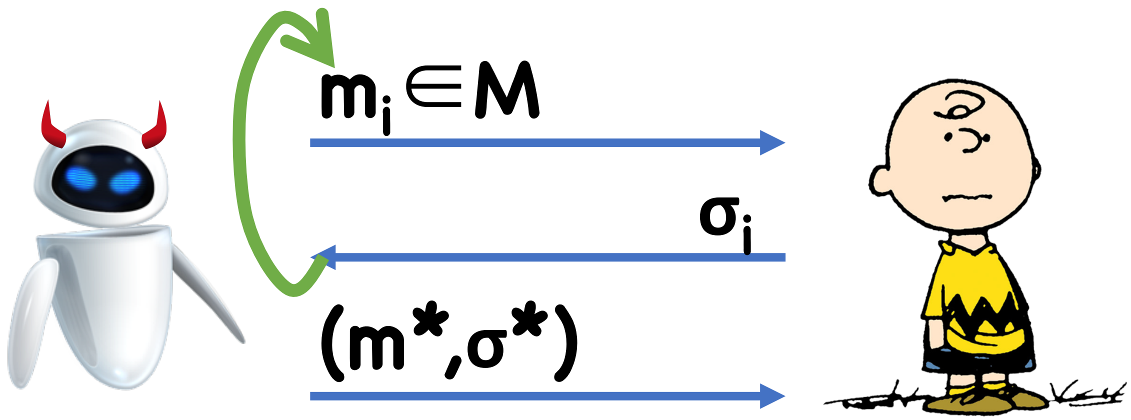
Output 1 iff:

- $m^* \notin \{m_1,\ldots\}$
- $F(k,m^*)=\sigma^*$

CMA Experiment

Security Proof

Hybrid 1



$H \leftarrow Funcs(X,Y)$

$\sigma \leftarrow H(m_i)$

Output 1 iff:
- $m^* \notin \{m_1,\ldots\}$
- $H(m^*)=\sigma^*$

Security Proof

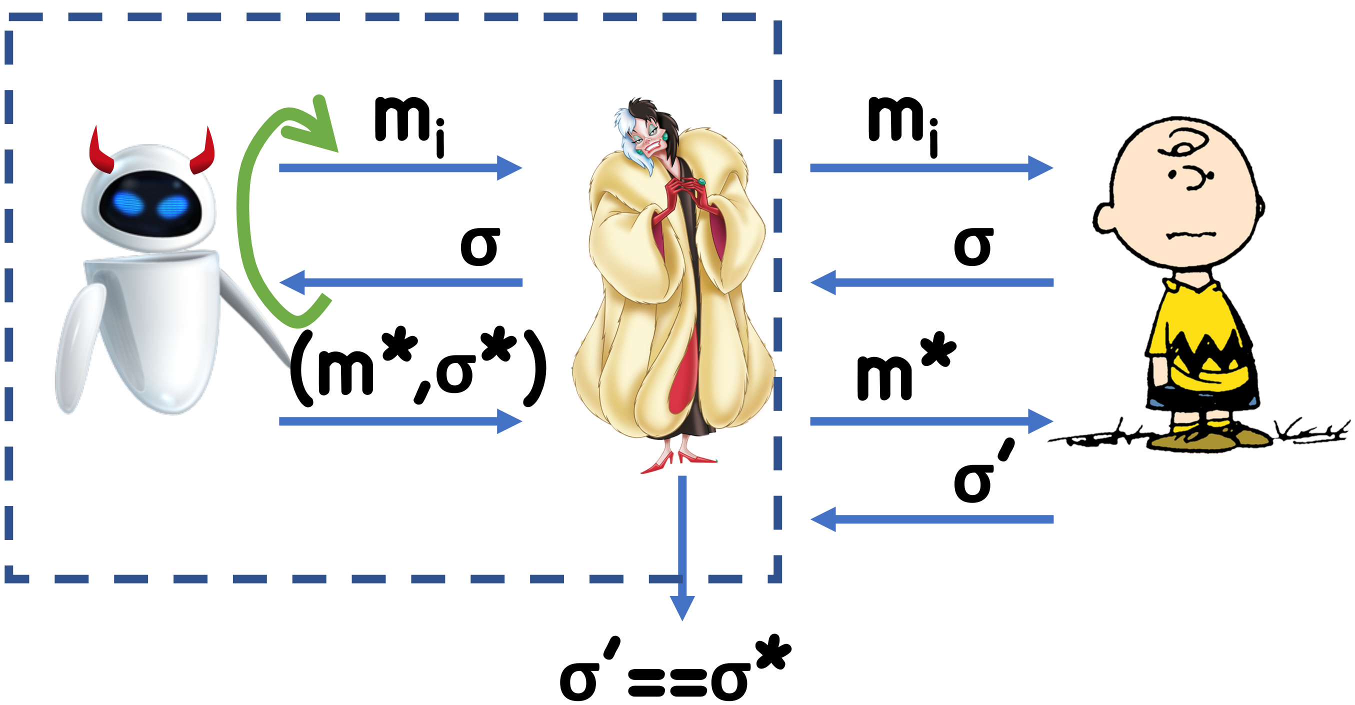
Claim: in Hybrid 1, output 1 with probability $1/|T|$

-  sees values of H on points m_i
- Value on m^* independent of  's view
- Therefore, probability $\sigma^* = H(m^*) = 1/|T|$

