CS 161: Design and Analysis of Algorithms

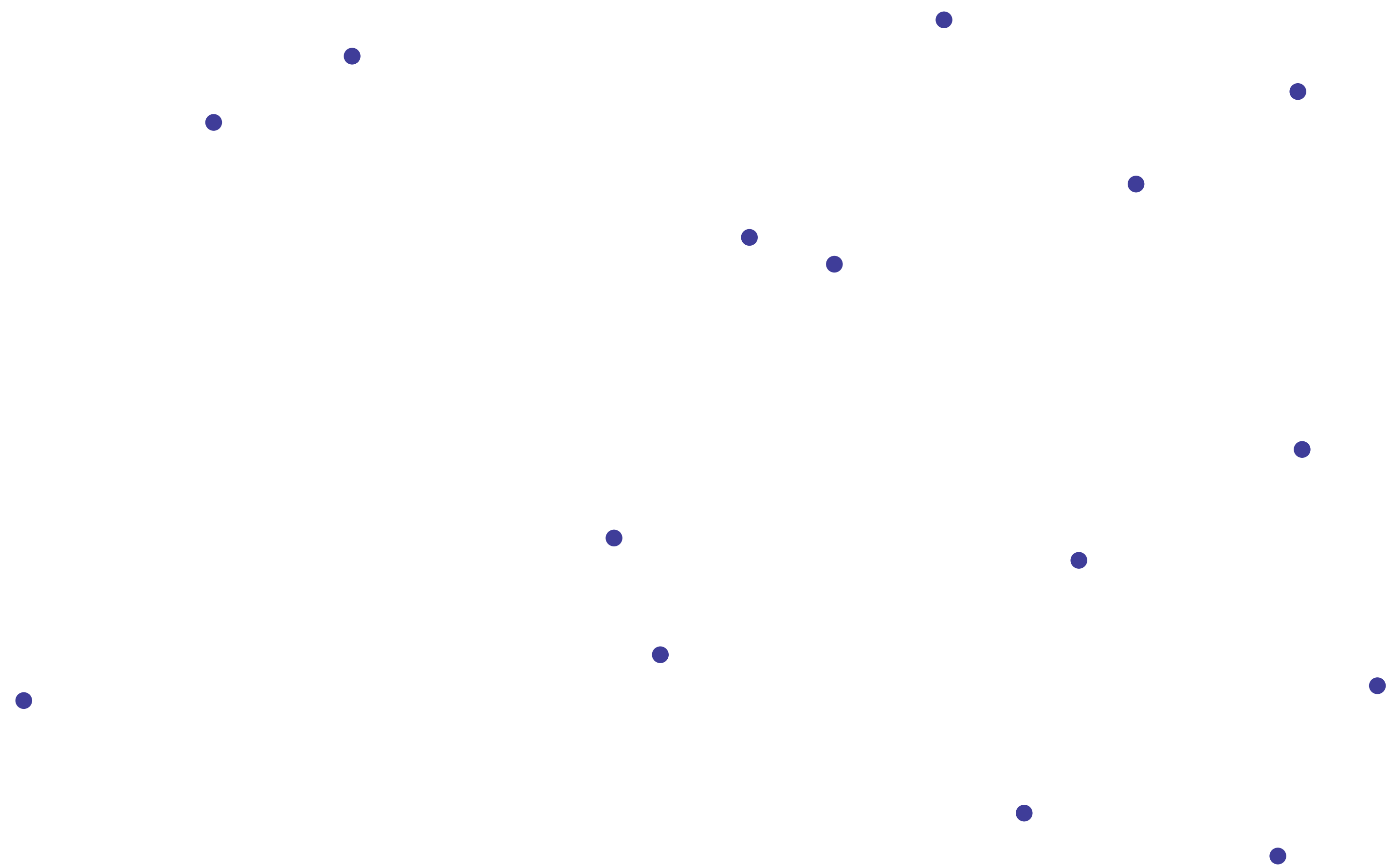
Beyond CS 161

- Computational Geometry
- Algorithmic Game Theory
- Complexity
- Quantum Computing

Computational Geometry (CS 268)

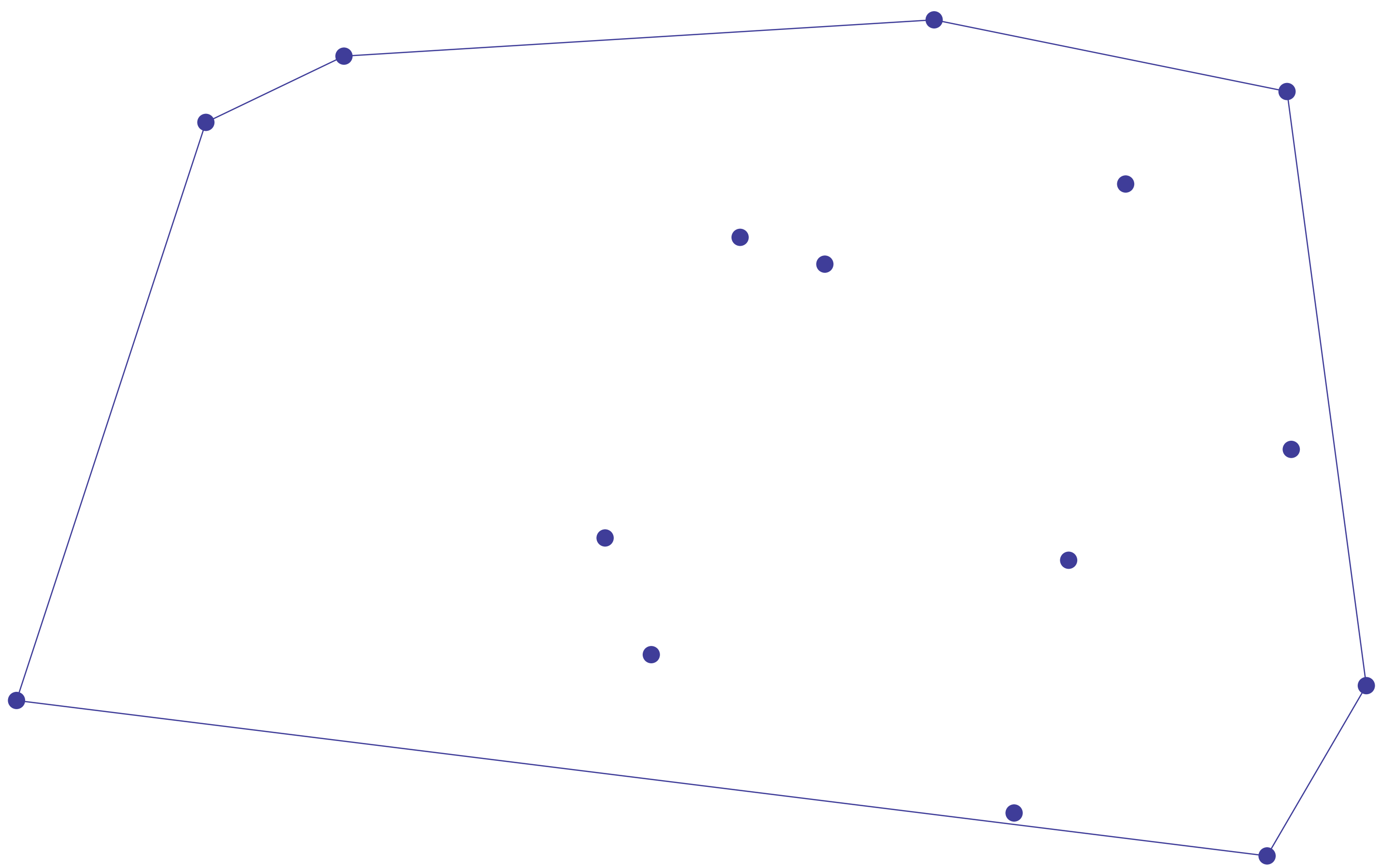
Computational Geometry (CS 268)

- Convex hull



Computational Geometry (CS 268)

- Convex hull



Computational Geometry (CS 268)