Security Proof

Claim: $|\Pr[1 \leftarrow \text{Hyb1}] - \Pr[1 \leftarrow \text{Hyb2}]| < \varepsilon$

Suppose not, construct PRF adversary 



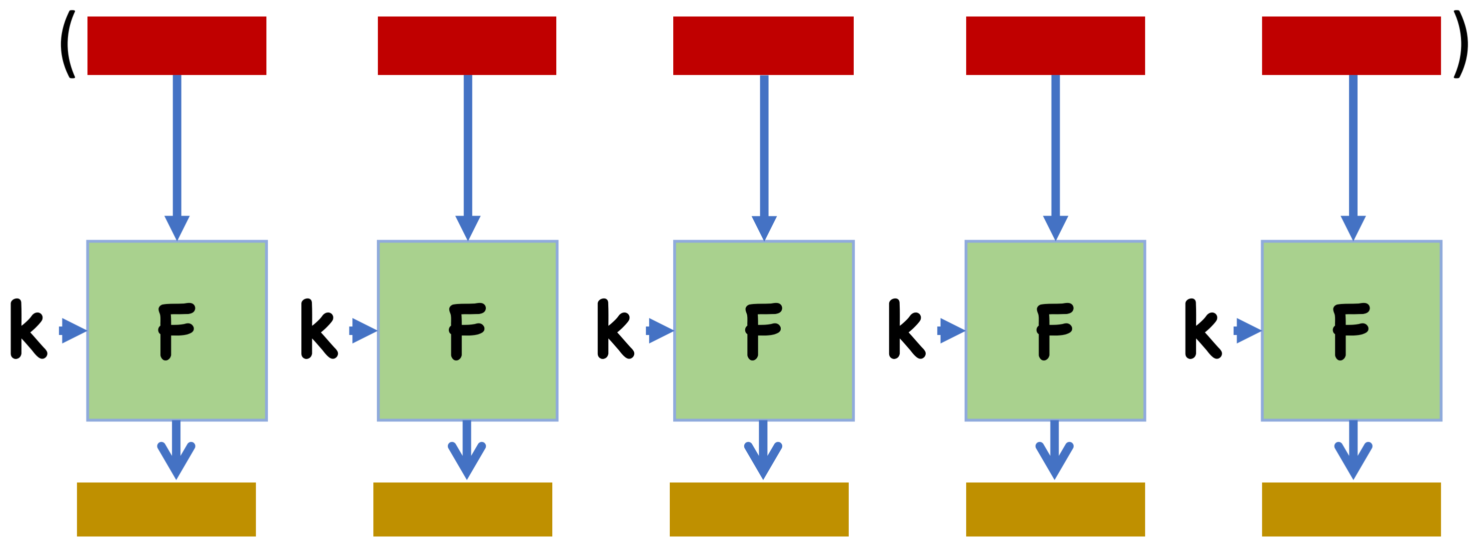
Constructing MACs/PRFs

We saw that block ciphers are good PRFs

However, the input length is generally fixed
- For example, AES maximum block length is 128 bits

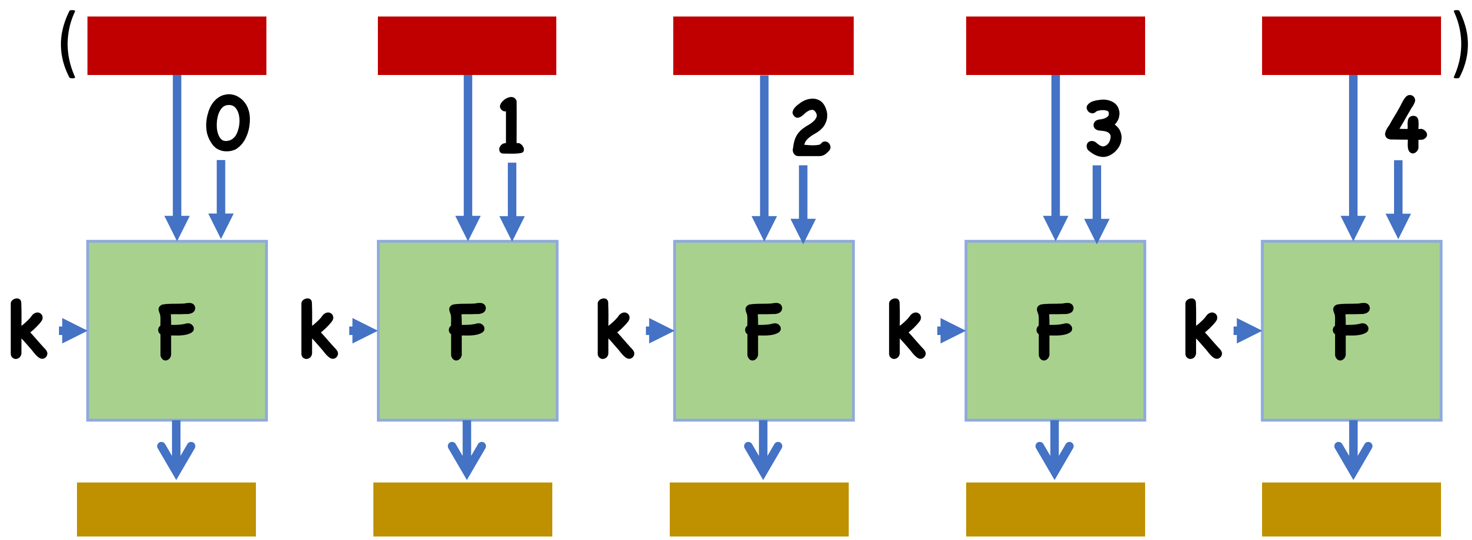
How do we handle larger messages?