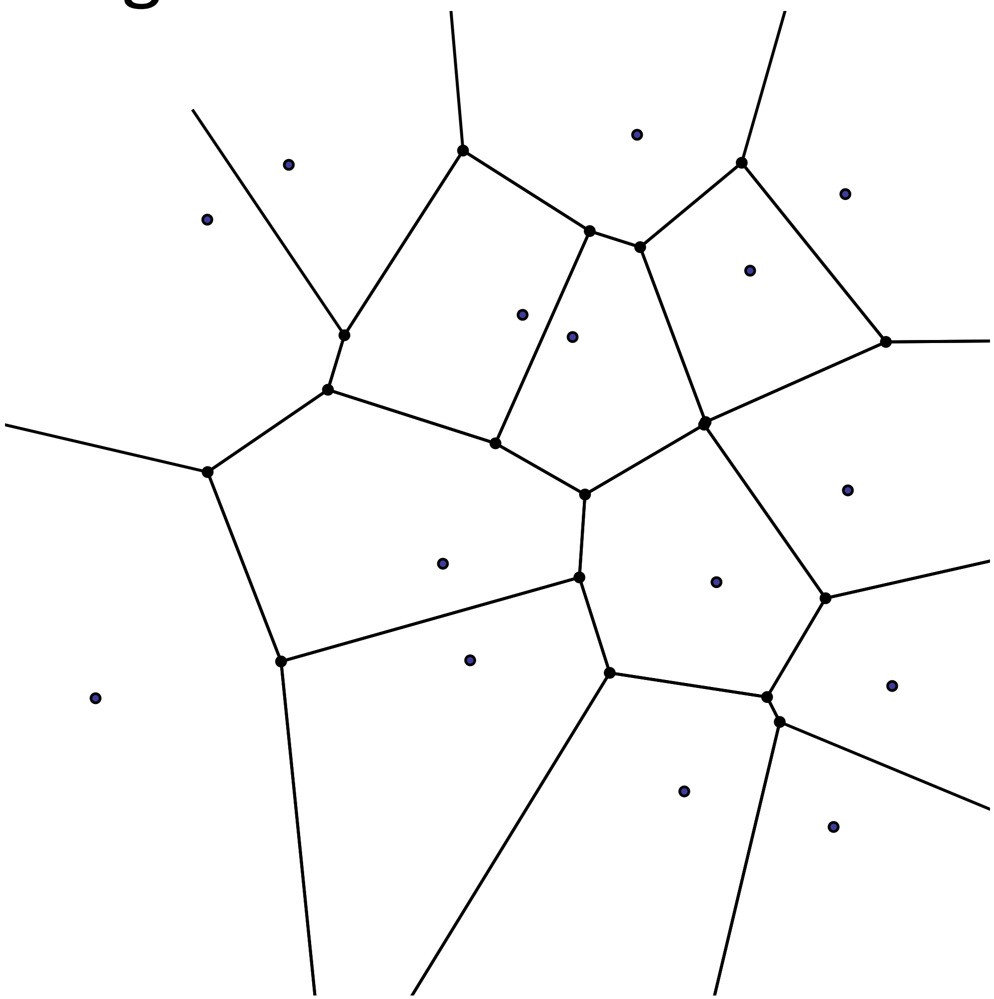
- Convex hull
  - Say you have a bunch of complicated 3D objects, and you want to determine if they collide
  - Take convex hull of each object
  - Only look at pairs of objects where convex hulls collide

Computational Geometry (CS 268)

- Simple algorithm for convex hull:
  - Find left-most point, x. Must be in convex hull
  - Scan all points to find the one that is next going clockwise around hull
  - Repeat until back at the starting point.
  - If hull has h points, O(n h)

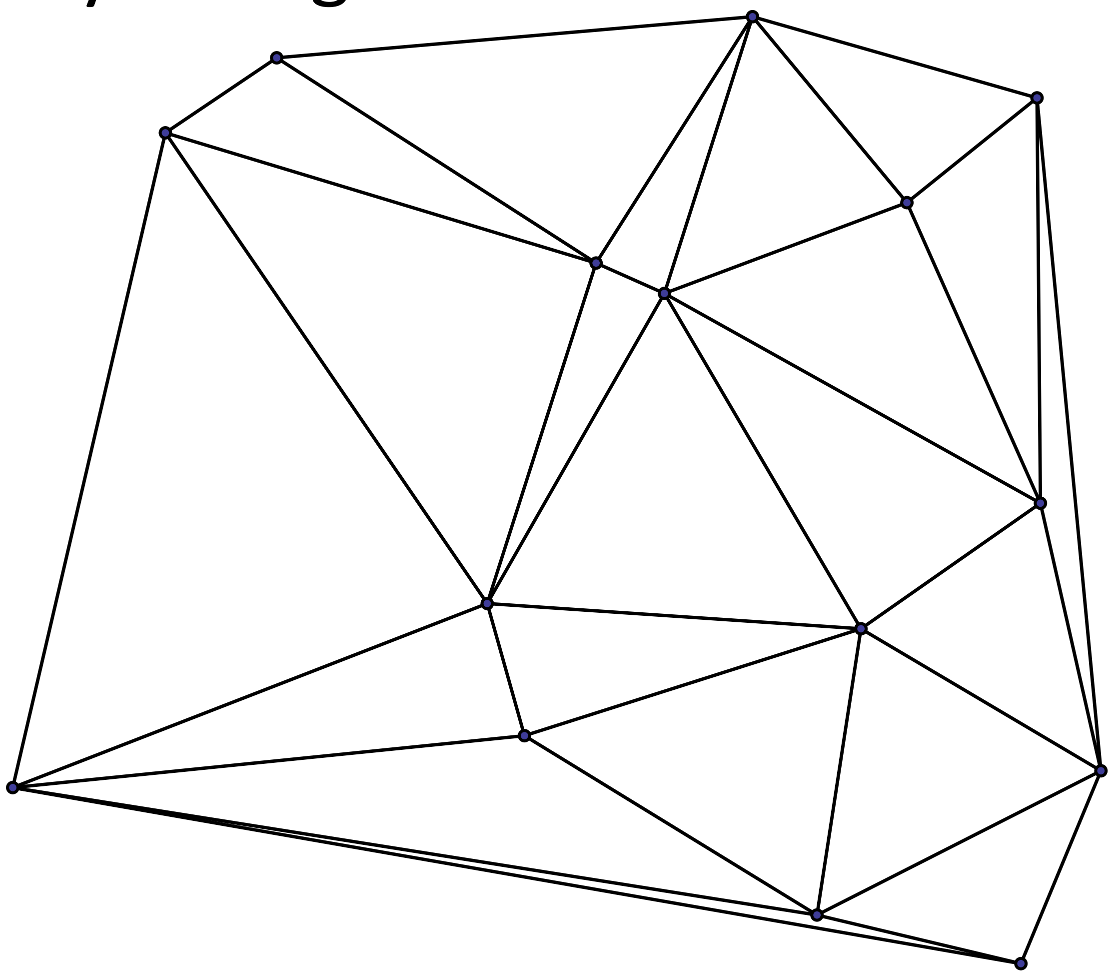
Computational Geometry (CS 268)

- Voronoi Diagram



Computational Geometry (CS 268)