Block-wise Authentication?



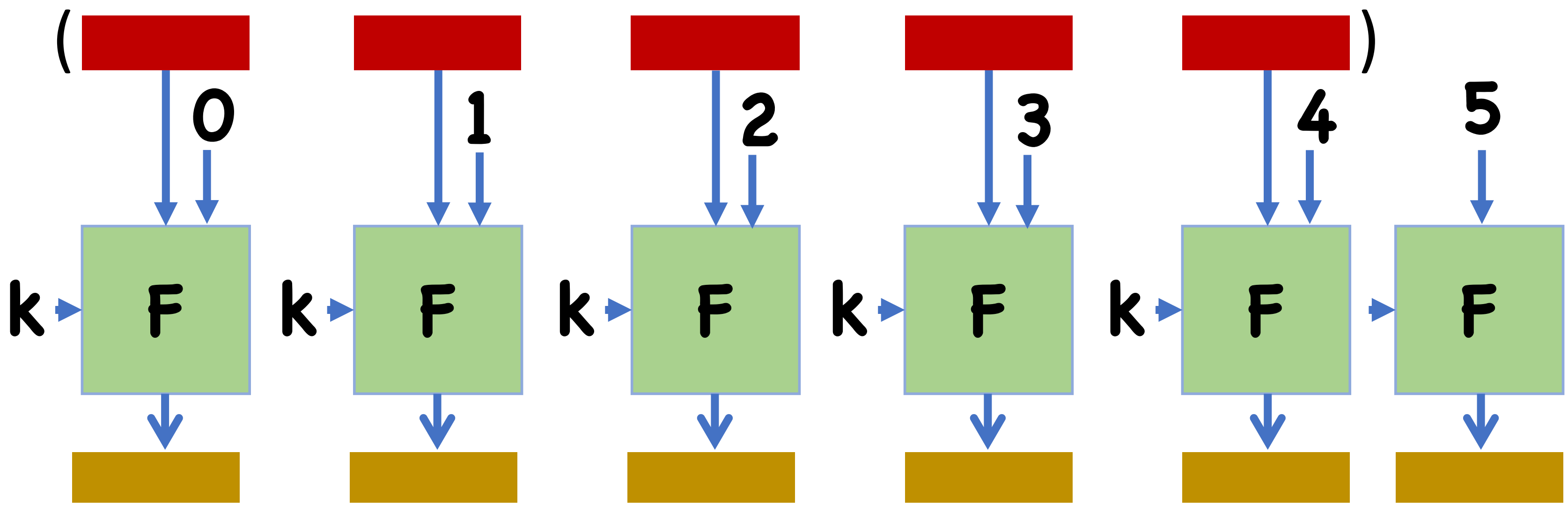
Why is this insecure?

Block-wise Authentication?



Why is this insecure?

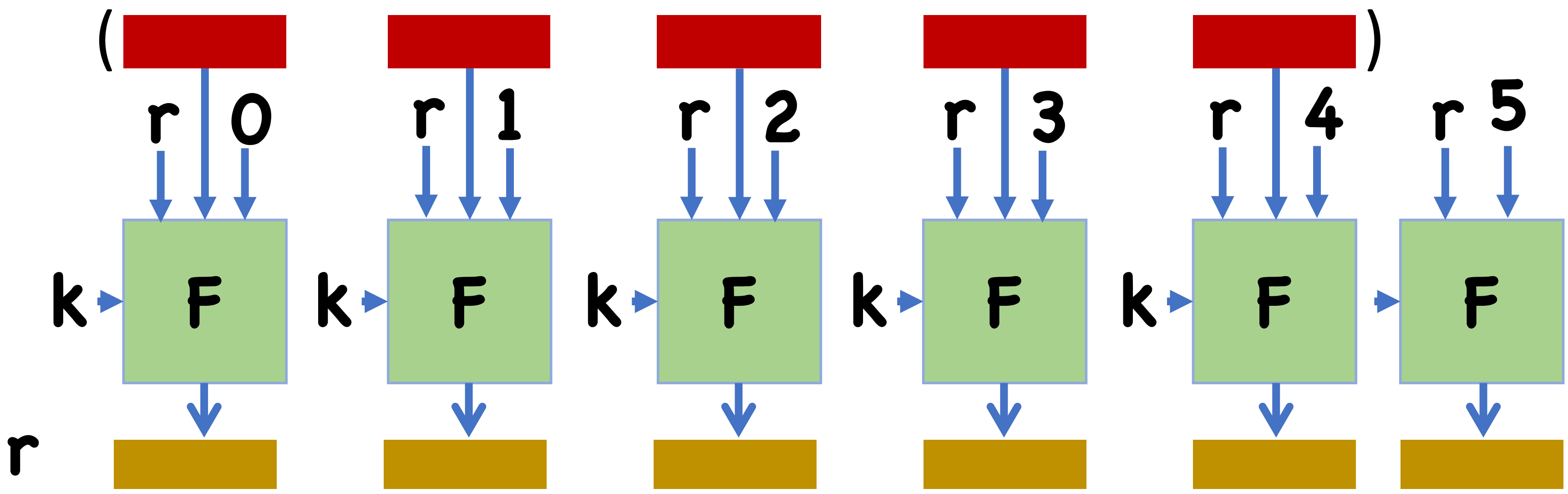
Block-wise Authentication?