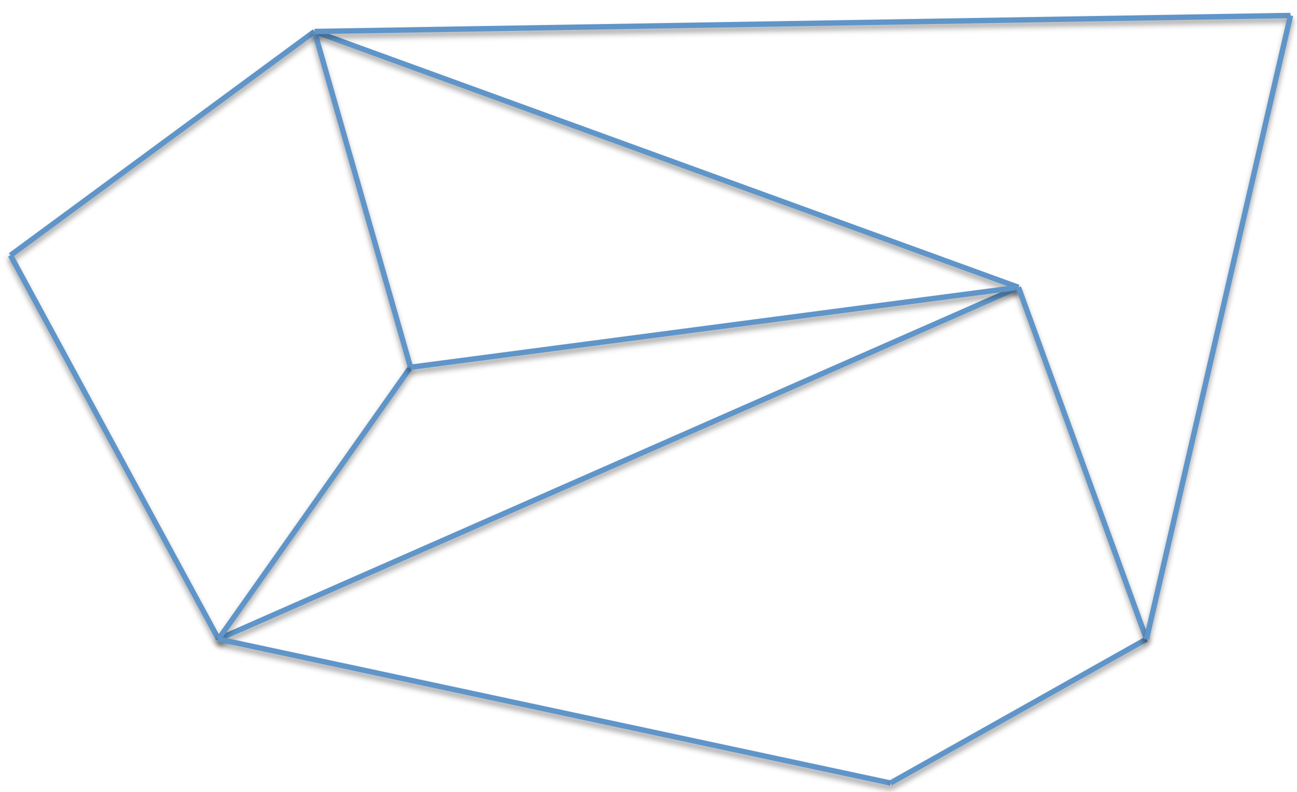
- Delaunay Triangulation



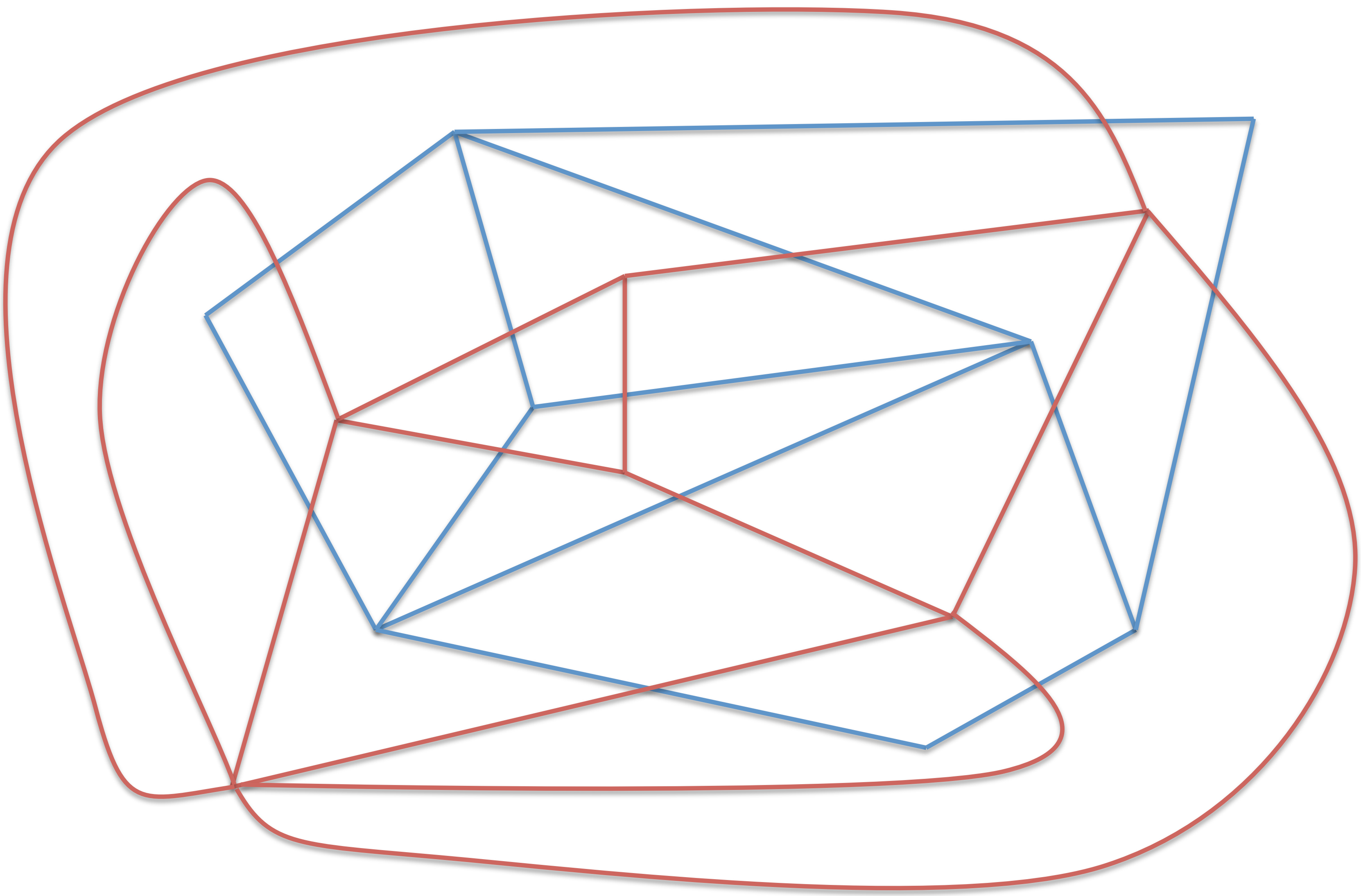
Computational Geometry (CS 268)