Why is this insecure?

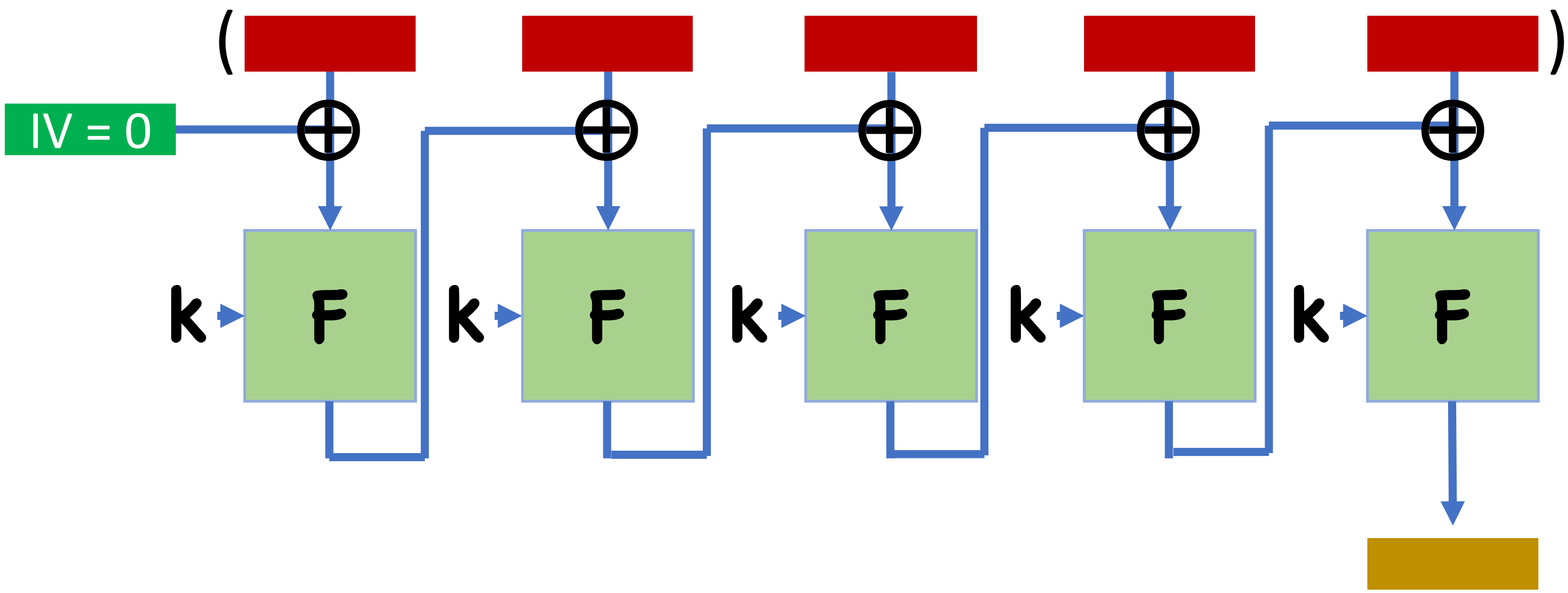
Block-wise Authentication?

r a random nonce



Secure, but not very useful in practice

CBC-MAC

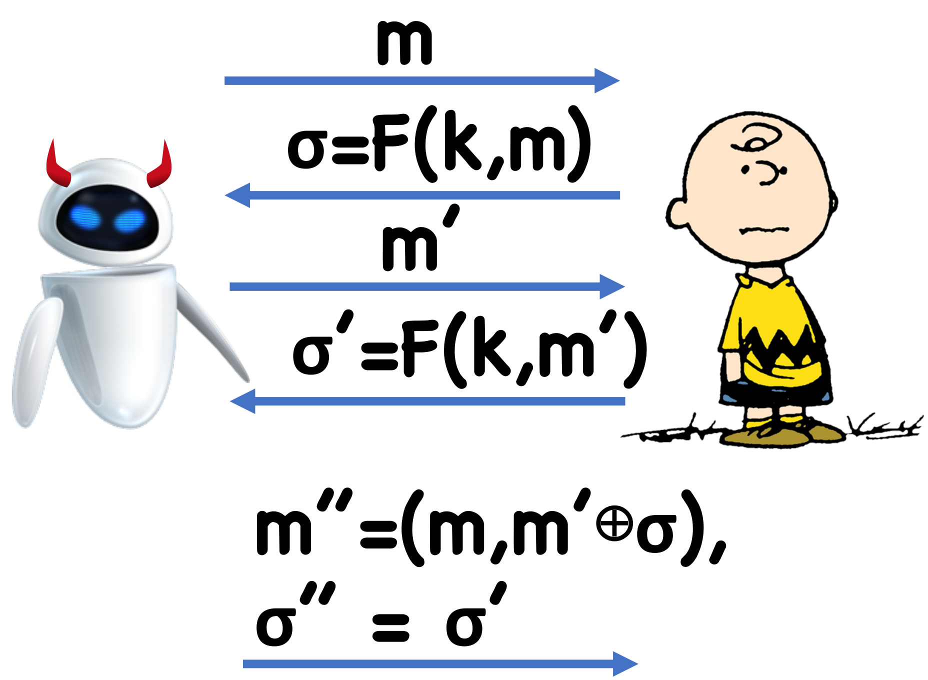


Theorem: CBC-MAC is a secure PRF for **fixed-length** messages

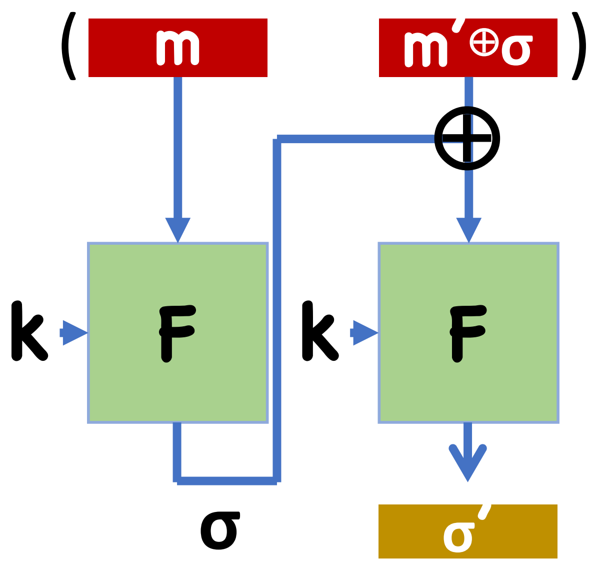
Variable Length Messages?

Basic CBC-MAC is insecure for variable length messages

Attack:



CBC-MAC



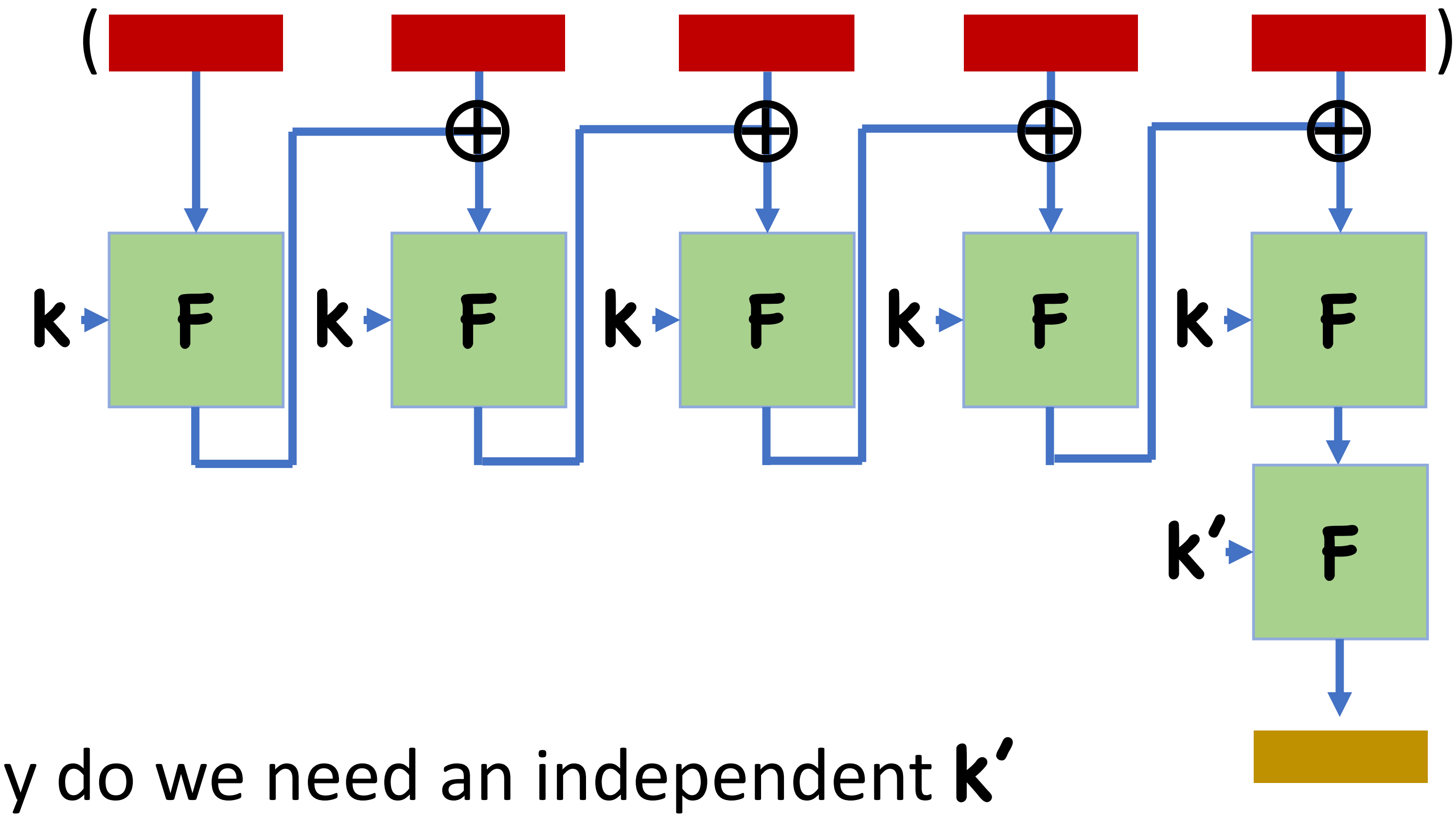
Handling Variable-Length Messages

Option 1:

- Prepend with msg length before applying CBC-MAC
  - $\Rightarrow$ No two messages will have the same prefix
- Limitation: must know message length when you start computing MAC
  - Not always reasonable if you are authenticating a stream of data
- Why is appending msg length to end not good?