- Turns out that Voronoi diagrams and Delaunay triangulations are “duals” of each other
  - Each face becomes an node
  - Each boundary between faces becomes an edge

Computational Geometry (CS 268)

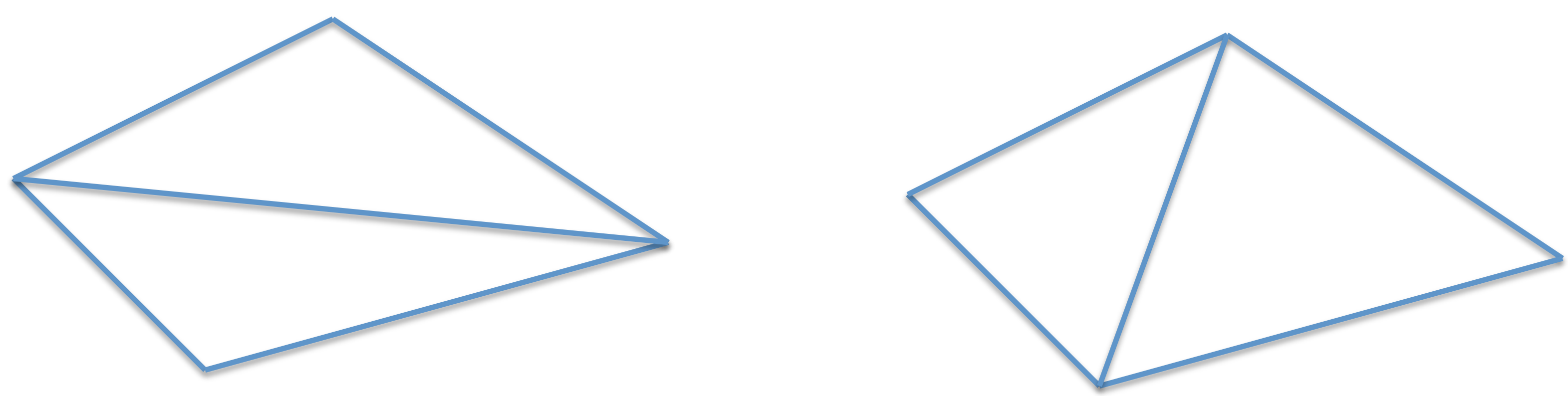


Computational Geometry (CS 268)



Computational Geometry (CS 268)

- To compute Delaunay triangulation:
  - Start with some triangulation
  - Repeatedly swap edges until Delaunay



Computational Geometry (CS 268)

- Faster Algorithm:
  - Divide points in half
  - Recursively compute Delaunay of each half
  - Glue results back together
  - O(n log n)

Computational Geometry (CS 268)

- Linear Programming
  - Feasible region is a geometric object

Algorithmic Game Theory (CS 364)

Algorithmic Game Theory (CS 364)

- Zero-Sum Games
  - Rock-Paper-Scissors: each person picks either rock paper or scissors, and announce their choice at the same time
  - Rock beats scissors, scissors beats paper, paper beats rock
  - Whoever wins gets $1 form the other person

Algorithmic Game Theory (CS 364)

- Zero-Sum Games

		Person A		
		Rock	Paper	Scissors
Person B	Rock	0	-1	1
	Paper	1	0	-2
	Scissors	-1	1	0

Algorithmic Game Theory (CS 364)

- Zero-Sum Games
  - What is the optimal strategy?
  - If person A always picks Rock, then person B will catch on an pick paper
  - A’s best strategy is to mix it up