Handling Variable-Length Messages

Option 2: Encrypt-Last-Block



Q: Why do we need an independent **k'**

Timing Attacks on MACs

How do you implement check $\mathbf{F(k,m)==\sigma}$?

String comparison often optimized for performance

Compare(A,B):
- **For i = 1,…,A.length**
  - **If A[i] != B[i], abort and return False;**
- **Return True;**

Time depends on number of initial bytes that match

Timing Attacks on MACs

To forge a message **m**:

For each candidate first byte $\boldsymbol{\sigma_0}$:

- Query server on $\boldsymbol{(m,\ \sigma)}$ where first byte of $\boldsymbol{\sigma}$ is $\boldsymbol{\sigma_0}$
- See how long it takes to reject

First byte is $\boldsymbol{\sigma_0}$ that causes the longest response

- If wrong, server rejects when comparing first byte
- If right, server rejects when comparing second

Timing Attacks on MACs

To forge a message $\mathbf{m}$:

Now we have first byte $\boldsymbol{\sigma}_0$

For each candidate second byte $\boldsymbol{\sigma}_1$:

- Query server on $(\mathbf{m},\ \boldsymbol{\sigma})$ where first two bytes of $\boldsymbol{\sigma}$ are $\boldsymbol{\sigma}_0,\boldsymbol{\sigma}_1$
- See how long it takes to reject

Second byte is $\boldsymbol{\sigma}_1$ that causes the longest response

•••

Holiwudd Criptoe!

Most likely not what was meant
Hollywood, but conceivable

Thwarting Timing Attacks

Possibility:

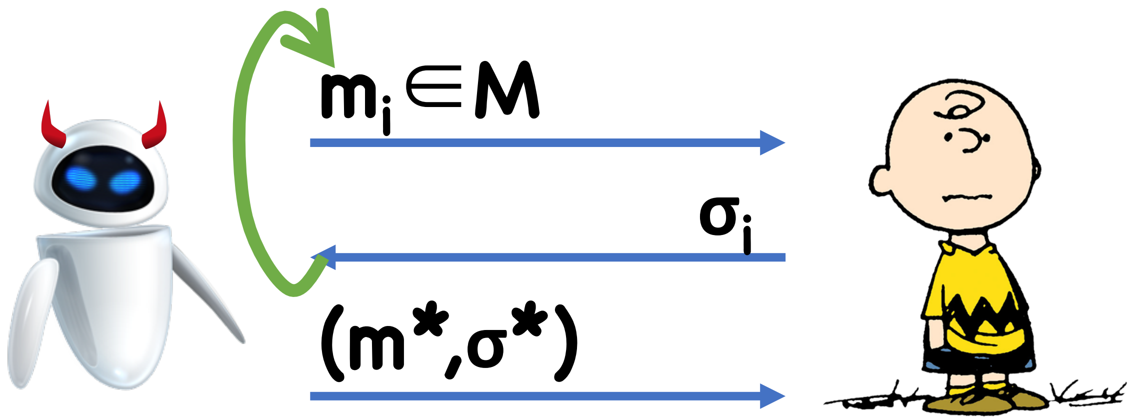
- Use a string comparison that is guaranteed to take constant time
- Unfortunately, this is hard in practice, as optimized compilers could still try to shortcut the comparison

Possibility:

- Choose random block cipher key **k'**
- Compare by testing **F(k',A) == F(k', B)**
- Timing of "**==**" independent of how many bytes **A** and **B** share

Alternate security notions

Strongly Secure MACs

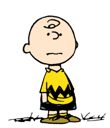


$k \leftarrow K$

$\sigma \leftarrow MAC(k, m_i)$

Output 1 iff:

- $(m^*, \sigma^*) \notin \{(m_1, \sigma_1), \ldots\}$
- $Ver(k, m^*, \sigma^*) = 1$

SCMA-Adv() = Pr[ outputs 1]