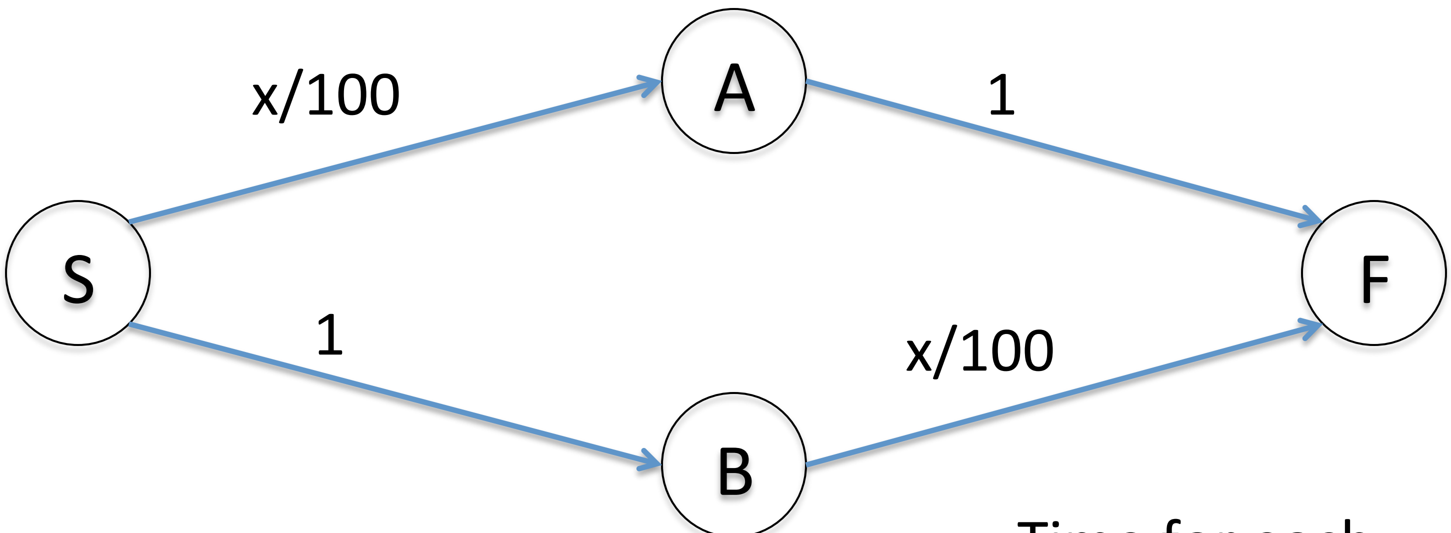
Algorithmic Game Theory (CS 364)

- Zero-Sum Games
  - Want a **mixed strategy**: pick each item randomly with some probabilities
  - Turns out, optimal mixed strategy for A can be solved by a linear program
  - The dual: the LP for the optimal mixed strategy for B
  - For zero-sum games, the optimal strategy for A is the one that yields the same expected profit, no matter what B does.

Algorithmic Game Theory (CS 364)

- Braess's Paradox
  - Have 100 drivers

Half the drivers will go each way

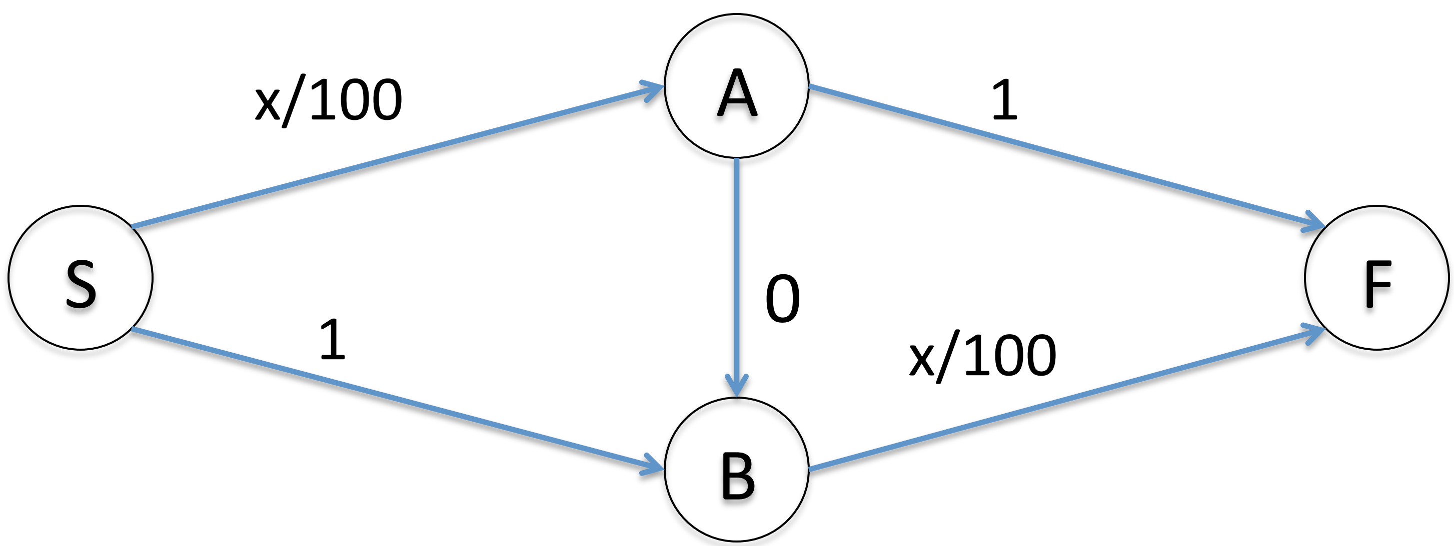


Time for each driver: 1.5

Algorithmic Game Theory (CS 364)

- Braess's Paradox
  - Add a new edge

All drivers will take route S – A – B – F



Time for each driver: 2

Algorithmic Game Theory (CS 364)

- If everyone makes selfish choices, outcome may be suboptimal for everyone
- Goal of mechanism design: force selfish choices to result in optimal outcome for everyone

Complexity (CS 254)

Complexity (CS 254)

- We already covered P, NP
- Other complexity classes?
  - L = Log space
  - PSPACE = polynomial memory
  - EXPTIME = exponential time
  - BPP = poly time randomized algorithms
  - Non-deterministic versions of each,

Complexity (CS 254)

- Very little is known about separations
  - Know that P ≠ EXPTIME
  - Know that L ≠ P
  - Don't know if P = BPP
  - Don't know how to compare BPP and NP
  - Don't know even if BPP = NEXPTIME
  - Don't know if L = P

Quantum Computing (CS 259Q)

Quantum Computing (CS 259Q)

- State of classical computer: bunch of bits
- Each step of algorithm: change values of bits according to some rules

Quantum Computing (CS 259Q)

- Quantum computer: bunch of qubits
  - Whereas a bit is either 0 or 1, a qubit is some superposition

$$\alpha|0\rangle + \beta|1\rangle \qquad \alpha^2 + \beta^2 = 1$$

  - Measure: get 0 with probability α^2, 1 with probability β^2. Left in state corresponding to result of measurement

Quantum Computing (CS 259Q)

- Example: two qubits:

$$(\alpha_1|0\rangle + \beta_1|1\rangle)(\alpha_2|0\rangle + \beta_2|1\rangle)$$

$$\alpha_1\alpha_2|0\rangle|0\rangle + \alpha_1\beta_2|0\rangle|1\rangle + \beta_1\alpha_2|1\rangle|0\rangle + \beta_1\beta_2|1\rangle|1\rangle$$

$$\alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

Quantum Computing (CS 259Q)

- In the state

$$\alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

- If we measure the first bit, we get 0 with prob

$$|\alpha_1\alpha_2|^2 + |\alpha_1\beta_2|^2 = |\alpha_1|^2$$

Quantum Computing (CS 259Q)

- After measurement, left with terms where first bit is 0

$$\alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle$$

- Must renormalize by dividing by

$$\sqrt{|\alpha_1\alpha_2|^2 + |\alpha_1\beta_2|^2} = \alpha_1$$

- Arrive in state

$$|0\rangle(\alpha_2|0\rangle + \beta_2|1\rangle)$$

Quantum Computing

- For these states, measuring the first qubit does not affect the second
- These states are unentangled
- In general, a quantum algorithm is in an entangled state

$$\alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

$$|\alpha_{00}|^2 + |\alpha_{01}|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 = 1$$

Quantum Computing (CS 259Q)

- n qubits:

$$\sum_{x=0}^{2^n-1} \alpha_x |x\rangle \qquad \sum_{x=0}^{2^n-1} |\alpha_x|^2 = 1$$

- Each step of of quantum algorithm: apply transformations to the state
- At the end of the algorithm: measure, obtain x with probability $|\alpha_x|^2$

Quantum Computing (CS 259Q)

- Classical computer on n bits: described by a n-bit string
- Quantum computer on n qubits: described by 2^n complex values
- Quantum operations act on exponentially larger state
- However, this state is invisible to us, must be measured first

Quantum Computing (CS 259Q)

- Quantum Fourier Transform:
  - Applies DFT to coefficients

$$\sum_{x=0}^{2^n-1} \alpha_x |x\rangle \xrightarrow{QFT} \sum_{y=0}^{2^n-1} A_y |y\rangle$$

$$A_y = \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} \alpha_x e^{\frac{-i2\pi xy}{2^n}}$$

Quantum Computing (CS 259Q)

- Two main quantum algorithms:
  - Shor's Algorithm: uses QFT to factor integers in poly time
  - Grover's Algorithm: search a database of N items in time $O(N^{1/2})$