Strongly Secure MACs

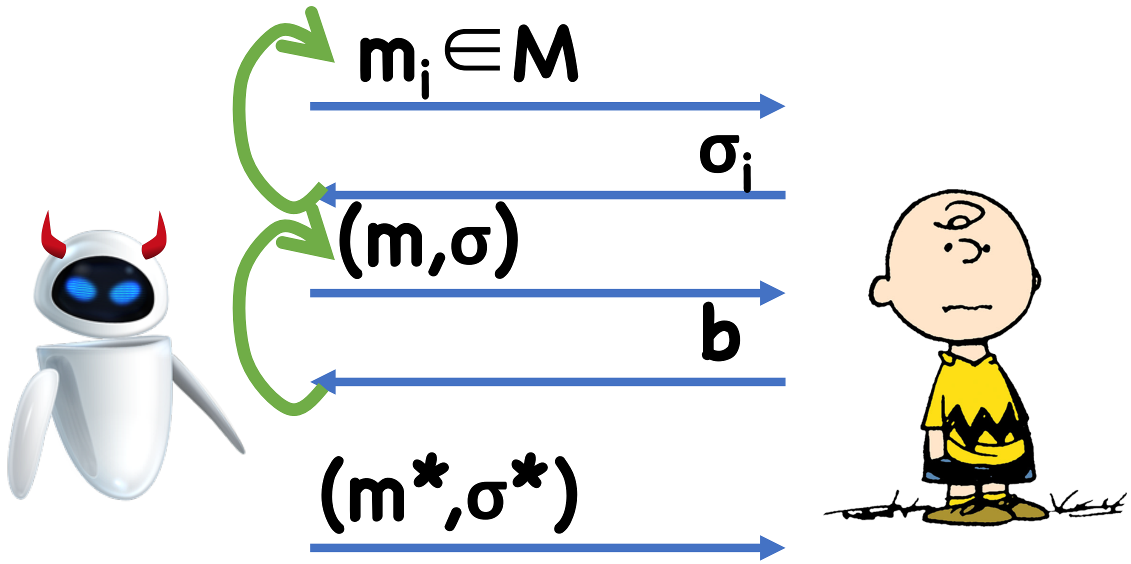
Useful when you don’t want to allow the adversary to change *any* part of the communication

If there is only a single valid tag for each message (such as in the PRF-based MAC), then (weak) security also implies strong security

In general, though, strong security is stronger than weak security

Adding Verification Queries

$k \leftarrow K$



$\sigma_i \leftarrow MAC(k,m_i)$

$b \leftarrow Ver(k,m,\sigma)$

Output 1 iff:

- $m^* \notin \{m_1,\ldots\}$
- $Ver(k,m^*,\sigma^*) = 1$

CMA'-Adv() = Pr[ outputs 1]

Theorem: (MAC,Ver) is strongly CMA secure if and only if it is strongly CMA' secure

Proof Sketch

Strong CMA' → strong CMA: trivial

Strong CMA → strong CMA'
Idea: adversary could have always answered verification queries for himself

- If adv previously received the message/signature pair from challenger, then it must be valid
- If adv did not previously receive pair, almost surely invalid

(if not, then we have a strong forgery)

Improving efficiency

Limitations of CBC-MAC

Many block cipher evaluations

Sequential

Carter Wegman MAC

k' = (k,h) where:

- **k** is a PRF key for $F: K \times R \rightarrow Y$
- **h** is sampled from a pairwise independent function family

MAC(k',m):

- Choose a random $r \leftarrow R$
- Set $\sigma = (r,\ F(k,r) \oplus h(m))$

Theorem: If F is **(t,q,ε)-**secure, then the Carter Wegman MAC is **$(t-t',q-1,\varepsilon+1/|T|+q^2/|R|)$-** strongly CMA secure

Proof

Assume toward contradiction a PPT 

Hybrids…

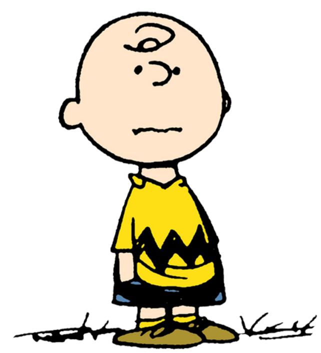
Proof

<u>Hybrid 0</u>

$m_i \in M$

$\sigma_i = (r_i, t_i)$

(m^*, r^*, t^*)



$k \leftarrow K$

h

$r_i \leftarrow R$

$t_i \leftarrow F(k,r) \oplus h(m)$

Output 1 iff:

- $(m^*, r^*, t^*) \notin \{(m_i, r_i, t_i)\}$
- $F(k, r^*) \oplus h(m^*) = t^*$

Proof

Hybrid 1

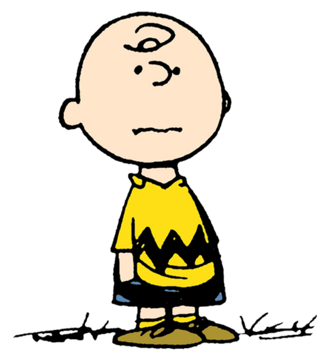
$k \leftarrow K$

h

$m_i \in M$

$\sigma_i = (r_i, t_i)$

(m^*, r^*, t^*)



(Distinct r_i)

$r_i \leftarrow R$

$t_i \leftarrow F(k,r) \oplus h(m)$

Output 1 iff:

- $(m^*, r^*, t^*) \notin \{(m_i, r_i, t_i)\}$
- $F(k, r^*) \oplus h(m^*) = t^*$

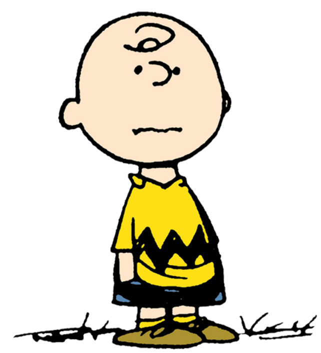
Proof

<u>Hybrid 2</u>

$m_i \in M$

$\sigma_i = (r_i, t_i)$

(m^*, r^*, t^*)



$H \leftarrow \text{Funcs}$

h

(Distinct r_i)

$r_i \leftarrow R$

$t_i \leftarrow H(r) \oplus h(m)$

Output 1 iff:

- $(m^*, r^*, t^*) \notin \{(m_i, r_i, t_i)\}$
- $H(r^*) \oplus h(m^*) = t^*$

Proof

Claim: In Hybrid 2, negligible success probability

Possibilities:

- $r^* \notin \{r_i\}$: then value of $H(r^*)$ hidden from adversary, so $\Pr[H(r^*) \oplus h(m^*) = t^*]$ is $1/|Y|$

- $r^* = r_i$ for some i: then $m^* \neq m_i$ (why?)

  h completely hidden from adversary

  $\Pr[H(r^*) \oplus h(m^*) = t^*]$

  $= \Pr[h(m^*) = t^* \oplus t_i \oplus h(m_i)] = 1/|Y|$

Proof

Hybrid 1 and 2 are indistinguishable

- PRF security

Hybrid 0 and 1 are indistinguishable

- W.h.p. random $\mathbf{r}_i$ will be distinct

Therefore, negligible success probability in Hybrid 0

Efficiency of CW MAC

MAC(k',m):

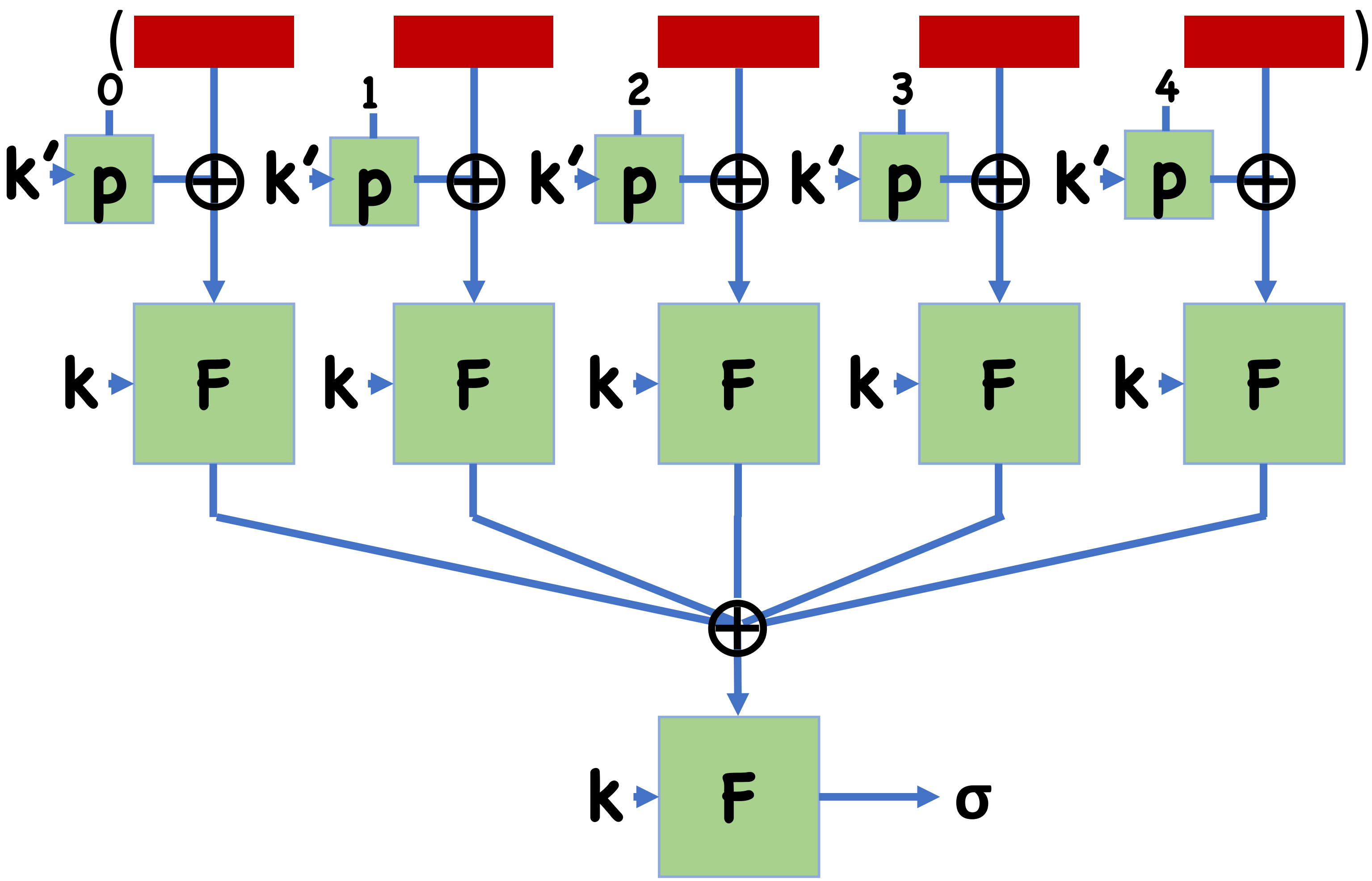
- Choose a random $r \leftarrow R$
- Set $\sigma = (r, F(k,r) \oplus h(m))$

h much more efficient that PRFs

PRF applied only to small nonce **r**

h applied to large message **m**

PMAC: A Parallel MAC



Reminder

HW3 Due Tomorrow